AUFMANN • BARKER • LOCKWOOD

ALGEBRA

INTRODUCTORY AND INTERMEDIATE

A COMBINED TEXT,
STUDENT SOLUTIONS
MANUAL & WORKSHEETS



PART TWO
CHAPTERS 6-10

SPECIAL EDITION
FOR NORTHERN ESSEX
COMMUNITY COLLEGE


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PART II: CHAPTERS 6-10

ALGEBRA

INTRODUCTORY AND INTERMEDIATE

RICHARD AUFMANN
PALOMAR COLLEGE, CALIFORNIA

VERNON C. BARKER
PALOMAR COLLEGE, CALIFORNIA

JOANNE S. LOCKWOOD
PLYMOUTH STATE COLLEGE,
NEW HAMPSHIRE

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STUDENT SOLUTIONS MANUAL
& WORKSHEETS

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Chapter 6 Polynomials



The Pathfinder mission produced the closest look to date at the surface of Mars. This photo of the Pathfinder Sojourner rover was taken at Mermaid Dune on Mars on August 8, 1997. Images such as this, transmitted from Mars to Earth, traveled a distance of over 100 million miles. Distances this great, as well as very small measurements, are generally expressed in scientific notation, as illustrated in **Exercise 119 on page 340.**

Objectives

Section 6.1

- **A** To multiply monomials
- **B** To divide monomials and simplify expressions with negative exponents
- **C** To write a number using scientific notation
- **D** To solve application problems

Section 6.2

- **A** To evaluate polynomial functions
- **B** To add or subtract polynomials

Section 6.3

- **A** To multiply a polynomial by a monomial
- **B** To multiply two polynomials
- **C** To multiply two binomials
- **D** To multiply binomials that have special products
- **E** To solve application problems

Section 6.4

- **A** To divide polynomials
- **B** To divide polynomials using synthetic division
- **C** To evaluate a polynomial using synthetic division



Need help? For on-line student resources, such as section quizzes, visit this textbook's web site at **college.hmco.com/students.**

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Prep Test

1. Subtract: $-2 - (-3)$

2. Multiply: $-3(6)$

For Exercises 3 to 8, simplify.

3. $-\dfrac{24}{-36}$

4. $-4(3y)$

5. $(-2)^3$

6. $-4a - 8b + 7a$

7. $3x - 2[y - 4(x + 1) + 5]$

8. $-4y + 4y$

9. Are $2x^2$ and $2x$ like terms?

Go Figure



A pair of perpendicular lines are drawn through the interior of a rectangle, dividing it into four smaller rectangles. The area of the smaller rectangles are x, 2, 3, and 6. Find the possible values of x.

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6.1 Exponential Expressions

Objective A

To multiply monomials

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A monomial is a number, a variable, or a product of a number and variables.

Point of Interest

Around A.D. 250, the monomial $3x^2$ shown at the right would have been written $\Delta^Y 3$ or at least approximately like that. In A.D. 250, the symbol for 3 was not the one we use today.

The examples at the right are monomials. The **degree of a monomial** is the sum of the exponents of the variables.

x	degree 1 ($x = x^1$)
$3x^2$	degree 2
$4x^2y$	degree 3
$6x^3y^4z^2$	degree 9

In this chapter, the variable n is considered a positive integer when used as an exponent.

x^n	degree n

The degree of a nonzero constant term is zero.

6	degree 0

The expression $5\sqrt{x}$ is not a monomial because $\sqrt{x}$ cannot be written as a product of variables. The expression $\frac{x}{y}$ is not a monomial because it is a quotient of variables.

The expression x^4 is an exponential expression. The exponent, 4, indicates the number of times the base, x, occurs as a factor.

The product of exponential expressions with the *same* base can be simplified by writing each expression in factored form and writing the result with an exponent.

$$x^3 \cdot x^4 = \overbrace{\underbrace{(x \cdot x \cdot x)}_{3 \text{ factors}} \cdot \underbrace{(x \cdot x \cdot x \cdot x)}_{4 \text{ factors}}}^{7 \text{ factors}}$$
$$= x^7$$

Note that adding the exponents results in the same product.

$$x^3 \cdot x^4 = x^{3+4} = x^7$$

Rule for Multiplying Exponential Expressions

If m and n are positive integers, then $x^m \cdot x^n = x^{m+n}$.

Multiply: $(-4x^5y^3)(3xy^2)$

$$(-4x^5y^3)(3xy^2) = (-4 \cdot 3)(x^5 \cdot x)(y^3 \cdot y^2)$$
$$= -12(x^{5+1})(y^{3+2})$$
$$= -12x^6y^5$$

- Use the Commutative and Associative Properties of Multiplication to rearrange and group factors.
- Multiply variables with the same base by adding their exponents. Do these steps mentally.
- Simplify.

TAKE NOTE

The Rule for Multiplying Exponential Expressions requires the bases to be the same. The expression x^6y^5 cannot be simplified.

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Point of Interest

One of the first symbolic representations of powers was given by Diophantus (c. A.D. 250) in his book *Arithmetica*. He used Δ^γ for x^2 and κ^γ for x^3. The symbol Δ^γ was the first two letters of the Greek word *dunamis*, meaning "power"; κ^γ was from the Greek word *kubos*, meaning "cube." He also combined these symbols to denote higher powers. For instance, $\Delta\kappa^\gamma$ was the symbol for x^5.

The power of a monomial can be simplified by writing the power in factored form and then using the Rule for Multiplying Exponential Expressions.

$$(x^4)^3 = x^4 \cdot x^4 \cdot x^4 = x^{4+4+4} = x^{12} \qquad (a^2b^3)^2 = (a^2b^3)(a^2b^3) = a^{2+2}b^{3+3} = a^4b^6$$

- Write in factored form.
- Use the Rule for Multiplying Exponential Expressions.

Note that multiplying each exponent inside the parentheses by the exponent outside the parentheses results in the same product.

$$(x^4)^3 = x^{4\cdot3} = x^{12} \qquad (a^2b^3)^2 = a^{2\cdot2}b^{3\cdot2} = a^4b^6$$

- Multiply each exponent inside the parentheses by the exponent outside the parentheses.

Rule for Simplifying Powers of Exponential Expressions

If m and n are positive integers, then $(x^m)^n = x^{mn}$.

Rule for Simplifying Powers of Products

If m, n, and p are positive integers, then $(x^m y^n)^p = x^{mp}y^{np}$.

Simplify: $(x^4)^5$

$$(x^4)^5 = x^{4\cdot5} = x^{20}$$

- Use the Rule for Simplifying Powers of Exponential Expressions to multiply the exponents.

Simplify: $(2a^3b^4)^3$

$$(2a^3b^4)^3 = 2^{1\cdot3}a^{3\cdot3}b^{4\cdot3} = 2^3a^9b^{12} = 8a^9b^{12}$$

- Use the Rule for Simplifying Powers of Products to multiply each exponent inside the parentheses by the exponent outside the parentheses. Note that $2 = 2^1$.

Example 1 Simplify: $(2xy^2)(-3xy^4)^3$

Solution

$$(2xy^2)(-3xy^4)^3 = (2xy^2)[(-3)^3x^3y^{12}] = (2xy^2)(-27x^3y^{12}) = -54x^4y^{14}$$

You Try It 1 Simplify: $(-3a^2b^4)(-2ab^3)^4$

Your solution

Example 2 Simplify: $(x^{n+2})^5$

Solution

$(x^{n+2})^5 = x^{5(n+2)} = x^{5n+10}$

You Try It 2 Simplify: $(y^{n-3})^2$

Your solution

Solutions on p. S17

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Example 3 Simplify: $[(2xy^2)^2]^3$

Solution $[(2xy^2)^2]^3 = [2^2x^2y^4]^3$
$= 2^6x^6y^{12} = 64x^6y^{12}$

You Try It 3 Simplify: $[(ab^3)^3]^4$

Your solution

Solution on p. S17

Objective B

To divide monomials and simplify expressions with negative exponents

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The quotient of two exponential expressions with the same base can be simplified by writing each expression in factored form, dividing by the common factors, and then writing the result with an exponent.

$$\frac{x^5}{x^2} = \frac{\overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot x \cdot x \cdot x}{\underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}}} = x^3$$

Note that subtracting the exponents gives the same result.

$$\frac{x^5}{x^2} = x^{5-2} = x^3$$

To divide two monomials with the same base, subtract the exponents of the like bases.

➡ Simplify: $\frac{z^8}{z^2}$

$\frac{z^8}{z^2} = \boxed{z^{8-2}}$

$= z^6$

• The bases are the same. Subtract the exponents. This step is often done mentally.

➡ Simplify: $\frac{a^5b^9}{a^4b}$

$\frac{a^5b^9}{a^4b} = \boxed{a^{5-4}b^{9-1}}$

$= ab^8$

• Subtract the exponents of the like bases. This step is often done mentally.

TAKE NOTE
A number divided by itself is 1. Therefore, $\frac{8}{8} = 1$, $\frac{3^4}{3^4} = 1$, and for $x \neq 0$, $\frac{x^4}{x^4} = 1$.

Consider the expression $\frac{x^4}{x^4}$, $x \neq 0$. This expression can be simplified, as shown below, by (1) subtracting exponents and (2) dividing by common factors.

$$\frac{x^4}{x^4} = x^{4-4} = x^0 \qquad \frac{x^4}{x^4} = \frac{\overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}}}{\underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}}} = 1$$

The equations $\frac{x^4}{x^4} = x^0$ and $\frac{x^4}{x^4} = 1$ suggest the following definition of x^0.

Definition of Zero as an Exponent

If $x \neq 0$, then $x^0 = 1$. The expression 0^0 is not defined.

TAKE NOTE
For the example below, we indicated that $z \neq 0$. If we try to evaluate $(16z^5)^0$ when $z = 0$, we have $[16(0)^5]^0 = [16(0)]^0 = 0^0$. However, 0^0 is not defined. Therefore, we must assume that $z \neq 0$. To avoid stating this for every example or exercise, we will assume that variables do not have values that result in the expression 0^0.

➡ Simplify: $(16z^5)^0$, $z \neq 0$

$(16z^5)^0 = 1$

• Any nonzero expression to the zero power is 1.

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Point of Interest

In the 15th century, the expression 12^{2m} was used to mean $12x^{-2}$. The use of m reflects an Italian influence, where m was used for minus and p was used for plus. It was understood that $2m$ referred to an unnamed variable. Isaac Newton, in the 17th century, advocated the use of a negative exponent, the symbol we use today.

➡ Simplify: $-(7x^4y^3)^0$, $x \neq 0, y \neq 0$

$-(7x^4y^3)^0 = -(1) = -1$ • The negative outside the parentheses is not affected by the exponent.

Consider the expression $\frac{x^4}{x^6}$, $x \neq 0$. This expression can be simplified, as shown below, by (1) subtracting exponents and (2) dividing by common factors.

$$\frac{x^4}{x^6} = x^{4-6} = x^{-2} \qquad \frac{x^4}{x^6} = \frac{\overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}}}{\underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}} \cdot x \cdot x} = \frac{1}{x^2}$$

The equations $\frac{x^4}{x^6} = x^{-2}$ and $\frac{x^4}{x^6} = \frac{1}{x^2}$ suggest that $x^{-2} = \frac{1}{x^2}$.

Definition of a Negative Exponent

If $x \neq 0$ and n is a positive integer, then

$$x^{-n} = \frac{1}{x^n} \quad \text{and} \quad \frac{1}{x^{-n}} = x^n$$

TAKE NOTE

Note from the example at the right that 2^{-4} is a *positive* number. A negative exponent does not indicate a negative number.

➡ Evaluate: 2^{-4}

$2^{-4} = \frac{1}{2^4}$ • Use the Definition of a Negative Exponent.

$= \frac{1}{16}$ • Evaluate the expression.

A power of the quotient of two exponential expressions can be simplified by multiplying each exponent in the quotient by the exponent outside the parentheses.

Rule for Simplifying Powers of Quotients

If m, n, and p are integers and $y \neq 0$, then $\left(\frac{x^m}{y^n}\right)^p = \frac{x^{mp}}{y^{np}}$.

➡ Simplify: $\left(\frac{a^2}{b^3}\right)^{-2}$

$\left(\frac{a^2}{b^3}\right)^{-2} = \frac{a^{2(-2)}}{b^{3(-2)}}$ • Use the Rule for Simplifying Powers of Quotients.

$= \frac{a^{-4}}{b^{-6}} = \frac{b^6}{a^4}$ • Use the Definition of a Negative Exponent.

Simplest Form of an Exponential Expression

An exponential expression is in simplest form when it is written with only positive exponents.

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TAKE NOTE

The exponent on n is -5 (*negative* 5). The n^{-5} is written in the denominator as n^5. The exponent on 3 is 1 (*positive* 1). The 3 remains in the numerator.

Also, we indicated that $n \neq 0$. This is necessary because division by zero is not defined. In this textbook, we will assume that values of the variables are chosen so that division by zero does not occur.

➡ Simplify: $3n^{-5}$, $n \neq 0$

$$3n^{-5} = 3 \cdot \frac{1}{n^5} = \frac{3}{n^5}$$

- Use the Definition of a Negative Exponent to rewrite the expression with a positive exponent.

➡ Simplify: $\dfrac{2}{5a^{-4}}$

$$\frac{2}{5a^{-4}} = \frac{2}{5} \cdot \frac{1}{a^{-4}} = \frac{2}{5} \cdot a^4 = \frac{2a^4}{5}$$

- Use the Definition of a Negative Exponent to rewrite the expression with a positive exponent.

Now that zero and negative exponents have been defined, a rule for dividing exponential expressions can be stated.

Rule for Dividing Exponential Expressions

If m and n are integers and $x \neq 0$, then $\dfrac{x^m}{x^n} = x^{m-n}$.

➡ Simplify: $\dfrac{x^4}{x^9}$

$$\frac{x^4}{x^9} = \boxed{x^{4-9}}$$
$$= x^{-5}$$
$$= \frac{1}{x^5}$$

- Use the Rule for Dividing Exponential Expressions. This step is often done mentally.
- Subtract the exponents.
- Use the Definition of a Negative Exponent to rewrite the expression with a positive exponent.

The rules for simplifying exponential expressions and powers of exponential expressions are true for all integer exponents. These rules are restated here for your convenience.

Rules of Exponents

If m, n, and p are integers, then

$x^m \cdot x^n = x^{m+n}$ $\qquad (x^m)^n = x^{mn}$ $\qquad (x^m y^n)^p = x^{mp} y^{np}$

$\dfrac{x^m}{x^n} = x^{m-n}, x \neq 0$ $\qquad \left(\dfrac{x^m}{y^n}\right)^p = \dfrac{x^{mp}}{y^{np}}, y \neq 0$ $\qquad x^{-n} = \dfrac{1}{x^n}, x \neq 0$

$x^0 = 1, x \neq 0$

➡ Simplify: $(3ab^{-4})(-2a^{-3}b^7)$

$$(3ab^{-4})(-2a^{-3}b^7) = \boxed{[3 \cdot (-2)](a^{1+(-3)}b^{-4+7})}$$
$$= -6a^{-2}b^3$$
$$= \frac{-6b^3}{a^2}$$

- When multiplying expressions, add the exponents on like bases. Do this step mentally.

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Simplify: $\dfrac{4a^{-2}b^5}{6a^5b^2}$

$$\frac{4a^{-2}b^5}{6a^5b^2} = \frac{2 \cdot 2a^{-2}b^5}{2 \cdot 3a^5b^2} = \frac{2a^{-2}b^5}{3a^5b^2}$$

- Divide the coefficients by their common factor.

$$= \frac{2a^{-2-5}b^{5-2}}{3}$$

- Use the Rule for Dividing Exponential Expressions.

$$= \frac{2a^{-7}b^3}{3} = \frac{2b^3}{3a^7}$$

- Use the Definition of a Negative Exponent to rewrite the expression with a positive exponent.

Simplify: $\left(\dfrac{6m^2n^3}{8m^7n^2}\right)^{-3}$

$$\left(\frac{6m^2n^3}{8m^7n^2}\right)^{-3} = \left(\frac{3m^{2-7}n^{3-2}}{4}\right)^{-3}$$

- Simplify inside the brackets.

$$= \left(\frac{3m^{-5}n}{4}\right)^{-3}$$

- Subtract the exponents.

$$= \frac{3^{-3}m^{15}n^{-3}}{4^{-3}}$$

- Use the Rule for Simplifying Powers of Quotients.

$$= \frac{4^3m^{15}}{3^3n^3} = \frac{64m^{15}}{27n^3}$$

- Use the Definition of a Negative Exponent to rewrite the expression with positive exponents. Then simplify.

Example 4

Simplify: $\dfrac{-28x^6z^{-3}}{42x^{-1}z^4}$

Solution

$$\frac{-28x^6z^{-3}}{42x^{-1}z^4} = -\frac{14 \cdot 2x^{6-(-1)}z^{-3-4}}{14 \cdot 3}$$

$$= -\frac{2x^7z^{-7}}{3} = -\frac{2x^7}{3z^7}$$

You Try It 4

Simplify: $\dfrac{20r^{-2}t^{-5}}{-16r^{-3}s^{-2}}$

Your solution

Example 5

Simplify: $\dfrac{(3a^{-1}b^4)^{-3}}{(6^{-1}a^{-3}b^{-4})^3}$

Solution

$$\frac{(3a^{-1}b^4)^{-3}}{(6^{-1}a^{-3}b^{-4})^3} = \frac{3^{-3}a^3b^{-12}}{6^{-3}a^{-9}b^{-12}}$$

$$= 3^{-3} \cdot 6^3a^{12}b^0$$

$$= \frac{6^3a^{12}}{3^3} = \frac{216a^{12}}{27} = 8a^{12}$$

You Try It 5

Simplify: $\dfrac{(9u^{-6}v^4)^{-1}}{(6u^{-3}v^{-2})^{-2}}$

Your solution

Example 6

Simplify: $\dfrac{x^{4n-2}}{x^{2n-5}}$

Solution

$$\frac{x^{4n-2}}{x^{2n-5}} = x^{4n-2-(2n-5)}$$

$$= x^{4n-2-2n+5} = x^{2n+3}$$

You Try It 6

Simplify: $\dfrac{a^{2n+1}}{a^{n+3}}$

Your solution

Solutions on pp. S17–S18

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Objective C — To write a number using scientific notation

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Point of Interest

Astronomers measure the distance of some stars by using the parsec. One parsec is approximately 1.91×10^{13} mi.

Integer exponents are used to represent the very large and very small numbers encountered in the fields of science and engineering. For example, the mass of the electron is 0.000000000000000000000000000009 g. Numbers such as this are difficult to read and write, so a more convenient system for writing such numbers has been developed. It is called **scientific notation**.

To express a number in scientific notation, write the number as the product of a number between 1 and 10 and a power of 10. The form for scientific notation is $a \times 10^n$, where $1 \le a < 10$.

TAKE NOTE

The numbers 9.65, 3.6, 9.2, 2, 9.74, and 8.6 are all numbers between 1 and 10. Each is multiplied by a power of 10. Note that there are two steps in writing a number in scientific notation: (1) Determine the number between 1 and 10, and (2) determine the exponent on 10.

For numbers greater than 10, move the decimal point to the right of the first digit. The exponent n is positive and equal to the number of places the decimal point has been moved.

$965{,}000 = 9.65 \times 10^5$

$3{,}600{,}000 = 3.6 \times 10^6$

$92{,}000{,}000{,}000 = 9.2 \times 10^{10}$

For numbers less than 1, move the decimal point to the right of the first nonzero digit. The exponent n is negative. The absolute value of the exponent is equal to the number of places the decimal point has been moved.

$0.0002 = 2 \times 10^{-4}$

$0.0000000974 = 9.74 \times 10^{-8}$

$0.000000000086 = 8.6 \times 10^{-11}$

Converting a number written in scientific notation to decimal notation requires moving the decimal point.

When the exponent is positive, move the decimal point to the right the same number of places as the exponent.

$1.32 \times 10^4 = 13{,}200$

$1.4 \times 10^8 = 140{,}000{,}000$

When the exponent is negative, move the decimal point to the left the same number of places as the absolute value of the exponent.

$1.32 \times 10^{-2} = 0.0132$

$1.4 \times 10^{-4} = 0.00014$

Numerical calculations involving numbers that have more digits than a hand-held calculator is able to handle can be performed using scientific notation.

Simplify: $\dfrac{220{,}000 \times 0.000000092}{0.0000011}$

$$\frac{220{,}000 \times 0.000000092}{0.0000011} = \frac{2.2 \times 10^5 \times 9.2 \times 10^{-8}}{1.1 \times 10^{-6}}$$

• Write the numbers in scientific notation.

$$= \frac{(2.2)(9.2) \times 10^{5+(-8)-(-6)}}{1.1}$$

• Simplify.

$$= 18.4 \times 10^3 = 18{,}400$$

Example 7 Write 0.000041 in scientific notation.

Solution $0.000041 = 4.1 \times 10^{-5}$

You Try It 7 Write 942,000,000 in scientific notation.

Your solution

Solution on p. S18

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Example 8 Write 3.3×10^7 in decimal notation.

Solution $3.3 \times 10^7 = 33{,}000{,}000$

You Try It 8 Write 2.7×10^{-5} in decimal notation.

Your solution

Example 9

Simplify: $\dfrac{2{,}400{,}000{,}000 \times 0.0000063}{0.00009 \times 480}$

Solution

$$\frac{2{,}400{,}000{,}000 \times 0.0000063}{0.00009 \times 480}$$

$$= \frac{2.4 \times 10^9 \times 6.3 \times 10^{-6}}{9 \times 10^{-5} \times 4.8 \times 10^2}$$

$$= \frac{(2.4)(6.3) \times 10^{9+(-6)-(-5)-2}}{(9)(4.8)}$$

$$= 0.35 \times 10^6 = 350{,}000$$

You Try It 9

Simplify: $\dfrac{5{,}600{,}000 \times 0.000000081}{900 \times 0.000000028}$

Your solution

Solutions on p. S18

To solve application problems



Example 10

How many miles does light travel in one day? The speed of light is 186,000 mi/s. Write the answer in scientific notation.

Strategy

To find the distance traveled:

- Write the speed of light in scientific notation.
- Write the number of seconds in one day in scientific notation.
- Use the equation $d = rt$, where r is the speed of light and t is the number of seconds in one day.

Solution

$r = 186{,}000 = 1.86 \times 10^5$

$24 \cdot 60 \cdot 60 = 86{,}400 = 8.64 \times 10^4$

rt

$(1.86 \times 10^5)(8.64 \times 10^4)$

$1.86 \times 8.64 \times 10^9$

16.0704×10^9

$= 1.60704 \times 10^{10}$

Light travels 1.60704×10^{10} mi in one day.

You Try It 10

A computer can do an arithmetic operation in 1×10^{-7} s. In scientific notation, how many arithmetic operations can the computer perform in 1 min?

Your strategy

Your solution

Solution on p. S18

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6.1 Exercises

Objective A

1. Which expression is the multiplication of two exponential expressions and which is the power of an exponential expression?
 a. $q^4 \cdot q^{10}$ **b.** $(q^4)^{10}$

2. In the expression $(a^8b^6)^5$, what is the product and what is the power?

Simplify.

3. $(ab^3)(a^3b)$
4. $(-2ab^4)(-3a^2b^4)$
5. $(9xy^2)(-2x^2y^2)$
6. $(x^2y)^2$
7. $(x^2y^4)^4$
8. $(-2ab^2)^3$
9. $(-3x^2y^3)^4$
10. $(2^2a^2b^3)^3$
11. $(3^3a^5b^3)^2$
12. $(xy)(x^2y)^4$
13. $(x^2y^2)(xy^3)^3$
14. $[(2x)^4]^2$
15. $[(3x)^3]^2$
16. $[(x^2y)^4]^5$
17. $[(ab)^3]^6$
18. $[(2ab)^3]^2$
19. $[(2xy)^3]^4$
20. $[(3x^2y^3)^2]^2$
21. $[(2a^4b^3)^3]^2$
22. $y^n \cdot y^{2n}$
23. $x^n \cdot x^{n+1}$
24. $y^{2n} \cdot y^{4n+1}$
25. $y^{3n} \cdot y^{3n-2}$
26. $(a^n)^{2n}$
27. $(a^{n-3})^{2n}$
28. $(y^{2n-1})^3$
29. $(x^{3n+2})^5$
30. $(b^{2n-1})^n$
31. $(2xy)(-3x^2yz)(x^2y^3z^3)$
32. $(x^2z^4)(2xyz^4)(-3x^3y^2)$
33. $(3b^5)(2ab^2)(-2ab^2c^2)$
34. $(-c^3)(-2a^2bc)(3a^2b)$
35. $(-2x^2y^3z)(3x^2yz^4)$
36. $(2a^2b)^3(-3ab^4)^2$
37. $(-3ab^3)^3(-2^2a^2b)^2$
38. $(4ab)^2(-2ab^2c^3)^3$
39. $(-2ab^2)(-3a^4b^5)^3$

Objective B

Simplify.

40. 2^{-3}
41. $\frac{1}{3^{-5}}$
42. $\frac{1}{x^{-4}}$
43. $\frac{1}{y^{-3}}$

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44. $\frac{2x^{-2}}{y^4}$

45. $\frac{a^3}{4b^{-2}}$

46. $x^{-3}y$

47. xy^{-4}

48. $-5x^0$

49. $\frac{1}{2x^0}$

50. $\frac{(2x)^0}{-2^3}$

51. $\frac{-3^{-2}}{(2y)^0}$

52. $\frac{y^{-7}}{y^{-8}}$

53. $\frac{y^{-2}}{y^6}$

54. $(x^2y^{-4})^2$

55. $(x^3y^5)^{-2}$

56. $\frac{x^{-2}y^{11}}{xy^{-2}}$

57. $\frac{x^4y^3}{x^{-1}y^{-2}}$

58. $\frac{a^{-1}b^{-3}}{a^4b^{-5}}$

59. $\frac{a^6b^{-4}}{a^{-2}b^5}$

60. $(2a^{-1})^{-2}(2a^{-1})^4$

61. $(3a)^{-3}(9a^{-1})^{-2}$

62. $(x^{-2}y)^2(xy)^{-2}$

63. $(x^{-1}y^2)^{-3}(x^2y^{-4})^{-3}$

64. $\frac{50b^{10}}{70b^5}$

65. $\frac{x^3y^6}{x^6y^2}$

66. $\frac{x^{17}y^5}{-x^7y^{10}}$

67. $\frac{-6x^2y}{12x^4y}$

68. $\frac{2x^2y^4}{(3xy^2)^3}$

69. $\frac{-3ab^2}{(9a^2b^4)^3}$

70. $\left(\frac{-12a^2b^3}{9a^5b^9}\right)^3$

71. $\left(\frac{12x^3y^2z}{18xy^3z^4}\right)^4$

72. $\frac{(4x^2y)^2}{(2xy^3)^3}$

73. $\frac{(3a^2b)^3}{(-6ab^3)^2}$

74. $\frac{(-4x^2y^3)^2}{(2xy^2)^3}$

75. $\frac{(-3a^2b^3)^2}{(-2ab^4)^3}$

76. $\frac{(-4xy^3)^3}{(-2x^7y)^4}$

77. $\frac{(-8x^2y^2)^4}{(16x^3y^7)^2}$

78. $\frac{a^{5n}}{a^{3n}}$

79. $\frac{b^{6n}}{b^{10n}}$

80. $\frac{-x^{5n}}{x^{2n}}$

81. $\frac{y^{2n}}{-y^{8n}}$

82. $\frac{x^{2n-1}}{x^{n-3}}$

83. $\frac{y^{3n+2}}{y^{2n+4}}$

84. $\frac{a^{3n}b^n}{a^nb^{2n}}$

85. $\frac{x^ny^{3n}}{x^ny^{5n}}$

86. $\frac{a^{3n-2}b^{n+1}}{a^{2n+1}b^{2n+2}}$

87. $\frac{x^{2n-1}y^{n-3}}{x^{n+4}y^{n+3}}$

88. $\left(\frac{4^{-2}xy^{-3}}{x^{-3}y}\right)^3\left(\frac{8^{-1}x^{-2}y}{x^{-1}y^{-1}}\right)^{-2}$

89. $\left(\frac{9ab^{-2}}{8a^{-2}b}\right)^{-2}\left(\frac{3a^{-2}b}{2a^2b^{-2}}\right)^3$

90. $\left(\frac{2ab^{-1}}{ab}\right)^{-1}\left(\frac{3a^{-2}b}{a^2b^2}\right)^{-2}$

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Objective C

Write in scientific notation.

91. 0.00000467 **92.** 0.00000005 **93.** 0.00000000017

94. 4,300,000 **95.** 200,000,000,000 **96.** 9,800,000,000

Write in decimal notation.

97. 1.23×10^{-7} **98.** 6.2×10^{-12} **99.** 8.2×10^{15}

100. 6.34×10^{5} **101.** 3.9×10^{-2} **102.** 4.35×10^{9}

Simplify. Write the answer in decimal notation.

103. $(3 \times 10^{-12})(5 \times 10^{16})$ **104.** $(8.9 \times 10^{-5})(3.2 \times 10^{-6})$

105. (0.0000065)(3,200,000,000,000) **106.** (480,000)(0.0000000096)

107. $\dfrac{9 \times 10^{-3}}{6 \times 10^{5}}$ **108.** $\dfrac{2.7 \times 10^{4}}{3 \times 10^{-6}}$ **109.** $\dfrac{0.0089}{500{,}000{,}000}$

110. $\dfrac{4800}{0.00000024}$ **111.** $\dfrac{0.00056}{0.000000000004}$ **112.** $\dfrac{0.000000346}{0.0000005}$

113. $\dfrac{(3.3 \times 10^{-11})(2.7 \times 10^{15})}{8.1 \times 10^{-3}}$ **114.** $\dfrac{(6.9 \times 10^{27})(8.2 \times 10^{-13})}{4.1 \times 10^{15}}$

115. $\dfrac{(0.00000004)(84{,}000)}{(0.0003)(1{,}400{,}000)}$ **116.** $\dfrac{(720)(0.0000000039)}{(26{,}000{,}000{,}000)(0.18)}$

Objective D *Application Problems*

Solve. Write the answer in scientific notation.

117. How many kilometers does light travel in one day? The speed of light is 300,000 km/s.

118. How many meters does light travel in 8 h? The speed of light is 300,000,000 m/s.

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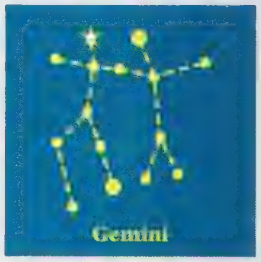
119. The rover Sojourner landed on Mars in July of 1997. It took 11 min for the commands from a computer on Earth to travel to the Sojourner, a distance of 119 million miles. How fast did the signals from Earth to Mars travel? Write the answer in scientific notation.

120. In the summer of 2001, the federal debt was 3×10^{12} dollars. How much would each American citizen have to contribute in order to pay off the debt? Use 300,000,000 as the number of citizens.

121. A high-speed centrifuge makes 9×10^7 revolutions each minute. Find the time in seconds for the centrifuge to make one revolution.

122. How long does it take light to travel to Earth from the sun? The sun is 9.3×10^7 mi from Earth, and light travels 1.86×10^5 mi/s.

123. The mass of Earth is 5.9×10^{27} g. The mass of the sun is 2×10^{33} g. How many times heavier is the sun than Earth?



124. One astronomical unit (A.U.) is 9.3×10^7 mi. The star Pollux in the constellation Gemini is 1.8228×10^{12} mi from Earth. Find the distance from Pollux to Earth in astronomical units.

125. The weight of 31 million orchid seeds is one ounce. Find the weight of one orchid seed.

126. One light year, an astronomical unit of distance, is the distance that light will travel in one year. Light travels 1.86×10^5 mi/s. Find the measure of one light year in miles. Use a 365-day year.

127. The Coma cluster of galaxies is approximately 2.8×10^8 light years from Earth. Find the distance, in miles, from the Coma cluster to Earth. (See Exercise 126 for the definition of one light year. Use a 365-day year.)

APPLYING THE CONCEPTS

128. Simplify.

a. $1 + (1 + (1 + 2^{-1})^{-1})^{-1}$ **b.** $2 - (2 - (2 - 2^{-1})^{-1})^{-1}$

129. Correct the error in each of the following expressions. Explain which rule or property was used incorrectly.

a. $2x + 3x = 5x^2$ **b.** $a - (b - c) = a - b - c$

c. $x^0 = 0$ **d.** $(x^4)^5 = x^9$

e. $x^2 \cdot x^3 = x^6$ **f.** $b^{m+n} = b^m + b^n$

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6.2 Introduction to Polynomials

OBJECTIVE

To evaluate polynomial functions

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TAKE NOTE
An analogy may help you understand these terms. Polynomial is similar to the word *car*; Chevrolet and Ford are types of cars, just as monomials and binomials are types of polynomials.

A **polynomial** is a variable expression in which the terms are monomials.

A polynomial of one term is a **monomial**. $5x$

A polynomial of two terms is a **binomial**. $5x^2y + 6x$

A polynomial of three terms is a **trinomial**. $3x^2 + 9xy - 5y$

Polynomials with more than three terms do not have special names.

The **degree of a polynomial** is the greatest of the degrees of any of its terms.

$3x + 2$	degree 1
$3x^2 + 2x - 4$	degree 2
$4x^3y^2 + 6x^4$	degree 5
$3x^{2n} - 5x^n - 2$	degree $2n$

The terms of a polynomial in one variable are usually arranged so that the exponents of the variable decrease from left to right. This is called **descending order**.

$2x^2 - x + 8$

$3y^3 - 3y^2 + y - 12$

For a polynomial in more than one variable, descending order may refer to any one of the variables.

The polynomial at the right is shown first in descending order of the x variable and then in descending order of the y variable.

$2x^2 + 3xy + 5y^2$

$5y^2 + 3xy + 2x^2$

Polynomial functions have many applications in mathematics. In general, a **polynomial function** is an expression whose terms are monomials. The **linear function** given by $f(x) = mx + b$ is an example of a polynomial function. It is a polynomial function of degree one. A second-degree polynomial function, called a **quadratic function**, is given by the equation $f(x) = ax^2 + bx + c$, $a \neq 0$. A third-degree polynomial function is called a **cubic function**.

To evaluate a polynomial function, replace the variable by its value and simplify.

➡ Given $P(x) = x^3 - 3x^2 + 4$, evaluate $P(-3)$.

$$\begin{aligned} P(x) &= x^3 - 3x^2 + 4 \\ P(-3) &= (-3)^3 - 3(-3)^2 + 4 \qquad \text{• Substitute } -3 \text{ for } x \text{ and simplify.} \\ &= -27 - 3(9) + 4 \\ &= -27 - 27 + 4 \\ &= -50 \end{aligned}$$

The **leading coefficient** of a polynomial function is the coefficient of the variable with the largest exponent. The constant term is the term without a variable.

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Find the leading coefficient, the constant term, and the degree of the polynomial function $P(x) = 7x^4 - 3x^2 + 2x - 4$.

The leading coefficient is 7, the constant term is -4, and the degree is 4.

The three equations below do not represent polynomial functions.

$f(x) = 3x^2 + 2x^{-1}$ A polynomial function does not have a variable raised to a negative power.

$g(x) = 2\sqrt{x} - 3$ A polynomial function does not have a variable expression within a radical.

$h(x) = \dfrac{x}{x - 1}$ A polynomial function does not have a variable in the denominator of a fraction.

The graph of a linear function is a straight line and can be found by plotting just two points. The graph of a polynomial function of degree greater than one is a curve. Consequently, many points may have to be found before an accurate graph can be drawn.

Evaluating the quadratic function given by the equation $f(x) = x^2 - x - 6$ when $x = -3, -2, -1, 0, 1, 2, 3$, and 4 gives the points shown in Figure 1 below. For instance, $f(-3) = 6$, so $(-3, 6)$ is graphed; $f(2) = -4$, so $(2, -4)$ is graphed; and $f(4) = 6$, so $(4, 6)$ is graphed. Evaluating the function when x is not an integer, such as $x = -\frac{3}{2}$ and $x = \frac{5}{2}$, produces more points to graph, as shown in Figure 2. Connecting the points with a smooth curve results in Figure 3, which is the graph of f.

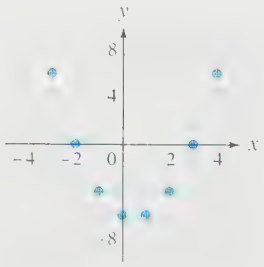


Figure 1

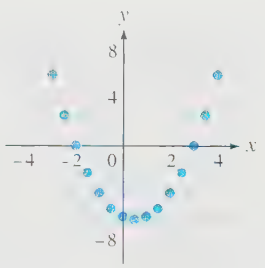


Figure 2

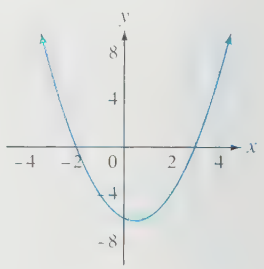


Figure 3

You can verify the graphs of these polynomial functions by using a graphing calculator. See the Appendix: Guidelines for Using a Graphing Calculator for instructions on graphing functions.

Here is an example of graphing a cubic function, $P(x) = x^3 - 2x^2 - 5x + 6$. Evaluating the function when $x = -2, -1, 0, 1, 2, 3$, and 4 gives the graph in Figure 4 below. Evaluating at some noninteger values gives the graph in Figure 5. Finally, connecting the dots with a smooth curve gives the graph in Figure 6.

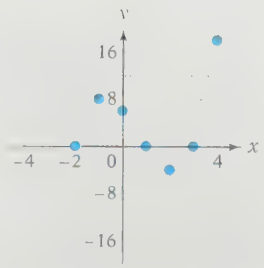


Figure 4



Figure 5

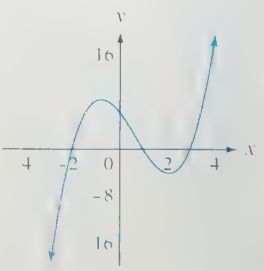


Figure 6

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Example 1

Given $P(x) = x^3 + 3x^2 - 2x + 8$, evaluate $P(-2)$.

Solution

$$\begin{aligned} P(x) &= x^3 + 3x^2 - 2x + 8 \\ P(-2) &= (-2)^3 + 3(-2)^2 - 2(-2) + 8 \\ &= (-8) + 3(4) + 4 + 8 \\ &= -8 + 12 + 4 + 8 \\ &= 16 \end{aligned}$$

You Try It 1

Given $R(x) = -2x^4 - 5x^3 + 2x - 8$, evaluate $R(2)$.

Your solution

Example 2

Find the leading coefficient, the constant term, and the degree of the polynomial.

$P(x) = 5x^6 - 4x^5 - 3x^2 + 7$

Solution

The leading coefficient is 5, the constant term is 7, and the degree is 6.

You Try It 2

Find the leading coefficient, the constant term, and the degree of the polynomial.

$r(x) = -3x^4 + 3x^3 + 3x^2 - 2x - 12$

Your solution

Example 3

Which of the following is a polynomial function?

a. $P(x) = 3x^{\frac{1}{2}} + 21x^2 - 3$

b. $T(x) = 3\sqrt{x} - 2x^2 - 3x + 2$

c. $R(x) = 14x^3 - \pi x^2 + 3x + 2$

Solution

a. This is not a polynomial function. A polynomial function does not have a variable raised to a fractional power.

b. This is not a polynomial function. A polynomial function does not have a variable expression within a radical.

c. This is a polynomial function.

You Try It 3

Which of the following is a polynomial function?

a. $R(x) = 5x^{14} - 5$

b. $V(x) = -x^{-1} + 2x - 7$

c. $P(x) = 2x^4 - 3\sqrt{x} - 3$

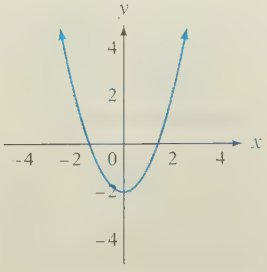
Your solution

Example 4

Graph $f(x) = x^2 - 2$.

Solution

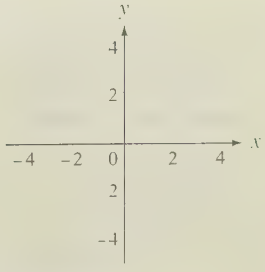
x	$y = f(x)$
-3	7
-2	2
-1	-1
0	-2
1	-1
2	2
3	7



You Try It 4

Graph $f(x) = x^2 + 2x - 3$.

Your solution



Solutions on p. S18

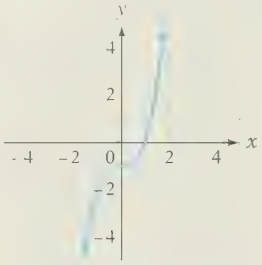
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Example 5

Graph $f(x) = x^3 - 1$.

Solution

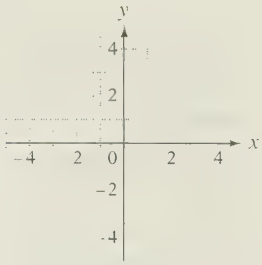
x	$y = f(x)$
-2	-9
-1	-2
0	-1
1	0
2	7



You Try It 5

Graph $f(x) = -x^3 + 1$.

Your solution



Solution on p. S18

To add or subtract polynomials

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Polynomials can be added by combining like terms. Either a vertical or a horizontal format can be used.

Add $(3x^3 - 7x + 2) + (7x^2 + 2x - 7)$. Use a horizontal format.

$$(3x^3 - 7x + 2) + (7x^2 + 2x - 7)$$
$$= 3x^3 + 7x^2 + (-7x + 2x) + (2 - 7)$$
$$= 3x^3 + 7x^2 - 5x - 5$$

- Use the Commutative and Associative Properties of Addition to rearrange and group like terms.
- Combine like terms.

Add $(4x^2 + 5x - 3) + (7x^3 - 7x + 1) + (2x - 3x^2 + 4x^3 + 1)$. Use a vertical format.

$$\begin{array}{r} 4x^2 + 5x - 3 \\ 7x^3 \qquad - 7x + 1 \\ \underline{4x^3 - 3x^2 + 2x + 1} \\ 11x^3 + x^2 \qquad - 1 \end{array}$$

- Arrange the terms of each polynomial in descending order with like terms in the same column.
- Add the terms in each column.

TAKE NOTE

The opposite of a polynomial is the polynomial with the sign of every term changed.

The additive inverse of the polynomial $x^2 + 5x - 4$ is $-(x^2 + 5x - 4)$.

To simplify the additive inverse of a polynomial, change the sign of every term inside the parentheses.

$$-(x^2 + 5x - 4) = -x^2 - 5x + 4$$

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TAKE NOTE
This is the same definition used for subtraction of integers: Subtraction is addition of the opposite.

To subtract two polynomials, add the additive inverse of the second polynomial to the first.

➡ Subtract $(3x^2 - 7xy + y^2) - (-4x^2 + 7xy - 3y^2)$. Use a horizontal format.

$$\begin{aligned}(3x^2 - 7xy + y^2) - (-4x^2 + 7xy - 3y^2) &\\ = (3x^2 - 7xy + y^2) + (4x^2 - 7xy + 3y^2) &\quad \bullet \text{ Rewrite the subtraction as the addition of the additive inverse.}\\ = 7x^2 - 14xy + 4y^2 &\quad \bullet \text{ Combine like terms.}\end{aligned}$$

➡ Subtract $(6x^3 - 3x + 7) - (3x^2 - 5x + 12)$. Use a vertical format.

$$\begin{aligned}(6x^3 - 3x + 7) - (3x^2 - 5x + 12) &\\ = (6x^3 - 3x + 7) + (-3x^2 + 5x - 12) &\quad \bullet \text{ Rewrite subtraction as the addition of the additive inverse.}\end{aligned}$$

$$\begin{array}{rl} 6x^3 - 3x + 7 & \bullet \text{ Arrange the terms of each polynomial in descending order with like terms in the same column.}\\ \underline{-3x^2 + 5x - 12} & \\ 6x^3 - 3x^2 + 2x - 5 & \bullet \text{ Combine the terms in each column.}\end{array}$$

Functional notation can be used when adding or subtracting polynomials.

➡ Given $P(x) = 3x^2 - 2x + 4$ and $R(x) = -5x^3 + 4x + 7$, find $P(x) + R(x)$.

$$\begin{aligned}P(x) + R(x) &= (3x^2 - 2x + 4) + (-5x^3 + 4x + 7)\\ &= -5x^3 + 3x^2 + 2x + 11\end{aligned}$$

➡ Given $P(x) = -5x^2 + 8x - 4$ and $R(x) = -3x^2 - 5x + 9$, find $P(x) - R(x)$.

$$\begin{aligned}P(x) - R(x) &= (-5x^2 + 8x - 4) - (-3x^2 - 5x + 9)\\ &= (-5x^2 + 8x - 4) + (3x^2 + 5x - 9)\\ &= -2x^2 + 13x - 13\end{aligned}$$

➡ Given $P(x) = 3x^2 - 5x + 6$ and $R(x) = 2x^2 - 5x - 7$, find $S(x)$, the sum of the two polynomials.

$$\begin{aligned}S(x) = P(x) + R(x) &= (3x^2 - 5x + 6) + (2x^2 - 5x - 7)\\ &= 5x^2 - 10x - 1\end{aligned}$$

Note that evaluating $P(x) = 3x^2 - 5x + 6$ and $R(x) = 2x^2 - 5x - 7$ at, for example, $x = 3$, and then adding the values, is the same as evaluating $S(x) = 5x^2 - 10x - 1$ at 3.

$P(3) = 3(3)^2 - 5(3) + 6 = 27 - 15 + 6 = 18$

$R(3) = 2(3)^2 - 5(3) - 7 = 18 - 15 - 7 = -4$

$P(3) + R(3) = 18 + (-4) = 14$

$S(3) = 5(3)^2 - 10(3) - 1 = 45 - 30 - 1 = 14$

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Example 6

Add
$(4x^2 - 3xy + 7y^2) + (-3x^2 + 7xy + y^2)$.
Use a vertical format.

Solution

$$\begin{array}{r} 4x^2 - 3xy + 7y^2 \\ -3x^2 + 7xy + y^2 \\ \hline x^2 + 4xy + 8y^2 \end{array}$$

You Try It 6

Add
$(-3x^2 - 4x + 9) + (-5x^2 - 7x + 1)$.
Use a vertical format.

Your solution

Example 7

Subtract
$(3x^2 - 2x + 4) - (7x^2 + 3x - 12)$.
Use a vertical format.

Solution

Add the additive inverse of $7x^2 + 3x - 12$ to $3x^2 - 2x + 4$.

$$\begin{array}{r} 3x^2 - 2x + 4 \\ -7x^2 - 3x + 12 \\ \hline -4x^2 - 5x + 16 \end{array}$$

You Try It 7

Subtract
$(-5x^2 + 2x - 3) - (6x^2 + 3x - 7)$.
Use a vertical format.

Your solution

Example 8

Given $P(x) = -3x^2 + 2x - 6$ and $R(x) = 4x^3 - 3x + 4$, find $S(x) = P(x) + R(x)$. Evaluate $S(-2)$.

Solution

$$\begin{aligned} S(x) &= P(x) + R(x) \\ &= (-3x^2 + 2x - 6) + (4x^3 - 3x + 4) \\ &= 4x^3 - 3x^2 - x - 2 \end{aligned}$$

$$\begin{aligned} S(-2) &= 4(-2)^3 - 3(-2)^2 - (-2) - 2 \\ &= 4(-8) - 3(4) - (-2) - 2 \\ &= \quad 32 \quad 12 + 2 - 2 \\ &= -44 \end{aligned}$$

You Try It 8

Given $P(x) = 4x^3 - 3x^2 + 2$ and $R(x) = -2x^2 + 2x - 3$, find $S(x) = P(x) + R(x)$. Evaluate $S(-1)$.

Your solution

Example 9

Given $P(x) = 2x^{2n} - 3x^n + 7$ and $R(x) = 3x^{2n} + 3x^n + 5$, find $D(x) = P(x) - R(x)$.

Solution

$$\begin{aligned} D(x) &= P(x) - R(x) \\ &= (2x^{2n} - 3x^n + 7) - (3x^{2n} + 3x^n + 5) \\ &= (2x^{2n} - 3x^n + 7) + (-3x^{2n} - 3x^n - 5) \\ &= -x^{2n} - 6x^n + 2 \end{aligned}$$

You Try It 9

Given $P(x) = 5x^{2n} - 3x^n - 7$ and $R(x) = -2x^{2n} - 5x^n + 8$, find $D(x) = P(x) - R(x)$.

Your solution

Solutions on p. S18

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6.2 Exercises

Objective 2

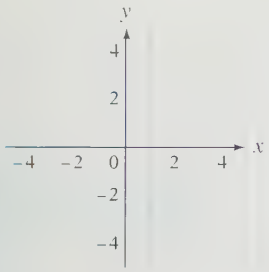
1. Given $P(x) = 3x^2 - 2x - 8$, evaluate $P(3)$.
2. Given $P(x) = -3x^2 - 5x + 8$, evaluate $P(-5)$.
3. Given $R(x) = 2x^3 - 3x^2 + 4x - 2$, evaluate $R(2)$.
4. Given $R(x) = -x^3 + 2x^2 - 3x + 4$, evaluate $R(-1)$.
5. Given $f(x) = x^4 - 2x^2 - 10$, evaluate $f(-1)$.
6. Given $f(x) = x^5 - 2x^3 + 4x$, evaluate $f(2)$.

Which of the following define a polynomial function? For those that are polynomial functions, identify **a.** the leading coefficient, **b.** the constant term, and **c.** the degree.

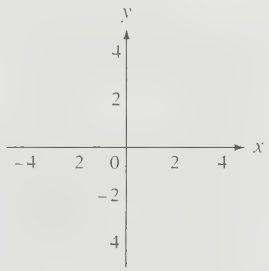
7. $P(x) = -x^2 + 3x + 8$
8. $P(x) = 3x^4 - 3x - 7$
9. $R(x) = \dfrac{x}{x+1}$
10. $R(x) = \dfrac{3x^2 - 2x + 1}{x}$
11. $f(x) = \sqrt{x} - x^2 + 2$
12. $f(x) = x^2 - \sqrt{x+2} - 8$
13. $g(x) = 3x^5 - 2x^2 + \pi$
14. $g(x) = -4x^5 - 3x^2 + x - \sqrt{7}$
15. $P(x) = 3x^2 - 5x^3 + 2$
16. $P(x) = x^2 - 5x^4 - x^6$
17. $R(x) = 14$
18. $R(x) = \dfrac{1}{x} + 2$

Graph.

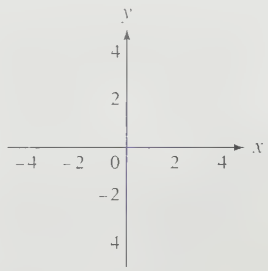
19. $P(x) = x^2 - 1$



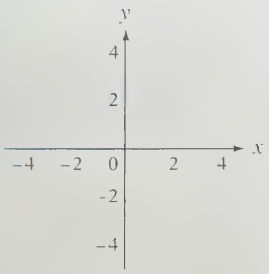
20. $P(x) = 2x^2 + 3$



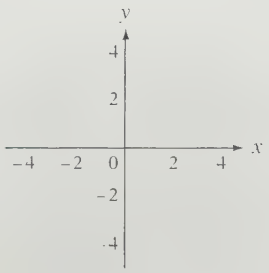
21. $R(x) = x^3 + 2$



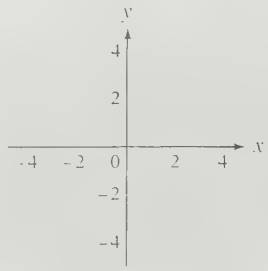
22. $R(x) = x^4 + 1$



23. $f(x) = x^3 - 2x$



24. $f(x) = x^2 - x - 2$



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Add or subtract. Use a vertical format.

25. $(5x^2 + 2x - 7) + (x^2 - 8x + 12)$

26. $(3x^2 - 2x + 7) + (-3x^2 + 2x - 12)$

27. $(x^2 - 3x + 8) - (2x^2 - 3x + 7)$

28. $(2x^2 + 3x - 7) - (5x^2 - 8x - 1)$

Add or subtract. Use a horizontal format for Exercises 29–32.

29. $(3y^2 - 7y) + (2y^2 - 8y + 2)$

30. $(-2y^2 - 4y - 12) + (5y^2 - 5y)$

31. $(2a^2 - 3a - 7) - (-5a^2 - 2a - 9)$

32. $(3a^2 - 9a) - (-5a^2 + 7a - 6)$

33. Given $P(x) = x^2 - 3x + 1$ and $R(x) = 2x^2 - 3$, find $P(x) + R(x)$.

34. Given $P(x) = x^{2n} + 7x^n - 3$ and $R(x) = -x^{2n} + 2x^n + 8$, find $P(x) + R(x)$.

35. Given $P(x) = 3x^2 + 2$ and $R(x) = -5x^2 + 2x - 3$, find $P(x) - R(x)$.

36. Given $P(x) = 2x^{2n} - x^n - 1$ and $R(x) = 5x^{2n} + 7x^n + 1$, find $P(x) - R(x)$.

37. Given $P(x) = 3x^4 - 3x^3 - x^2$ and $R(x) = 3x^3 - 7x^2 + 2x$, find $S(x) = P(x) + R(x)$. Evaluate $S(2)$.

38. Given $P(x) = 3x^4 - 2x + 1$ and $R(x) = 3x^5 - 5x - 8$, find $S(x) = P(x) + R(x)$. Evaluate $S(-1)$.

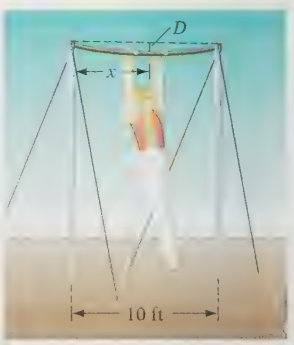
APPLYING THE CONCEPTS

39. For what value of k is the given equation an identity?

a. $(2x^3 + 3x^2 + kx + 5) - (x^3 + 2x^2 + 3x + 7) = x^3 + x^2 + 5x - 2$

b. $(6x^3 + kx^2 - 2x - 1) - (4x^3 - 3x^2 + 1) = 2x^3 - x^2 - 2x - 2$

40. The deflection D (in inches) of a beam that is uniformly loaded is given by the polynomial function $D(x) = 0.005x^4 - 0.1x^3 + 0.5x^2$, where x is the distance in feet from one end of the beam. See the figure at the right. The maximum deflection occurs when x is the midpoint of the beam. Determine the maximum deflection for the beam in the diagram.



41. If $P(x)$ is a third-degree polynomial and $Q(x)$ is a fourth-degree polynomial, what can be said about the degree of $P(x) + Q(x)$? Give some examples of polynomials that support your answer.

42. If $P(x)$ is a fifth-degree polynomial and $Q(x)$ is a fourth-degree polynomial, what can be said about the degree of $P(x) - Q(x)$? Give some examples of polynomials that support your answer.

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6.3 Multiplication of Polynomials

To multiply a polynomial by a monomial

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To multiply a polynomial by a monomial, use the Distributive Property and the Rule for Multiplying Exponential Expressions.

➡ Multiply: $-3a(4a^2 - 5a + 6)$

$$-3a(4a^2 - 5a + 6) = \boxed{-3a(4a^2) - (-3a)(5a) + (-3a)(6)}$$
$$= -12a^3 + 15a^2 - 18a$$

• Use the Distributive Property. This step is frequently done mentally.

Example 1
Multiply: $(5x + 4)(-2x)$

Solution
$(5x + 4)(-2x) = -10x^2 - 8x$

You Try It 1
Multiply: $(-2y + 3)(-4y)$

Your solution

Example 2
Multiply: $2a^2b(4a^2 - 2ab + b^2)$

Solution
$2a^2b(4a^2 - 2ab + b^2)$
$= 8a^4b - 4a^3b^2 + 2a^2b^3$

You Try It 2
Multiply: $-a^2(3a^2 + 2a - 7)$

Your solution

Solutions on p. S18

To multiply two polynomials

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Multiplication of two polynomials requires the repeated application of the Distributive Property.

$$(y - 2)(y^2 + 3y + 1) = (y - 2)(y^2) + (y - 2)(3y) + (y - 2)(1)$$
$$= y^3 - 2y^2 + 3y^2 - 6y + y - 2$$
$$= y^3 + y^2 - 5y - 2$$

A convenient method of multiplying two polynomials is to use a vertical format similar to that used for multiplication of whole numbers.

$$\begin{array}{r} y^2 + 3y + 1 \\ \underline{\qquad\quad y - 2} \\ -2y^2 - 6y - 2 \\ \underline{y^3 + 3y^2 + y\quad\;\;} \\ y^3 + y^2 - 5y - 2 \end{array}$$

• Multiply $-2(y^2 + 3y + 1)$.
• Multiply $y(y^2 + 3y + 1)$.
• Add the terms in each column.

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Multiply: $(2a^3 + a - 3)(a + 5)$

$$\begin{array}{r} 2a^3 + a - 3 \\ a + 5 \\ \hline 10a^3 + 5a - 15 \\ 2a^4 + a^2 - 3a \\ \hline 2a^4 + 10a^3 + a^2 + 2a - 15 \end{array}$$

- Note that spaces are provided in each product so that like terms are in the same column.
- Add the terms in each column.

Example 3

Multiply: $(2b^3 - b + 1)(2b + 3)$

Solution

$$\begin{array}{r} 2b^3 - b + 1 \\ 2b + 3 \\ \hline 6b^3 - 3b + 3 \\ 4b^4 - 2b^2 + 2b \\ \hline 4b^4 + 6b^3 - 2b^2 - b + 3 \end{array}$$

You Try It 3

Multiply: $(2y^3 + 2y^2 - 3)(3y - 1)$

Your solution

Solution on p. S19

To multiply two binomials

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It is frequently necessary to find the product of two binomials. The product can be found using a method called **FOIL**, which is based on the Distributive Property. The letters of FOIL stand for **F**irst, **O**uter, **I**nner, and **L**ast.

TAKE NOTE

FOIL is not really a different way of multiplying. It is based on the Distributive Property.

$(2x + 3)(x + 5)$
$= 2x(x + 5) + 3(x + 5)$
F O I L
$= 2x^2 + 10x + 3x + 15$
$= 2x^2 + 13x + 15$

FOIL is an efficient way to remember how to do binomial multiplication.

Multiply: $(2x + 3)(x + 5)$

Multiply the First terms.	$(2x + 3)(x + 5)$	$2x \cdot x = 2x^2$
Multiply the Outer terms.	$(2x + 3)(x + 5)$	$2x \cdot 5 = 10x$
Multiply the Inner terms.	$(2x + 3)(x + 5)$	$3 \cdot x = 3x$
Multiply the Last terms.	$(2x + 3)(x + 5)$	$3 \cdot 5 = 15$
		F O I L
Add the products.	$(2x + 3)(x + 5)$	$= 2x^2 + 10x + 3x + 15$
Combine like terms.		$= 2x^2 + 13x + 15$

Multiply: $(4x - 3)(3x - 2)$

$$(4x - 3)(3x - 2) = \boxed{4x(3x) + 4x(-2) + (-3)(3x) + (-3)(-2)}$$
$$= 12x^2 - 8x - 9x + 6$$
$$= 12x^2 - 17x + 6$$

- Do this step mentally.

Multiply: $(3x - 2y)(x + 4y)$

$$(3x - 2y)(x + 4y) = \boxed{3x(x) + 3x(4y) + (-2y)(x) + (-2y)(4y)}$$
$$= 3x^2 + 12xy - 2xy - 8y^2$$
$$= 3x^2 + 10xy - 8y^2$$

- Do this step mentally.

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Example 4
Multiply: $(2a - 1)(3a - 2)$

Solution
$(2a - 1)(3a - 2) = 6a^2 - 4a - 3a + 2$
$= 6a^2 - 7a + 2$

You Try It 4
Multiply: $(4y - 5)(2y - 3)$

Your solution

Example 5
Multiply: $(3x - 2)(4x + 3)$

Solution
$(3x - 2)(4x + 3) = 12x^2 + 9x - 8x - 6$
$= 12x^2 + x - 6$

You Try It 5
Multiply: $(3b + 2)(3b - 5)$

Your solution

Solutions on p. S19

Objective 3

To multiply binomials that have special products

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Using FOIL, it is possible to find a pattern for the product of the sum and difference of two terms and for the square of a binomial.

The Sum and Difference of Two Terms

$$(a + b)(a - b) = a^2 - ab + ab - b^2$$
$$= a^2 - b^2$$

Square of the first term
Square of the second term

The Square of a Binomial

$$(a + b)^2 = (a + b)(a + b) = a^2 + ab + ab + b^2$$
$$= a^2 + 2ab + b^2$$

Square of the first term
Twice the product of the two terms
Square of the last term

Multiply: $(2x + 3)(2x - 3)$

$(2x + 3)(2x - 3)$ is the sum and difference of two terms.

$(2x + 3)(2x - 3) = (2x)^2 - 3^2$ • Do this step mentally.

$= 4x^2 - 9$

Multiply: $(3x - 2)^2$

$(3x - 2)^2$ is the square of a binomial.

$(3x - 2)^2 = (3x)^2 + 2(3x)(-2) + (-2)^2$ • Do this step mentally.

$= 9x^2 - 12x + 4$

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Example 6
Multiply: $(4z - 2w)(4z + 2w)$

Solution
$(4z - 2w)(4z + 2w) = 16z^2 - 4w^2$

You Try It 6
Multiply: $(2a + 5c)(2a - 5c)$

Your solution

Example 7
Simplify: $(2r - 3s)^2$

Solution
$(2r - 3s)^2 = 4r^2 - 12rs + 9s^2$

You Try It 7
Simplify: $(3x + 2y)^2$

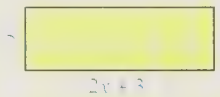
Your solution

Solutions on p. S19

To solve application problems

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Example 8
The length of a rectangle is $(2x + 3)$ ft. The width is $(x - 5)$ ft. Find the area of the rectangle in terms of the variable x.

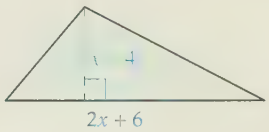


Strategy
To find the area, replace the variables L and W in the equation $A = L \cdot W$ by the given values and solve for A.

Solution
$A = L \cdot W$
$A = (2x + 3)(x - 5)$
$= 2x^2 - 10x + 3x - 15$
$= 2x^2 - 7x - 15$

The area is $(2x^2 - 7x - 15)$ ft^2.

You Try It 8
The base of a triangle is $(2x + 6)$ ft. The height is $(x - 4)$ ft. Find the area of the triangle in terms of the variable x.



Your strategy

Your solution

Solution on p. S19

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6.3 Exercises

Objective A

Multiply.

1. $x(x - 2)$ **2.** $y(3 - y)$ **3.** $-x(x + 7)$ **4.** $-y(7 - y)$

5. $3a^2(a - 2)$ **6.** $4b^2(b + 8)$ **7.** $-5x^2(x^2 - x)$ **8.** $-6y^2(y + 2y^2)$

9. $-x^3(3x^2 - 7)$ **10.** $-y^4(2y^2 - y^6)$ **11.** $2x(6x^2 - 3x)$ **12.** $3y(4y - y^2)$

13. $(2x - 4)3x$ **14.** $(3y - 2)y$ **15.** $(3x + 4)x$ **16.** $(2x + 1)2x$

17. $-xy(x^2 - y^2)$ **18.** $-x^2y(2xy - y^2)$ **19.** $x(2x^3 - 3x + 2)$ **20.** $y(-3y^2 - 2y + 6)$

21. $-a(-2a^2 - 3a - 2)$ **22.** $-b(5b^2 + 7b - 35)$ **23.** $x^2(3x^4 - 3x^2 - 2)$

24. $y^3(-4y^3 - 6y + 7)$ **25.** $2y^2(-3y^2 - 6y + 7)$ **26.** $4x^2(3x^2 - 2x + 6)$

27. $(a^2 + 3a - 4)(-2a)$ **28.** $(b^3 - 2b + 2)(-5b)$ **29.** $-3y^2(-2y^2 + y - 2)$

30. $-5x^2(3x^2 - 3x - 7)$ **31.** $xy(x^2 - 3xy + y^2)$ **32.** $ab(2a^2 - 4ab - 6b^2)$

Objective B

Multiply.

33. $(x^2 + 3x + 2)(x + 1)$ **34.** $(x^2 - 2x + 7)(x - 2)$ **35.** $(a^2 - 3a + 4)(a - 3)$

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36. $(x^2 - 3x + 5)(2x - 3)$ **37.** $(-2b^2 - 3b + 4)(b - 5)$ **38.** $(-a^2 + 3a - 2)(2a - 1)$

39. $(-2x^2 + 7x - 2)(3x - 5)$ **40.** $(-a^2 - 2a + 3)(2a - 1)$ **41.** $(x^2 + 5)(x - 3)$

42. $(y^2 - 2y)(2y + 5)$ **43.** $(x^3 - 3x + 2)(x - 4)$ **44.** $(y^3 + 4y^2 - 8)(2y - 1)$

45. $(5y^2 + 8y - 2)(3y - 8)$ **46.** $(3y^2 + 3y - 5)(4y - 3)$ **47.** $(5a^3 - 5a + 2)(a - 4)$

48. $(3b^3 - 5b^2 + 7)(6b - 1)$ **49.** $(y^3 + 2y^2 - 3y + 1)(y + 2)$ **50.** $(2a^3 - 3a^2 + 2a - 1)(2a - 3)$

Objective 2

Multiply.

51. $(x + 1)(x + 3)$ **52.** $(y + 2)(y + 5)$ **53.** $(a - 3)(a + 4)$ **54.** $(b - 6)(b + 3)$

55. $(y + 3)(y - 8)$ **56.** $(x + 10)(x - 5)$ **57.** $(y - 7)(y - 3)$ **58.** $(a - 8)(a - 9)$

59. $(2x + 1)(x + 7)$ **60.** $(y + 2)(5y + 1)$ **61.** $(3x - 1)(x + 4)$ **62.** $(7x - 2)(x + 4)$

63. $(4x - 3)(x - 7)$ **64.** $(2x - 3)(4x - 7)$ **65.** $(3y - 8)(y + 2)$ **66.** $(5y - 9)(y + 5)$

67. $(3x + 7)(3x + 11)$ **68.** $(5a + 6)(6a + 5)$ **69.** $(7a - 16)(3a - 5)$ **70.** $(5a - 12)(3a - 7)$

71. $(3a - 2b)(2a - 7b)$ **72.** $(5a - b)(7a - b)$ **73.** $(a - 9b)(2a + 7b)$

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74. $(2a + 5b)(7a - 2b)$ **75.** $(10a - 3b)(10a - 7b)$ **76.** $(12a - 5b)(3a - 4b)$

77. $(5x + 12y)(3x + 4y)$ **78.** $(11x + 2y)(3x + 7y)$ **79.** $(2x - 15y)(7x + 4y)$

80. $(5x + 2y)(2x - 5y)$ **81.** $(8x - 3y)(7x - 5y)$ **82.** $(2x - 9y)(8x - 3y)$

Objective D

83. What does it mean to square a binomial?

84. Why is $(a + b)^2$ not equal to $a^2 + b^2$?

Multiply.

85. $(y - 5)(y + 5)$ **86.** $(y + 6)(y - 6)$ **87.** $(2x + 3)(2x - 3)$ **88.** $(4x - 7)(4x + 7)$

89. $(x + 1)^2$ **90.** $(y - 3)^2$ **91.** $(3a - 5)^2$ **92.** $(6x - 5)^2$

93. $(3x - 7)(3x + 7)$ **94.** $(9x - 2)(9x + 2)$ **95.** $(2a + b)^2$ **96.** $(x + 3y)^2$

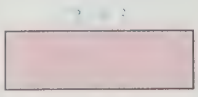
97. $(4 - 3y)(4 + 3y)$ **98.** $(4x - 9y)(4x + 9y)$ **99.** $(5x + 2y)^2$ **100.** $(2a - 9b)^2$

Objective E Application Problems

101. The length of a rectangle is $5x$ ft. The width is $(2x - 7)$ ft. Find the area of the rectangle in terms of the variable x.



102. The width of a rectangle is $(x - 6)$ m. The length is $(2x + 3)$ m. Find the area of the rectangle in terms of the variable x.



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103. The length of a side of a square is $(2x + 1)$ km. Find the area of the square in terms of the variable x.



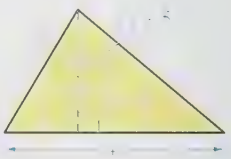
104. The length of a side of a square is $(2x - 3)$ yd. Find the area of the square in terms of the variable x.

105. The width of a rectangle is $(3x + 1)$ in. The length of the rectangle is twice the width. Find the area of the rectangle in terms of the variable x.

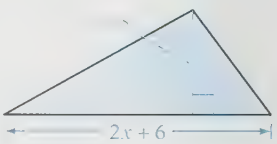


106. The width of a rectangle is $(4x - 3)$ cm. The length of the rectangle is twice the width. Find the area of the rectangle in terms of the variable x.

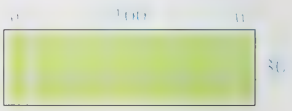
107. The base of a triangle is $4x$ m, and the height is $(2x + 5)$ m. Find the area of the triangle in terms of the variable x.



108. The base of a triangle is $(2x + 6)$ in., and the height is $(x - 8)$ in. Find the area of the triangle in terms of the variable x.



109. An athletic field has dimensions 30 yd by 100 yd. An end zone that is w yd wide borders each end of the field. Express the total area of the field and the end zones in terms of the variable w.



110. A softball diamond has dimensions 45 ft by 45 ft. A base path border x ft wide lies on both the first-base side and the third-base side of the diamond. Express the total area of the softball diamond and the base path in terms of the variable x.



APPLYING THE CONCEPTS

Simplify.

111. $(a + b)^2 - (a - b)^2$ **112.** $(x^2 + x - 3)^2$ **113.** $(a + 3)^3$

114. **a.** What polynomial has quotient $3x - 4$ when divided by $4x + 5$?
b. What polynomial has quotient $x^2 + 2x - 1$ when divided by $x + 3$?

115. **a.** Add $x^2 + 2x - 3$ to the product of $2x - 5$ and $3x + 1$.
b. Subtract $4x^2 - x - 5$ from the product of $x^2 + x + 3$ and $x - 4$.

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6.4 Division of Polynomials

Objective To divide polynomials

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To divide two polynomials, use a method similar to that used for long division of whole numbers. To check division of polynomials, use

Dividend = (quotient × divisor) + remainder

➡ Divide: $(x^2 + 5x - 7) \div (x + 3)$

Step 1

$$\begin{array}{r} x \\ x + 3 \overline{)\, x^2 + 5x - 7} \\ \underline{x^2 + 3x} \quad \downarrow \\ 2x - 7 \end{array}$$

Think: $x \overline{)x^2} = \frac{x^2}{x} = x$

Multiply: $x(x + 3) = x^2 + 3x$

Subtract: $(x^2 + 5x) - (x^2 + 3x) = 2x$

Bring down the -7.

Step 2

$$\begin{array}{r} x + 2 \\ x + 3 \overline{)\, x^2 + 5x - 7} \\ \underline{x^2 + 3x} \\ 2x - 7 \\ \underline{2x + 6} \\ -13 \end{array}$$

Think: $x \overline{)2x} = \frac{2x}{x} = 2$

Multiply: $2(x + 3) = 2x + 6$

Subtract: $(2x - 7) - (2x + 6) = -13$

The remainder is -13.

Check: (Quotient)(divisor) + remainder

$$= (x + 2)(x + 3) + (-13) = x^2 + 3x + 2x + 6 - 13 = x^2 + 5x - 7$$

$$(x^2 + 5x - 7) \div (x + 3) = x + 2 - \frac{13}{x + 3}$$

➡ Divide: $\dfrac{6 - 6x^2 + 4x^3}{2x + 3}$

Arrange the terms in descending order. Note that there is no term of x in $4x^3 - 6x^2 + 6$. Insert a $0x$ for the missing term so that like terms will be in the same columns.

$$\begin{array}{r} 2x^2 - 6x + 9 \\ 2x + 3 \overline{)\, 4x^3 - 6x^2 + 0x + 6} \\ \underline{4x^3 + 6x^2} \\ -12x^2 + 0x \\ \underline{-12x^2 - 18x} \\ 18x + 6 \\ \underline{18x + 27} \\ -21 \end{array}$$

$$\frac{4x^3 - 6x^2 + 6}{2x + 3} = 2x^2 - 6x + 9 - \frac{21}{2x + 3}$$

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Example 1

Divide: $\dfrac{12x^2 - 11x + 10}{4x - 5}$

Solution

$$\begin{array}{r} 3x + 1 \\ 4x - 5 \overline{)\,12x^2 - 11x + 10} \\ \underline{12x^2 - 15x} \\ 4x + 10 \\ \underline{4x - 5} \\ 15 \end{array}$$

$$\frac{12x^2 - 11x + 10}{4x - 5} = 3x + 1 + \frac{15}{4x - 5}$$

You Try It 1

Divide: $\dfrac{15x^2 + 17x - 20}{3x + 4}$

Your solution

Example 2

Divide: $\dfrac{x^3 + 1}{x + 1}$

Solution

$$\begin{array}{r} x^2 - x + 1 \\ x + 1 \overline{)\,x^3 + 0x^2 + 0x + 1} \\ \underline{x^3 + x^2} \\ -x^2 + 0x \\ \underline{-x^2 - x} \\ x + 1 \\ \underline{x + 1} \\ 0 \end{array}$$

- Insert zeros for the coefficients of the missing terms.

$$\frac{x^3 + 1}{x + 1} = x^2 - x + 1$$

You Try It 2

Divide: $\dfrac{3x^3 + 8x^2 - 6x + 2}{3x - 1}$

Your solution

Example 3

Divide:

$(2x^4 - 7x^3 + 3x^2 + 4x - 5) \div (x^2 - 2x - 2)$

Solution

$$\begin{array}{r} 2x^2 - 3x + 1 \\ x^2 - 2x - 2 \overline{)\,2x^4 - 7x^3 + 3x^2 + 4x - 5} \\ \underline{2x^4 - 4x^3 - 4x^2} \\ -3x^3 + 7x^2 + 4x \\ \underline{-3x^3 + 6x^2 + 6x} \\ x^2 - 2x - 5 \\ \underline{x^2 - 2x - 2} \\ -3 \end{array}$$

$$(2x^4 - 7x^3 + 3x^2 + 4x - 5) \div (x^2 - 2x - 2)$$

$$= 2x^2 - 3x + 1 - \frac{3}{x^2 - 2x - 2}$$

You Try It 3

Divide:

$(3x^4 - 11x^3 + 16x^2 - 16x + 8) \div (x^2 - 3x + 2)$

Your solution

Solutions on p. S19

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Objective B To divide polynomials using synthetic division

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Synthetic division is a shorter method of dividing a polynomial by a binomial of the form $x - a$.

Divide $(3x^2 - 4x + 6) \div (x - 2)$ by using long division.

$$\begin{array}{r} 3x + 2 \\ x - 2 \overline{)\,3x^2 - 4x + 6} \\ \underline{3x^2 - 6x} \\ 2x + 6 \\ \underline{2x - 4} \\ 10 \end{array}$$

$$(3x^2 - 4x + 6) \div (x - 2) = 3x + 2 + \frac{10}{x - 2}$$

The variables can be omitted because the position of a term indicates the power of the term.

$$\begin{array}{r} 3 \quad 2 \\ -2 \overline{)\,3 \quad -4 \quad 6} \\ \underline{3 \quad -6} \\ 2 \quad 6 \\ \underline{2 \quad -4} \\ 10 \end{array}$$

Each number shown in color above is exactly the same as the number above it. Removing the colored numbers condenses the vertical spacing.

$$\begin{array}{r} 3 \quad 2 \\ -2 \overline{)\,3 \quad -4 \quad 6} \\ \underline{-6 \quad -4} \\ 2 \quad 10 \end{array}$$

The number in color on the top row is the same as the one in the bottom row. Writing the 3 from the top row in the bottom row allows the spacing to be condensed even further.

$$\begin{array}{r|lll} -2 & 3 & -4 & 6 \\ & & \underline{-6} & \underline{-4} \\ & \underbrace{3}_{\text{Terms of the quotient}} & \underbrace{2}_{} & \underbrace{10}_{\text{Remainder}} \end{array}$$

Because the degree of the dividend $(3x^2 - 4x + 6)$ is 2 and the degree of the divisor $(x - 2)$ is 1, the degree of the quotient is $2 - 1 = 1$. This means that, using the terms of the quotient given above, the quotient is $3x + 2$. The remainder is 10.

In general, the degree of the quotient of two polynomials is the difference between the degree of the dividend and the degree of the divisor.

By replacing the constant term in the divisor by its additive inverse, we may add rather than subtract terms. This is illustrated in the following example.

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Divide: $(3x^3 + 6x^2 - x - 2) \div (x + 3)$

The additive inverse of the binomial constant

$$
\begin{array}{r|cccc}
 & \multicolumn{4}{c}{\text{Coefficients of the polynomial}} \\
-3 & 3 & 6 & -1 & -2 \\
 & & -9 & 9 & -24 \\
\hline
 & 3 & -3 & 8 & -26 \\
 & \multicolumn{2}{c}{\text{Terms of the quotient}} & & \text{Remainder}
\end{array}
$$

- Bring down the 3.
- Multiply $-3(3)$ and add the product to 6.
- Multiply $-3(-3)$ and add the product to -1.
- Multiply $-3(8)$ and add the product to -2.

TAKE NOTE

Remember that you can check the answer to a synthetic-division problem in the same way that you check the answer to a long-division problem.

The degree of the dividend is 3 and the degree of the divisor is 1. Therefore, the degree of the quotient is $3 - 1 = 2$.

$$(3x^3 + 6x^2 - x - 2) \div (x + 3) = 3x^2 - 3x + 8 - \frac{26}{x + 3}$$

Divide: $(2x^3 - x + 2) \div (x - 2)$

The additive inverse of the binomial constant

$$
\begin{array}{r|cccc}
 & \multicolumn{4}{c}{\text{Coefficients of the polynomial}} \\
2 & 2 & 0 & -1 & 2 \\
 & & 4 & 8 & 14 \\
\hline
 & 2 & 4 & 7 & 16 \\
 & \multicolumn{2}{c}{\text{Terms of the quotient}} & & \text{Remainder}
\end{array}
$$

- Insert a 0 for the missing term and bring down the 2.
- Multiply $2(2)$ and add the product to 0.
- Multiply $2(4)$ and add the product to -1.
- Multiply $2(7)$ and add the product to 2.

$$(2x^3 - x + 2) \div (x - 2) = 2x^2 + 4x + 7 + \frac{16}{x - 2}$$

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Example 4

Divide by using synthetic division:
$(7 - 3x + 5x^2) \div (x - 1)$

Solution

Arrange the coefficients in decreasing powers of x.

$$\begin{array}{r|rrr} 1 & 5 & -3 & 7 \\ & & 5 & 2 \\ \hline & 5 & 2 & 9 \end{array}$$

$$(5x^2 - 3x + 7) \div (x - 1) = 5x + 2 + \frac{9}{x - 1}$$

You Try It 4

Divide by using synthetic division:
$(8x + 6x^2 - 5) \div (x + 2)$

Your solution

Example 5

Divide by using synthetic division:
$(2x^3 + 4x^2 - 3x + 12) \div (x + 4)$

Solution

$$\begin{array}{r|rrrr} -4 & 2 & 4 & -3 & 12 \\ & & -8 & 16 & -52 \\ \hline & 2 & -4 & 13 & -40 \end{array}$$

$$(2x^3 + 4x^2 - 3x + 12) \div (x + 4)$$
$$= 2x^2 - 4x + 13 - \frac{40}{x + 4}$$

You Try It 5

Divide by using synthetic division:
$(5x^3 - 12x^2 - 8x + 16) \div (x - 2)$

Your solution

Example 6

Divide by using synthetic division:
$(3x^4 - 8x^2 + 2x + 1) \div (x + 2)$

Solution

Insert a zero for the missing term.

$$\begin{array}{r|rrrrr} -2 & 3 & 0 & -8 & 2 & 1 \\ & & -6 & 12 & -8 & 12 \\ \hline & 3 & -6 & 4 & -6 & 13 \end{array}$$

$$(3x^4 - 8x^2 + 2x + 1) \div (x + 2)$$
$$= 3x^3 - 6x^2 + 4x - 6 + \frac{13}{x + 2}$$

You Try It 6

Divide by using synthetic division:
$(2x^4 - 3x^3 - 8x^2 - 2) \div (x - 3)$

Your solution

Solutions on p. S19

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To evaluate a polynomial using synthetic division

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A polynomial can be evaluated by using synthetic division. Consider the polynomial $P(x) = 2x^4 - 3x^3 + 4x^2 - 5x + 1$. One way to evaluate the polynomial when $x = 2$ is to replace x by 2 and then simplify the numerical expression.

$$P(x) = 2x^4 - 3x^3 + 4x^2 - 5x + 1$$

$$\begin{aligned} P(2) &= 2(2)^4 - 3(2)^3 + 4(2)^2 - 5(2) + 1 \\ &= 2(16) - 3(8) + 4(4) - 5(2) + 1 \\ &= 32 - 24 + 16 - 10 + 1 \\ &= 15 \end{aligned}$$

Now use synthetic division to divide $(2x^4 - 3x^3 + 4x^2 - 5x + 1)$ by $(x - 2)$.

$$\begin{array}{r|rrrrr} 2 & 2 & -3 & 4 & -5 & 1 \\ & & 4 & 2 & 12 & 14 \\ \hline & 2 & 1 & 6 & 7 & 15 \end{array}$$

Terms of the quotient — Remainder

Note that the remainder is 15, which is the same value as $P(2)$. This is not a coincidence. The following theorem states that this situation is always true.

Remainder Theorem

If the polynomial $P(x)$ is divided by $x - a$, the remainder is $P(a)$.

Use the Remainder Theorem to evaluate $P(x) = x^4 - 3x^2 + 4x - 5$ when $x = -2$.

The value at which the polynomial is evaluated

$$\begin{array}{r|rrrrr} -2 & 1 & 0 & -3 & 4 & -5 \\ & & -2 & 4 & -2 & -4 \\ \hline & 1 & -2 & 1 & 2 & -9 \end{array}$$

• A 0 is inserted for the x^3 term.

← The remainder

$P(-2) = -9$

Example 7

Use the Remainder Theorem to evaluate $P(x) = x^4 + 3x^3 + 2x^2 - x - 5$ when $x = 2$.

Solution

$$\begin{array}{r|rrrrr} 2 & 1 & 3 & 2 & -1 & 5 \\ & & 2 & 10 & 16 & 30 \\ \hline & 1 & 5 & 8 & 15 & 35 \end{array}$$

$P(2) = 35$

You Try It 7

Use the Remainder Theorem to evaluate $P(x) = 2x^3 - 5x^2 + 7$ when $x = -3$.

Your solution

Solution on p. S19

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6.4 Exercises

Objective A

Divide by using long division.

1. $(x^2 + 3x - 40) \div (x - 5)$

2. $(x^2 - 14x + 24) \div (x - 2)$

3. $(x^3 - 3x^2 + 2) \div (x - 3)$

4. $(x^3 + 4x^2 - 8) \div (x + 4)$

5. $(6x^2 + 13x + 8) \div (2x + 1)$

6. $(12x^2 + 13x - 14) \div (3x - 2)$

7. $(10x^2 + 9x - 5) \div (2x - 1)$

8. $(18x^2 - 3x + 2) \div (3x + 2)$

9. $(8x^3 - 9) \div (2x - 3)$

10. $(64x^3 + 4) \div (4x + 2)$

11. $(6x^4 - 13x^2 - 4) \div (2x^2 - 5)$

12. $(12x^4 - 11x^2 + 10) \div (3x^2 + 1)$

13. $\dfrac{-10 - 33x + 3x^3 - 8x^2}{3x + 1}$

14. $\dfrac{10 - 49x + 38x^2 - 8x^3}{1 - 4x}$

15. $\dfrac{x^3 - 5x^2 + 7x - 4}{x - 3}$

16. $\dfrac{2x^3 - 3x^2 + 6x + 4}{2x + 1}$

17. $\dfrac{16x^2 - 13x^3 + 2x^4 + 20 - 9x}{x - 5}$

18. $\dfrac{x - x^2 + 5x^3 + 3x^4 - 2}{x + 2}$

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19. $\dfrac{2x^3 + 4x^2 - x + 2}{x^2 + 2x - 1}$

20. $\dfrac{3x^3 - 2x^2 + 5x - 4}{x^2 - x + 3}$

21. $\dfrac{x^4 + 2x^3 - 3x^2 - 6x + 2}{x^2 - 2x - 1}$

22. $\dfrac{x^4 - 3x^3 + 4x^2 - x + 1}{x^2 + x - 3}$

23. $\dfrac{x^4 + 3x^2 - 4x + 5}{x^2 + 2x + 3}$

24. $\dfrac{x^4 + 2x^3 - x + 2}{x^2 - x - 1}$

Divide by using synthetic division.

25. $(2x^2 - 6x - 8) \div (x + 1)$

26. $(3x^2 + 19x + 20) \div (x + 5)$

27. $(3x^2 - 14x + 16) \div (x - 2)$

28. $(4x^2 - 23x + 28) \div (x - 4)$

29. $(3x^2 - 4) \div (x - 1)$

30. $(4x^2 - 8) \div (x - 2)$

31. $(2x^3 - x^2 + 6x + 9) \div (x + 1)$

32. $(3x^3 + 10x^2 + 6x - 4) \div (x + 2)$

33. $(18 + x - 4x^3) \div (2 - x)$

34. $(12 - 3x^2 + x^3) \div (x + 3)$

35. $(2x^3 + 5x^2 - 5x + 20) \div (x + 4)$

36. $(5x^3 + 3x^2 - 17x + 6) \div (x + 2)$

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37. $\dfrac{5 + 5x - 8x^2 + 4x^3 - 3x^4}{2 - x}$

38. $\dfrac{3 - 13x - 5x^2 + 9x^3 - 2x^4}{3 - x}$

39. $\dfrac{3x^4 + 3x^3 - x^2 + 3x + 2}{x + 1}$

40. $\dfrac{4x^4 + 12x^3 - x^2 - x + 2}{x + 3}$

41. $\dfrac{2x^4 - x^2 + 2}{x - 3}$

42. $\dfrac{x^4 - 3x^3 - 30}{x + 2}$

43. If the polynomial $2x^3 - 3x^2 - 4x - 15$ is divided by $x - 3$ and the remainder is 0, what do we know about $f(3)$ for the function $f(x) = 2x^3 - 3x^2 - 4x - 15$?

44. If the polynomial $3x^4 - 8x^2 + 2x + 1$ is divided by $x + 2$ and the remainder is 13, what do we know about $f(-2)$ for the function $f(x) = 3x^4 - 8x^2 + 2x + 1$?

Use the Remainder Theorem to evaluate the polynomial.

45. $P(x) = 2x^2 - 3x - 1$; $P(3)$

46. $Q(x) = 3x^2 - 5x - 1$; $Q(2)$

47. $R(x) = x^3 - 2x^2 + 3x - 1$; $R(4)$

48. $F(x) = x^3 + 4x^2 - 3x + 2$; $F(3)$

49. $P(z) = 2z^3 - 4z^2 + 3z - 1$; $P(-2)$

50. $R(t) = 3t^3 + t^2 - 4t + 2$; $R(-3)$

51. $Z(p) = 2p^3 - p^2 + 3$; $Z(-3)$

52. $P(y) = 3y^3 + 2y^2 - 5$; $P(-2)$

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53. $Q(x) = x^4 + 3x^3 - 2x^2 + 4x - 9$; $Q(2)$

54. $Y(z) = z^4 - 2z^3 - 3z^2 - z + 7$; $Y(3)$

55. $F(x) = 2x^4 - x^3 + 2x - 5$; $F(-3)$

56. $Q(x) = x^4 - 2x^3 + 4x - 2$; $Q(-2)$

57. $P(x) = x^3 - 3$; $P(5)$

58. $S(t) = 4t^3 + 5$; $S(-4)$

59. $R(t) = 4t^4 - 3t^2 + 5$; $R(-3)$

60. $P(z) = 2z^4 + z^2 - 3$; $P(-4)$

61. $Q(x) = x^5 - 4x^3 - 2x^2 + 5x - 2$; $Q(2)$

62. $T(x) = 2x^5 + 4x^4 - x^2 + 4$; $T(3)$

63. $R(x) = 2x^5 - x^3 + 4x - 1$; $R(-2)$

64. $P(x) = x^5 - x^3 + 4x + 1$; $P(-3)$

APPLYING THE CONCEPTS

65. Divide by using long division.

a. $\dfrac{a^3 + b^3}{a + b}$ **b.** $\dfrac{x^5 + y^5}{x + y}$ **c.** $\dfrac{x^6 - y^6}{x + y}$

66. For what value of k will the remainder be zero?

a. $(x^3 - x^2 - 3x + k) \div (x + 3)$ **b.** $(2x^3 - x + k) \div (x - 1)$

67. Divide.

a. $(2x^3 + 7x^2 + 2x - 8) \div (4x + 8)$ **b.** $(4x^3 + 13x^2 - 22x + 24) \div (6x - 12)$

c. $(2x^4 - 3x^3 + 4x^2 + x - 10) \div (x^2 - x + 1)$ **d.** $(x^4 + 4x^3 + 2x^2 - x + 5) \div (x^2 - 2x - 3)$

68. Show how synthetic division can be modified so that the divisor can be of the form $ax + b$.

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Focus on Problem Solving

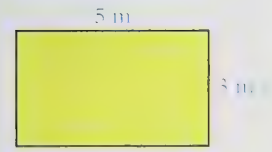
Dimensional Analysis

In solving application problems, it may be useful to include the units in order to organize the problem so that the answer is in the proper units. Using units to organize and check the correctness of an application is called **dimensional analysis**. We use the operations of multiplying units and dividing units in applying dimensional analysis to application problems.

The Rule for Multiplying Exponential Expressions states that we multiply two expressions with the same base by adding the exponents.

$$x^4 \cdot x^6 = x^{4+6} = x^{10}$$

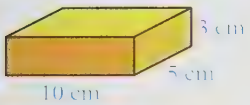
In calculations that involve quantities, the units are operated on algebraically.



➡ A rectangle measures 3 m by 5 m. Find the area of the rectangle.

$A = LW = (3\text{ m})(5\text{ m}) = (3 \cdot 5)(\text{m} \cdot \text{m}) = 15\text{ m}^2$

The area of the rectangle is 15 m^2 (square meters).



➡ A box measures 10 cm by 5 cm by 3 cm. Find the volume of the box.

$V = LWH = (10\text{ cm})(5\text{ cm})(3\text{ cm}) = (10 \cdot 5 \cdot 3)(\text{cm} \cdot \text{cm} \cdot \text{cm}) = 150\text{ cm}^3$

The volume of the box is 150 cm^3 (cubic centimeters).



➡ Find the area of a square whose side measures $(3x + 5)$ in.

$A = s^2 = [(3x + 5)\text{ in.}]^2 = (3x + 5)^2\text{ in}^2 = (9x^2 + 30x + 25)\text{ in}^2$

The area of the square is $(9x^2 + 30x + 25)\text{ in}^2$ (square inches).

Dimensional analysis is used in the conversion of units.



The following example converts the unit miles to feet. The equivalent measures 1 mi = 5280 ft are used to form the following rates, which are called conversion factors: $\frac{1\text{ mi}}{5280\text{ ft}}$ and $\frac{5280\text{ ft}}{1\text{ mi}}$. Because 1 mi = 5280 ft, both of the conversion factors $\frac{1\text{ mi}}{5280\text{ ft}}$ and $\frac{5280\text{ ft}}{1\text{ mi}}$ are equal to 1.

To convert 3 mi to feet, multiply 3 mi by the conversion factor $\frac{5280\text{ ft}}{1\text{ mi}}$.



$$3\text{ mi} = 3\text{ mi} \cdot 1 = \frac{3\text{ mi}}{1} \cdot \frac{5280\text{ ft}}{1\text{ mi}} = \frac{3\ \cancel{\text{mi}} \cdot 5280\text{ ft}}{1\ \cancel{\text{mi}}} = 3 \cdot 5280\text{ ft} = 15{,}840\text{ ft}$$

There are two important points in the above illustration. First, you can think of dividing the numerator and denominator by the common unit "mile" just as you would divide the numerator and denominator of a fraction by a common factor. Second, the conversion factor $\frac{5280\text{ ft}}{1\text{ mi}}$ is equal to 1, and multiplying an expression by 1 does not change the value of the expression.

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In the application problem that follows, the units are kept in the problem while the problem is worked.

In 2000, a horse named Fusaichi Pegasus ran a 1.25-mile race in 2.02 min. Find Fusaichi Pegasus's average speed for that race in miles per hour. Round to the nearest tenth.

Strategy To find the average speed, use the formula $r = \frac{d}{t}$, where r is the speed, d is the distance, and t is the time. Use the conversion factor $\frac{60 \text{ min}}{1 \text{ h}}$.

Solution
$$r = \frac{d}{t} = \frac{1.25 \text{ mi}}{2.02 \text{ min}} = \frac{1.25 \text{ mi}}{2.02 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ h}}$$
$$= \frac{75 \text{ mi}}{2.02 \text{ h}} \approx 37.1 \text{ mph}$$

Fusaichi Pegasus's average speed was 37.1 mph.

Try each of the following problems. Round to the nearest tenth.

1. Convert 88 ft/s to miles per hour.

2. Convert 8 m/s to kilometers per hour (1 km = 1000 m).

3. A carpet is to be placed in a meeting hall that is 36 ft wide and 80 ft long. At $29.50 per square yard, how much will it cost to carpet the meeting hall?

4. A carpet is to be placed in a room that is 20 ft wide and 30 ft long. At $32.25 per square yard, how much will it cost to carpet the area?

5. Find the number of gallons of water in a fish tank that is 36 in. long and 24 in. wide and is filled to a depth of 16 in. (1 gal = 231 in^3).

6. Find the number of gallons of water in a fish tank that is 24 in. long and 18 in. wide and is filled to a depth of 12 in. (1 gal = 231 in^3).

7. A $\frac{1}{4}$-acre commercial lot is on sale for $2.15 per square foot. Find the sale price of the commercial lot (1 acre = 43,560 ft^2).

8. A 0.75-acre industrial parcel was sold for $98,010. Find the parcel's price per square foot (1 acre = 43,560 ft^2).

9. A new driveway will require 800 ft^3 of concrete. Concrete is ordered by the cubic yard. How much concrete should be ordered?

10. A piston-engined dragster traveled 440 yd in 4.936 s at Ennis, Texas, on October 9, 1988. Find the average speed of the dragster in miles per hour.

11. The Marianas Trench in the Pacific Ocean is the deepest part of the ocean. Its depth is 6.85 mi. The speed of sound under water is 4700 ft/s. Find the time it takes sound to travel from the surface to the bottom of the Marianas Trench and back.

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Projects and Group Activities

Pascal's Triangle

Simplifying the power of a binomial is called *expanding the binomial.* The expansion of the first three powers of a binomial is shown below.

$$(a + b)^1 = a + b$$

$$(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$$

$$(a + b)^3 = (a + b)^2(a + b) = (a^2 + 2ab + b^2)(a + b) = a^3 + 3a^2b + 3ab^2 + b^3$$

Point of Interest

Pascal did not invent the triangle of numbers known as Pascal's Triangle. It was known to mathematicians in China probably as early as A.D. 1050. But Pascal's *Traite du triangle arithmetique* (*Treatise Concerning the Arithmetical Triangle*) brought together all the different aspects of the numbers for the first time.

Find $(a + b)^4$. [*Hint*: $(a + b)^4 = (a + b)^3(a + b)$]

Find $(a + b)^5$. [*Hint*: $(a + b)^5 = (a + b)^4(a + b)$]

If we continue in this way, the results for $(a + b)^6$ are

$$(a + b)^6 = a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$$

Now expand $(a + b)^8$. Before you begin, see whether you can find a pattern that will help you write the expansion of $(a + b)^8$ without having to multiply it out. Here are some hints.

1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

1. Write out the variable terms of each binomial expansion from $(a + b)^1$ through $(a + b)^6$. Observe how the exponents on the variables change.
2. Write out the coefficients of all the terms without the variable parts. It will be helpful to make a triangular arrangement as shown at the left. Note that each row begins and ends with a 1. Also note (in the two shaded regions, for example) that any number in a row is the sum of the two closest numbers above it. For instance, $1 + 5 = 6$ and $6 + 4 = 10$.

The triangle of numbers shown at the left is called Pascal's Triangle. To find the expansion of $(a + b)^8$, you need to find the eighth row of Pascal's Triangle. First find row seven. Then find row eight and use the patterns you have observed to write the expansion $(a + b)^8$.



Pascal's Triangle has been the subject of extensive analysis, and many patterns have been found. See whether you can find some of them. You might check the Internet, where you will find some web sites with information on Pascal's Triangle.

Chapter Summary

Key Words

A *monomial* is a number, a variable, or a product of a number and variables. The *degree of a monomial* is the sum of the exponents of the variables. [p. 329]

An *exponential expression is in simplest form* when it is written with only positive exponents. [p. 332]

A number written in *scientific notation* is a number written in the form $a \times 10^n$, where $1 \le a < 10$. [p. 335]

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A *polynomial* is a variable expression in which the terms are monomials. A polynomial of two terms is a *binomial.* A polynomial of three terms is a *trinomial.* The *degree of a polynomial* is the greatest of the degrees of any of its terms. A polynomial in one variable is usually written in *descending order,* which means that the terms are arranged so that the exponents of the variable decrease from left to right. [p. 341]

A *polynomial function* is an expression whose terms are monomials. A *linear function* is given by the equation $f(x) = mx + b$. A second-degree polynomial function, called a *quadratic function,* is given by the equation $f(x) = ax^2 + bx + c$, $a \neq 0$. A third-degree polynomial function is called a *cubic function*. The *leading coefficient* of a polynomial function is the coefficient of the variable with the largest exponent. [p. 341]

The product of two binomials can be found using the *FOIL* method, which is based on the Distributive Property. The letters of FOIL stand for First, Outer, Inner, and Last. [p. 350]

Synthetic division is a shorter method of dividing a polynomial by a binomial of the form $x - a$. This method uses only the coefficients of the variable terms. [p. 359]

Essential Rules

Rule for Multiplying Exponential Expressions [p. 329]
$$x^m \cdot x^n = x^{m+n}$$

Rule for Simplifying Powers of Exponential Expressions [p. 330]
$$(x^m)^n = x^{mn}$$

Rule for Simplifying Powers of Products [p. 330]
$$(x^m y^n)^p = x^{mp} y^{np}$$

Definition of Zero as an Exponent [p. 331]
If $x \neq 0$, then $x^0 = 1$.

Definition of a Negative Exponent [p. 332]
$$x^{-n} = \frac{1}{x^n} \text{ and } \frac{1}{x^{-n}} = x^n, \quad x \neq 0$$

Rule for Dividing Exponential Expressions [p. 333]
$$\frac{x^m}{x^n} = x^{m-n}, \quad x \neq 0$$

Rule for Simplifying Powers of Quotients [p. 332]
$$\left(\frac{x^m}{y^n}\right)^p = \frac{x^{mp}}{y^{np}}, \quad y \neq 0$$

The Sum and Difference of Two Terms [p. 351]
$$(a + b)(a - b) = a^2 - b^2$$

The Square of a Binomial [p. 351]
$$(a + b)^2 = a^2 + 2ab + b^2$$

Remainder Theorem [p. 362]
If the polynomial $P(x)$ is divided by $x - a$, then the remainder is $P(a)$.

Dividend = (quotient × divisor) + remainder [p. 357]

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Chapter Review

1. Multiply: $(-2a^2b^4)(3ab^2)$

2. Simplify: $(-3x^2y^3)^2$

3. Simplify: $\dfrac{(2a^4b^{-3}c^2)^3}{(2a^3b^2c^{-1})^4}$

4. Write 2.54×10^{-3} in decimal notation.

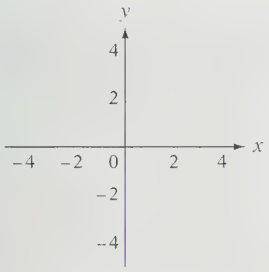
5. Given $P(x) = 2x^3 - x + 7$, find $P(-2)$.

6. Subtract: $(5x^2 - 8xy + 2y^2) - (x^2 - 3y^2)$

7. Simplify: $(2x^{-1}y^2z^5)(-3x^3yz^{-3})^2$

8. Multiply: $(2a^{12}b^3)(-9b^2c^6)(3ac)$

9. Graph: $f(x) = x^2 + 1$



10. Identify (a) the leading coefficient, (b) the constant term, and (c) the degree of the polynomial $P(x) = 3x^5 - 3x^2 + 7x + 8$.

11. Use the Remainder Theorem to evaluate $P(x) = x^3 - 2x^2 + 3x - 5$ when $x = 2$.

12. Multiply: $-2x(4x^2 + 7x - 9)$

13. Multiply: $2ab^3(4a^2 - 2ab + 3b^2)$

14. Multiply: $(3y^2 + 4y - 7)(2y + 3)$

15. Multiply: $(5a - 7)(2a + 9)$

16. Write 0.000000127 in scientific notation.

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17. Simplify: $(5y - 7)^2$

18. Divide: $\dfrac{15x^2 + 2x - 2}{3x - 2}$

19. Divide: $\dfrac{4x^3 + 27x^2 + 10x + 2}{x + 6}$

20. Divide: $\dfrac{x^4 - 4}{x - 4}$

21. Write 8.1×10^9 in decimal notation.

22. Simplify: $\dfrac{-18a^6b}{27a^3b^4}$

23. Multiply: $(5a + 2b)(5a - 2b)$

24. Use the Remainder Theorem to evaluate $P(x) = -2x^3 + 2x^2 - 4$ when $x = -3$.

25. Add: $(12y^2 + 17y - 4) + (9y^2 - 13y + 3)$

26. Multiply: $(6b^3 - 2b^2 - 5)(2b^2 - 1)$

27. Multiply: $(a + 7)(a - 7)$

28. Write 765,000,000,000 in scientific notation.

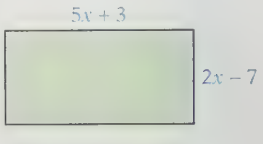
29. The length of a Ping-Pong table is 1 ft less than twice the width of the table. Let w represent the width of the Ping-Pong table. Express the area of the table in terms of the variable w.

30. The most distant object visible from Earth without the aid of a telescope is the Great Galaxy of Andromeda. It takes light from this galaxy 2.2×10^6 years to travel to Earth. Light travels about 6.7×10^8 mph. How far from Earth is the Great Galaxy of Andromeda? Use a 365-day year.

31. Write the number of seconds in one week in scientific notation.

32. The length of a side of a square checkerboard is $(3x - 2)$ in. Express the area of the checkerboard in terms of the variable x.

33. The length of a rectangle is $(5x + 3)$ cm. The width is $(2x - 7)$ cm. Find the area of the rectangle in terms of the variable x.



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Chapter Test

1. Multiply: $2x(2x^2 - 3x)$

2. Use the Remainder Theorem to evaluate $P(x) = -x^3 + 4x - 8$ when $x = -2$.

3. Simplify: $\dfrac{12x^2}{-3x^8}$

4. Simplify: $(-2xy^2)(3x^2y^4)$

5. Divide: $(x^2 + 1) \div (x + 1)$

6. Multiply: $(x - 3)(x^2 - 4x + 5)$

7. Simplify: $(-2a^2b)^3$

8. Simplify: $\dfrac{(3x^{-2}y^3)^3}{3x^4y^{-1}}$

9. Multiply: $(a - 2b)(a + 5b)$

10. Given $P(x) = 3x^2 - 8x + 1$, evaluate $P(2)$.

11. Divide: $(x^2 + 6x - 7) \div (x - 1)$

12. Multiply: $-3y^2(-2y^2 + 3y - 6)$

13. Multiply: $(-2x^3 + x^2 - 7)(2x - 3)$

14. Simplify: $(4y - 3)(4y + 3)$

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15. Simplify: $(ab^2)(a^3b^5)$

16. Simplify: $\dfrac{2a^{-1}b}{2^{-2}a^{-2}b^{-3}}$

17. Simplify: $\dfrac{(2a^{-4}b^2)^3}{4a^{-2}b^{-1}}$

18. Subtract: $(3a^2 - 2a - 7) - (5a^3 + 2a - 10)$

19. Simplify: $(2x - 5)^2$

20. Divide: $\dfrac{x^3 - 2x^2 - 5x + 7}{x + 3}$

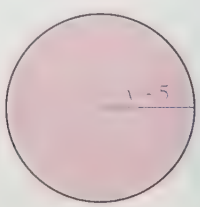
21. Multiply: $(2x - 7y)(5x - 4y)$

22. Add: $(3x^3 - 2x^2 - 4) + (8x^2 - 8x + 7)$

23. Write 0.00000000302 in scientific notation.

24. The mass of the moon is 3.7×10^{-8} times the mass of the sun. The mass of the sun is 2.19×10^{27} tons. Find the mass of the moon. Write the answer in scientific notation.

25. The radius of a circle is $(x - 5)$ m. Use the equation $A = \pi r^2$, where r is the radius, to find the area of the circle in terms of the variable x. Leave the answer in terms of π.



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Cumulative Review

1. Let $x \in \{-8, -3, 3\}$. For what values of x is the inequality $x \geq -3$ a true statement?

2. Find the additive inverse of 83.

3. Simplify: $8 - 2[-3 - (-1)]^2 \div 4$

4. Evaluate $\frac{2a - b}{b - c}$ when $a = 4$, $b = -2$, and $c = 6$.

5. Simplify: $-5\sqrt{300}$

6. Identify the property that justifies the statement $2x + (-2x) = 0$.

7. Simplify: $2x - 4[x - 2(3 - 23x) + 4]$

8. Solve: $\frac{2}{3} - y = \frac{5}{6}$

9. Solve: $8x - 3 - x = -6 + 3x - 8$

10. Solve: $3 - |2 - 3x| = -2$

11. Given $P(x) = 3x^2 - 2x + 2$, find $P(-2)$.

12. Is the relation $\{(-1, 0), (0, 0), (1, 0)\}$ a function?

13. Find the slope of the line containing the points $(-2, 3)$ and $(4, 2)$.

14. Find the equation of the line that contains the point $(-1, 2)$ and has slope $-\frac{3}{2}$.

15. Find the equation of the line that contains the point $(-2, 4)$ and is perpendicular to the line $3x + 2y = 4$.

16. Solve by using Cramer's Rule:

$$2x - 3y = 2$$
$$x + y = -3$$

17. Solve by the addition method:

$$x - y + z = 0$$
$$2x + y - 3z = -7$$
$$-x + 2y + 2z = 5$$

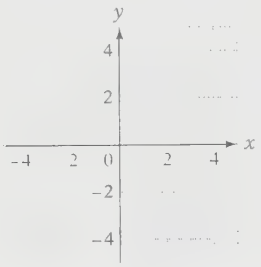
18. Simplify: $-2x - (-xy) + 7x - 4xy$

19. Multiply: $(2x + 3)(2x^2 - 3x + 1)$

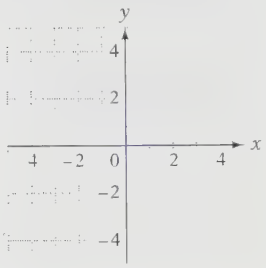
20. Write the number 0.00000501 in scientific notation.

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21. Graph: $3x - 4y = 12$



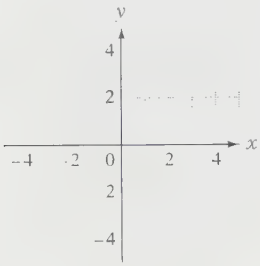
22. Graph: $-3x + 2y < 6$



23. Solve by graphing:

$x - 2y = 3$

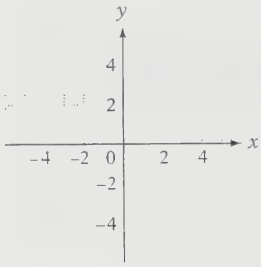
$-2x + y = -3$



24. Graph the solution set:

$2x + y < 2$

$-6x + 3y \geq 6$



25. Simplify: $(4a^{-2}b^3)(2a^3b^{-1})^{-2}$

26. Simplify: $\dfrac{(5x^3y^{-3}z)^{\,2}}{y^4z^{-2}}$

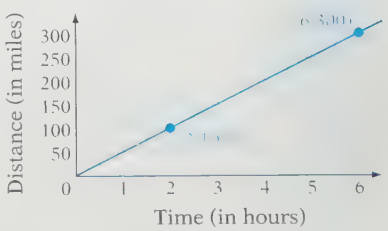
27. The sum of two integers is twenty-four. The difference between four times the smaller integer and nine is three less than twice the larger integer. Find the integers.

28. How many ounces of pure gold that costs \$360 per ounce must be mixed with 80 oz of an alloy that costs \$120 per ounce to make a mixture that costs \$200 per ounce?

29. Two bicycles are 25 mi apart and traveling toward each other. One cyclist is traveling at 1.5 times the rate of the other cyclist. They meet in 2 h. Find the rate of each cyclist.

30. If \$3000 is invested at an annual simple interest rate of 7.5%, how much additional money must be invested at an annual simple interest rate of 10% so that the total interest earned in one year is 9% of the total investment?

31. The graph shows the relationship between the distance traveled and the time of travel. Find the slope of the line between the two points on the graph. Write a sentence that states the meaning of the slope.



32. The width of a rectangle is 40% of the length. The perimeter of the rectangle is 42 m. Find the length and width of the rectangle.

33. The length of a side of a square is $(2x + 3)$ m. Find the area of the square in terms of the variable x.

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Chapter 7 Factoring



Coordinating the position of each skydiver in a jump involves a lot of careful planning. Position within the "pattern" is determined by when and where each chute is deployed, which is further dependent upon each diver's velocity during free fall. Factors such as the initial altitude of the jump, the size of the parachute, and the weight and body position of the diver affect velocity. The velocity of a falling object can be modeled by a quadratic equation, as shown in **Exercises 72 and 73 on page 417.**

Objectives

Section 7.1

A To factor a monomial from a polynomial
B To factor by grouping

Section 7.2

A To factor a trinomial of the form $x^2 + bx + c$
B To factor completely

Section 7.3

A To factor a trinomial of the form $ax^2 + bx + c$ by using trial factors
B To factor a trinomial of the form $ax^2 + bx + c$ by grouping

Section 7.4

A To factor the difference of two perfect squares or a perfect-square trinomial
B To factor the sum or the difference of two cubes
C To factor a trinomial that is quadratic in form
D To factor completely

Section 7.5

A To solve equations by factoring
B To solve application problems

WEB Need help? For on-line student resources, such as section quizzes, visit this textbook's web site at **college.hmco.com/students.**

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Prep Test

1. Write 30 as a product of prime numbers.

2. Simplify: $-3(4y - 5)$

3. Simplify: $-(a - b)$

4. $2(a - b) - 5(a - b)$

5. Solve: $4x = 0$

6. Solve: $2x + 1 = 0$

7. Multiply: $(x + 4)(x - 6)$

8. Multiply: $(2x - 5)(3x + 2)$

9. Simplify: $\frac{x^5}{x^2}$

10. Simplify: $\frac{6x^4y^3}{2xy^2}$

Go Figure



Find the values of x and y if $x + y$, xy, and $\frac{x}{y}$ all equal the same number.

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7.1 Common Factors

Objective A

To factor a monomial from a polynomial

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The **greatest common factor (GCF)** of two or more monomials is the product of the GCF of the coefficients and the common variable factors.

$6x^3y = 2 \cdot 3 \cdot x \cdot x \cdot x \cdot y$
$8x^2y^2 = 2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot y \cdot y$
$\text{GCF} = 2 \cdot x \cdot x \cdot y = 2x^2y$

Note that the exponent of each variable in the GCF is the same as the *smallest* exponent of that variable in either of the monomials.

The GCF of $6x^3y$ and $8x^2y^2$ is $2x^2y$.

➨ Find the GCF of $12a^4b$ and $18a^2b^2c$.

The common variable factors are a^2 and b; c is not a common variable factor.

$12a^4b = 2 \cdot 2 \cdot 3 \cdot a^4 \cdot b$
$18a^2b^2c = 2 \cdot 3 \cdot 3 \cdot a^2 \cdot b^2 \cdot c$
$\text{GCF} = 2 \cdot 3 \cdot a^2 \cdot b = 6a^2b$

TAKE NOTE
$2x^2 + 10x$ is a sum. $2x(x + 5)$ is a product. When we factor a polynomial, we write it as a product.

To **factor** a polynomial means to write the polynomial as a product of other polynomials.

Multiply

Polynomial $2x^2 + 10x$ = **Factors** $2x(x + 5)$

Factor

In the example above, $2x$ is the GCF of the terms $2x^2$ and $10x$. It is a **common factor** of the terms.

➨ Factor: $5x^3 - 35x^2 + 10x$

Find the GCF of the terms of the polynomial.

$5x^3 = 5 \cdot x^3$ $\quad 35x^2 = 5 \cdot 7 \cdot x^2$ $\quad 10x = 2 \cdot 5 \cdot x$

The GCF is $5x$.

Rewrite the polynomial, expressing each term as a product with the GCF as one of the factors.

$5x^3 - 35x^2 + 10x = 5x(x^2) + 5x(-7x) + 5x(2)$
$= 5x(x^2 - 7x + 2)$

• Use the Distributive Property to write the polynomial as a product of factors.

TAKE NOTE
At the right, the factors in parentheses are determined by dividing each term of the trinomial by the GCF, $5x$.

$\left(\frac{5x^3}{5x}\right) = x^2$,

$\left(\frac{-35x^2}{5x}\right) = -7x$, and

$\left(\frac{10x}{5x}\right) = 2$.

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Example 1

Factor: $8x^2 + 2xy$

Solution

The GCF is $2x$.

$$8x^2 + 2xy = 2x(4x) + 2x(y)$$
$$= 2x(4x + y)$$

You Try It 1

Factor: $14a^2 - 21a^4b$

Your solution

Example 2

Factor: $n^3 - 5n^2 + 2n$

Solution

The GCF is n.

$$n^3 - 5n^2 + 2n = n(n^2) + n(-5n) + n(2)$$
$$= n(n^2 - 5n + 2)$$

You Try It 2

Factor: $27b^2 + 18b + 9$

Your solution

Example 3

Factor: $16x^2y + 8x^4y^2 - 12x^4y^5$

Solution

The GCF is $4x^2y$.

$$16x^2y + 8x^4y^2 - 12x^4y^5$$
$$= 4x^2y(4) + 4x^2y(2x^2y) + 4x^2y(-3x^2y^4)$$
$$= 4x^2y(4 + 2x^2y - 3x^2y^4)$$

You Try It 3

Factor: $6x^4y^2 - 9x^3y^2 + 12x^2y^4$

Your solution

Example 4

Factor: $x^{5n} - 2x^{3n}$

Solution

The GCF is x^{3n}.

$$x^{5n} - 2x^{3n} = x^{3n}(x^{2n}) - x^{3n}(2)$$
$$= x^{3n}(x^{2n} - 2)$$

You Try It 4

Factor: $a^{n+4} + a^2$

Your solution

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To factor by grouping

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In the examples at the right, the binomials in parentheses are called **binomial factors**.

$2a(a + b)^2$
$3xy(x - y)$

The Distributive Property is used to factor a common binomial factor from an expression.

The common binomial factor of the expression $6x(x - 3) + y^2(x - 3)$ is $(x - 3)$. To factor that expression, use the Distributive Property to write the expression as a product of factors.

$$6x\underbrace{(x - 3)} + y^2\underbrace{(x - 3)} = \underbrace{(x - 3)}(6x + y^2)$$

Consider the following simplification of $-(a - b)$.

$$-(a - b) = -1(a - b) = -a + b = b - a$$

Thus, $$\boldsymbol{b - a = -(a - b)}$$

This equation is sometimes used to factor a common binomial from an expression.

➡ Factor: $2x(x - y) + 5(y - x)$

$$2x(x - y) + 5(y - x) = 2x(x - y) - 5(x - y)$$
$$= (x - y)(2x - 5)$$

- $5(y - x) = 5[(-1)(x - y)]$
 $= -5(x - y)$

Some polynomials can be factored by grouping terms in such a way that a common binomial factor is found.

➡ Factor: $ax + bx - ay - by$

$$ax + bx - ay - by = (ax + bx) - (ay + by)$$

- Group the first two terms and the last two terms. Note that $-ay - by = -(ay + by)$.

$$= x(a + b) - y(a + b)$$
$$= (a + b)(x - y)$$

- Factor the GCF from each group.

➡ Factor: $6x^2 - 9x - 4xy + 6y$

$$6x^2 - 9x - 4xy + 6y = (6x^2 - 9x) - (4xy - 6y)$$

- Group the first two terms and the last two terms. Note that $-4xy + 6y = -(4xy - 6y)$.

$$= 3x(2x - 3) - 2y(2x - 3)$$
$$= (2x - 3)(3x - 2y)$$

- Factor the GCF from each group.

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Example 5
Factor: $4x(3x - 2) - 7(3x - 2)$

Solution
$4x(3x - 2) - 7(3x - 2)$
$= (3x - 2)(4x - 7)$

You Try It 5
Factor: $2y(5x - 2) - 3(2 - 5x)$

Your solution

Example 6
Factor: $9x^2 - 15x - 6xy + 10y$

Solution
$9x^2 - 15x - 6xy + 10y$
$= (9x^2 - 15x) - (6xy - 10y)$
$= 3x(3x - 5) - 2y(3x - 5)$
$= (3x - 5)(3x - 2y)$

You Try It 6
Factor: $a^2 - 3a + 2ab - 6b$

Your solution

Example 7
Factor: $3x^2y - 4x - 15xy + 20$

Solution
$3x^2y - 4x - 15xy + 20$
$= (3x^2y - 4x) - (15xy - 20)$
$= x(3xy - 4) - 5(3xy - 4)$
$= (3xy - 4)(x - 5)$

You Try It 7
Factor: $2mn^2 - n + 8mn - 4$

Your solution

Example 8
Factor: $4ab - 6 + 3b - 2ab^2$

Solution
$4ab - 6 + 3b - 2ab^2$
$= (4ab - 6) + (3b - 2ab^2)$
$= 2(2ab - 3) + b(3 - 2ab)$
$= 2(2ab - 3) - b(2ab - 3)$
$= (2ab - 3)(2 - b)$

You Try It 8
Factor: $3xy - 9y - 12 + 4x$

Your solution

Solutions on p. S20

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7.1 Exercises

Objective A

Factor.

1. $5a + 5$ **2.** $7b - 7$ **3.** $16 - 8a^2$ **4.** $12 + 12y^2$ **5.** $8x + 12$

6. $16a - 24$ **7.** $30a - 6$ **8.** $20b + 5$ **9.** $7x^2 - 3x$ **10.** $12y^2 - 5y$

11. $3a^2 + 5a^5$ **12.** $9x - 5x^2$ **13.** $14y^2 + 11y$ **14.** $6b^3 - 5b^2$ **15.** $2x^4 - 4x$

16. $3y^4 - 9y$ **17.** $10x^4 - 12x^2$ **18.** $12a^5 - 32a^2$ **19.** $8a^8 - 4a^5$ **20.** $16y^4 - 8y^7$

21. $x^2y^2 - xy$ **22.** $a^2b^2 + ab$ **23.** $3x^2y^4 - 6xy$ **24.** $12a^2b^5 - 9ab$ **25.** $x^2y - xy^3$

26. $3x^3 + 6x^2 + 9x$ **27.** $5y^3 - 20y^2 + 10y$ **28.** $2x^4 - 4x^3 + 6x^2$ **29.** $3y^4 - 9y^3 - 6y^2$

30. $2x^3 + 6x^2 - 14x$ **31.** $3y^3 - 9y^2 + 24y$ **32.** $2y^5 - 3y^4 + 7y^3$ **33.** $6a^5 - 3a^3 - 2a^2$

34. $x^3y - 3x^2y^2 + 7xy^3$ **35.** $2a^2b - 5a^2b^2 + 7ab^2$ **36.** $5y^3 + 10y^2 - 25y$

37. $4b^5 + 6b^3 - 12b$ **38.** $3a^2b^2 - 9ab^2 + 15b^2$ **39.** $8x^2y^2 - 4x^2y + x^2$

40. $x^{2n} - x^n$ **41.** $2a^{5n} + a^{2n}$ **42.** $x^{3n} - x^{2n}$

43. $y^{4n} + y^{2n}$ **44.** $a^{2n+2} + a^2$ **45.** $b^{n+5} - b^5$

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Factor.

46. $x(b + 4) + 3(b + 4)$

47. $y(a + z) + 7(a + z)$

48. $a(y - x) - b(y - x)$

49. $3r(a - b) + s(a - b)$

50. $x(x - 2) + y(2 - x)$

51. $t(m - 7) + 7(7 - m)$

52. $2x(7 + b) - y(b + 7)$

53. $2y(4a - b) - (b - 4a)$

54. $8c(2m - 3n) + (3n - 2m)$

55. $x^2 + 2x + 2xy + 4y$

56. $x^2 - 3x + 4ax - 12a$

57. $p^2 - 2p - 3rp + 6r$

58. $t^2 + 4t - st - 4s$

59. $ab + 6b - 4a - 24$

60. $xy - 5y - 2x + 10$

61. $2z^2 - z + 2yz - y$

62. $2y^2 - 10y + 7xy - 35x$

63. $8v^2 - 12vy + 14v - 21y$

64. $21x^2 + 6xy - 49x - 14y$

65. $2x^2 - 5x - 6xy + 15y$

66. $4a^2 + 5ab - 10b - 8a$

67. $3y^2 - 6y - ay + 2a$

68. $2ar + a^2 - 2r - a$

69. $3xy - y^2 - y + 3x$

70. $2ab - 3b^2 - 3b + 2a$

71. $3st + t^2 - 2t - 6s$

72. $4x^2 + 3xy - 12y - 16x$

APPLYING THE CONCEPTS

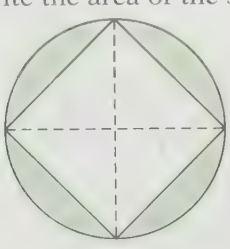
73. A whole number is a perfect number if it is the sum of all of its factors less than itself. For example, 6 is a perfect number because all the factors of 6 that are less than 6 are 1, 2, and 3, and $1 + 2 + 3 = 6$.

a. Find the one perfect number between 20 and 30.

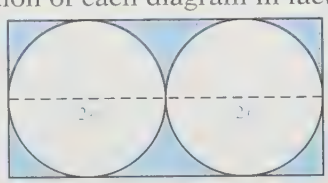
b. Find the one perfect number between 490 and 500.

74. Write the area of the shaded portion of each diagram in factored form.

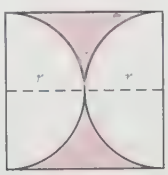
a.



b.



c.



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7.2 Factoring Polynomials of the Form $x^2 + bx + c$

Objective A

To factor a trinomial of the form $x^2 + bx + c$

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Trinomials of the form $x^2 + bx + c$, where b and c are integers, are shown at the right.

$x^2 + 8x + 12;\ b = 8,\ c = 12$
$x^2 - 7x + 12;\ b = -7,\ c = 12$
$x^2 - 2x - 15;\ b = -2,\ c = -15$

To factor a trinomial of this form means to express the trinomial as the product of two binomials.

Trinomials expressed as the product of binomials are shown at the right.

$x^2 + 8x + 12 = (x + 6)(x + 2)$
$x^2 - 7x + 12 = (x - 3)(x - 4)$
$x^2 - 2x - 15 = (x + 3)(x - 5)$

The method by which factors of a trinomial are found is based on FOIL. Consider the following binomial products, noting the relationship between the constant terms of the binomials and the terms of the trinomials.

The signs in the binomials are the same.

$(x + 6)(x + 2) = x^2 + 2x + 6x + (6)(2) = x^2 + 8x + 12$
sum of 6 and 2
product of 6 and 2

$(x - 3)(x - 4) = x^2 - 4x - 3x + (-3)(-4) = x^2 - 7x + 12$
sum of -3 and -4
product of -3 and -4

The signs in the binomials are opposite.

$(x + 3)(x - 5) = x^2 - 5x + 3x + (3)(-5) = x^2 - 2x - 15$
sum of 3 and -5
product of 3 and -5

$(x - 4)(x + 6) = x^2 + 6x - 4x + (-4)(6) = x^2 + 2x - 24$
sum of -4 and 6
product of -4 and 6

IMPORTANT RELATIONSHIPS

1. When the constant term of the trinomial is positive, the constant terms of the binomials have the same sign. They are both positive when the coefficient of the x term in the trinomial is positive. They are both negative when the coefficient of the x term in the trinomial is negative.
2. When the constant term of the trinomial is negative, the constant terms of the binomials have opposite signs.
3. In the trinomial, the coefficient of x is the sum of the constant terms of the binomials.
4. In the trinomial, the constant term is the product of the constant terms of the binomials.

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Factor: $x^2 - 7x + 10$

Because the constant term is positive and the coefficient of x is negative, the binomial constants will be negative. Find two negative factors of 10 whose sum is -7. The results can be recorded in a table.

Negative Factors of 10	*Sum*
$-1, -10$	-11
$-2, -5$	-7

- These are the correct factors.

$x^2 - 7x + 10 = (x - 2)(x - 5)$

- Write the trinomial as a product of its factors.

Check: $(x - 2)(x - 5) = x^2 - 5x - 2x + 10$
$= x^2 - 7x + 10$

- Check the proposed factorization by multiplying the two binomials.

TAKE NOTE
Always check your proposed factorization to ensure its accuracy.

Factor: $x^2 - 9x - 36$

The constant term is negative. The binomial constants will have opposite signs. Find two factors of -36 whose sum is -9.

Factors of -36	*Sum*
$+1, -36$	-35
$-1, +36$	35
$+2, -18$	-16
$-2, +18$	16
$+3, -12$	-9

- Once the correct factors are found, it is not necessary to try the remaining factors.

$x^2 - 9x - 36 = (x + 3)(x - 12)$

- Write the trinomial as a product of its factors.

Factor: $x^2 + 7x + 8$

Because the constant term is positive and the coefficient of x is positive, the binomial constants will be positive. Find two positive factors of 8 whose sum is 7.

Positive Factors of 8	*Sum*
1, 8	9
2, 4	6

There are no positive integer factors of 8 whose sum is 7. The trinomial $x^2 + 7x + 8$ is said to be **nonfactorable over the integers**. Just as 17 is a prime number, $x^2 + 7x + 8$ is a **prime polynomial**. Binomials of the form $x - a$ and $x + a$ are also prime polynomials.

Example 5
Factor: $x^2 - 8x + 15$

Solution

Factors	*Sum*
$-1, -15$	-16
$-3, -5$	-8

- Find two negative factors of 15 whose sum is -8.

$x^2 - 8x + 15 = (x - 3)(x - 5)$

You Try It 1
Factor: $x^2 + 9x + 20$

Your solution

Solution on p. S20

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Example 2
Factor: $x^2 + 6x - 27$

Solution

Factors	*Sum*
$+1, -27$	-26
$-1, +27$	26
$+3, -9$	-6

- Find two factors of -27 whose sum is 6.

$x^2 + 6x - 27 = (x - 3)(x + 9)$

You Try It 2
Factor: $x^2 + 7x - 18$

Your solution

Solution on p. S20

Objective B

To factor completely

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A polynomial is factored completely when it is written as a product of factors that are nonfactorable over the integers.

TAKE NOTE
The first step in *any* factoring problem is to determine whether the terms of the polynomial have a **common factor.** If they do, factor it out first.

➨ Factor: $4y^3 - 4y^2 - 24y$

$$4y^3 - 4y^2 - 24y = \boxed{4y(y^2) - 4y(y) - 4y(6)}$$
$$= 4y(y^2 - y - 6)$$
$$= 4y(y + 2)(y - 3)$$

- The GCF is $4y$. Do this step mentally.
- Use the Distributive Property to factor out the GCF.
- Factor $y^2 - y - 6$. The two factors of -6 whose sum is -1 are 2 and -3.

It is always possible to check the proposed factorization by multiplying the polynomials. Here is the check for the last example.

Check: $4y(y + 2)(y - 3) = 4y(y^2 - 3y + 2y - 6)$
$= 4y(y^2 - y - 6)$
$= 4y^3 - 4y^2 - 24y$

- This is the original polynomial.

➨ Factor: $5x^2 + 60xy + 100y^2$

$$5x^2 + 60xy + 100y^2 = \boxed{5(x^2) + 5(12xy) + 5(20y^2)}$$
$$= 5(x^2 + 12xy + 20y^2)$$
$$= 5(x + 2y)(x + 10y)$$

- The GCF is 5. Do this step mentally.
- Use the Distributive Property to factor out the GCF.
- Factor $x^2 + 12xy + 20y^2$. The two factors of 20 whose sum is 12 are 2 and 10.

TAKE NOTE
$2y$ and $10y$ are placed in the binomials. This is necessary so that the middle term contains xy and the last term contains y^2.

Note that $2y$ and $10y$ were placed in the binomials. The following check shows that this was necessary.

Check: $5(x + 2y)(x + 10y) = 5(x^2 + 10xy + 2xy + 20y^2)$
$= 5(x^2 + 12xy + 20y^2)$
$= 5x^2 + 60xy + 100y^2$

- The original polynomial

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Factor: $15 - 2x - x^2$

TAKE NOTE
When the coefficient of the highest power in a polynomial is negative, consider factoring out a negative GCF. Example 3 below is another example of this technique.

Because the coefficient of x^2 is -1, factor -1 from the trinomial and then write the resulting trinomial in descending order.

$15 - 2x - x^2 = -(x^2 + 2x - 15)$ • $15 - 2x - x^2 = -1(-15 + 2x + x^2)$ $= -(x^2 + 2x - 15)$

$= -(x + 5)(x - 3)$ • Factor $x^2 + 2x - 15$. The two factors of -15 whose sum is 2 are 5 and -3.

Check: $-(x + 5)(x - 3) = -(x^2 + 2x - 15)$

$= -x^2 - 2x + 15$

$= 15 - 2x - x^2$ • The original polynomial

Example 3
Factor: $-3x^3 + 9x^2 + 12x$

Solution
The GCF is $-3x$.

$-3x^3 + 9x^2 + 12x = -3x(x^2 - 3x - 4)$

Factor the trinomial $x^2 - 3x - 4$. Find two factors of -4 whose sum is -3.

Factors	*Sum*
$-2, +2$	0
$+1, -4$	-3

$-3x^3 + 9x^2 + 12x = -3x(x + 1)(x - 4)$

You Try It 3
Factor: $-2x^3 + 14x^2 - 12x$

Your solution

Example 4
Factor: $4x^2 - 40xy + 84y^2$

Solution
The GCF is 4.

$4x^2 - 40xy + 84y^2 = 4(x^2 - 10xy + 21y^2)$

Factor the trinomial $x^2 - 10xy + 21y^2$. Find two negative factors of 21 whose sum is -10.

Factors	*Sum*
$-1, -21$	-22
$-3, -7$	-10

$4x^2 - 40xy + 84y^2 = 4(x - 3y)(x - 7y)$

You Try It 4
Factor: $3x^2 - 9xy - 12y^2$

Your solution

Solutions on p. S20

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7.2 Exercises

Objective A

Factor.

1. $x^2 + 3x + 2$

2. $x^2 + 5x + 6$

3. $x^2 - x - 2$

4. $x^2 + x - 6$

5. $a^2 + a - 12$

6. $a^2 - 2a - 35$

7. $a^2 - 3a + 2$

8. $a^2 - 5a + 4$

9. $a^2 + a - 2$

10. $a^2 - 2a - 3$

11. $b^2 - 6b + 9$

12. $b^2 + 8b + 16$

13. $b^2 + 7b - 8$

14. $y^2 - y - 6$

15. $y^2 + 6y - 55$

16. $z^2 - 4z - 45$

17. $y^2 - 5y + 6$

18. $y^2 - 8y + 15$

19. $z^2 - 14z + 45$

20. $z^2 - 14z + 49$

21. $z^2 - 12z - 160$

22. $p^2 + 2p - 35$

23. $p^2 + 12p + 27$

24. $p^2 - 6p + 8$

25. $x^2 + 20x + 100$

26. $x^2 + 18x + 81$

27. $b^2 + 9b + 20$

28. $b^2 + 13b + 40$

29. $x^2 - 11x - 42$

30. $x^2 + 9x - 70$

31. $b^2 - b - 20$

32. $b^2 + 3b - 40$

33. $y^2 - 14y - 51$

34. $y^2 - y - 72$

35. $p^2 - 4p - 21$

36. $p^2 + 16p + 39$

37. $y^2 - 8y + 32$

38. $y^2 - 9y + 81$

39. $x^2 - 20x + 75$

40. $p^2 + 24p + 63$

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41. $x^2 - 15x + 56$ **42.** $x^2 + 21x + 38$ **43.** $x^2 + x - 56$ **44.** $x^2 + 5x - 36$

45. $a^2 - 21a - 72$ **46.** $a^2 - 7a - 44$ **47.** $a^2 - 15a + 36$ **48.** $a^2 - 21a + 54$

49. $z^2 - 9z - 136$ **50.** $z^2 + 14z - 147$ **51.** $c^2 - c - 90$ **52.** $c^2 - 3c - 180$

53. $z^2 + 15z + 44$ **54.** $p^2 + 24p + 135$ **55.** $c^2 + 19c + 34$ **56.** $c^2 + 11c + 18$

57. $x^2 - 4x - 96$ **58.** $x^2 + 10x - 75$ **59.** $x^2 - 22x + 112$ **60.** $x^2 + 21x - 100$

61. $b^2 + 8b - 105$ **62.** $b^2 - 22b + 72$ **63.** $a^2 - 9a - 36$ **64.** $a^2 + 42a - 135$

65. $b^2 - 23b + 102$ **66.** $b^2 - 25b + 126$ **67.** $a^2 + 27a + 72$ **68.** $z^2 + 24z + 144$

69. $x^2 + 25x + 156$ **70.** $x^2 - 29x + 100$ **71.** $x^2 - 10x - 96$ **72.** $x^2 + 9x - 112$

Factor.

73. $2x^2 + 6x + 4$ **74.** $3x^2 + 15x + 18$ **75.** $18 + 7x - x^2$

76. $12 - 4x - x^2$ **77.** $ab^2 + 2ab - 15a$ **78.** $ab^2 + 7ab - 8a$

79. $xy^2 - 5xy + 6x$ **80.** $xy^2 + 8xy + 15x$ **81.** $z^3 - 7z^2 + 12z$

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82. $-2a^3 - 6a^2 - 4a$

83. $-3y^3 + 15y^2 - 18y$

84. $4y^3 + 12y^2 - 72y$

85. $3x^2 + 3x - 36$

86. $2x^3 - 2x^2 + 4x$

87. $5z^2 - 15z - 140$

88. $6z^2 + 12z - 90$

89. $2a^3 + 8a^2 - 64a$

90. $3a^3 - 9a^2 - 54a$

91. $x^2 - 5xy + 6y^2$

92. $x^2 + 4xy - 21y^2$

93. $a^2 - 9ab + 20b^2$

94. $a^2 - 15ab + 50b^2$

95. $x^2 - 3xy - 28y^2$

96. $s^2 + 2st - 48t^2$

97. $y^2 - 15yz - 41z^2$

98. $y^2 + 85yz + 36z^2$

99. $z^4 - 12z^3 + 35z^2$

100. $z^4 + 2z^3 - 80z^2$

101. $b^4 - 22b^3 + 120b^2$

102. $b^4 - 3b^3 - 10b^2$

103. $2y^4 - 26y^3 - 96y^2$

104. $3y^4 + 54y^3 + 135y^2$

105. $-x^4 - 7x^3 + 8x^2$

106. $-x^4 + 11x^3 + 12x^2$

107. $4x^2y + 20xy - 56y$

108. $3x^2y - 6xy - 45y$

109. $c^3 + 18c^2 - 40c$

110. $-3x^3 + 36x^2 - 81x$

111. $-4x^3 - 4x^2 + 24x$

112. $x^2 - 8xy + 15y^2$

113. $y^2 - 7xy - 8x^2$

114. $a^2 - 13ab + 42b^2$

115. $y^2 + 4yz - 21z^2$

116. $y^2 + 8yz + 7z^2$

117. $y^2 - 16yz + 15z^2$

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118. $3x^2y + 60xy - 63y$

119. $4x^2y - 68xy - 72y$

120. $3x^3 + 3x^2 - 36x$

121. $4x^3 + 12x^2 - 160x$

122. $4z^3 + 32z^2 - 132z$

123. $5z^3 - 50z^2 - 120z$

124. $4x^3 + 8x^2 - 12x$

125. $5x^3 + 30x^2 + 40x$

126. $5p^2 + 25p - 420$

127. $4p^2 - 28p - 480$

128. $p^4 + 9p^3 - 36p^2$

129. $p^4 + p^3 - 56p^2$

130. $t^2 - 12ts + 35s^2$

131. $a^2 - 10ab + 25b^2$

132. $a^2 - 8ab - 33b^2$

133. $x^2 + 4xy - 60y^2$

134. $5x^4 - 30x^3 + 40x^2$

135. $6x^3 - 6x^2 - 120x$

APPLYING THE CONCEPTS

Factor.

136. $2 + c^2 + 9c$

137. $x^2y - 54y - 3xy$

138. $45a^2 + a^2b^2 - 14a^2b$

Find all integers k such that the trinomial can be factored over the integers.

139. $x^2 + kx + 35$

140. $x^2 + kx + 18$

141. $x^2 + kx + 21$

Determine the positive integer values of k for which the following polynomials are factorable over the integers.

142. $y^2 + 4y + k$

143. $z^2 + 7z + k$

144. $a^2 - 6a + k$

145. $c^2 - 7c + k$

146. $x^2 - 3x + k$

147. $y^2 + 5y + k$

148. In Exercises 142 to 147, there was the stated requirement that $k > 0$. If k is allowed to be any integer, how many different values of k are possible for each polynomial?

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7.3 Factoring Polynomials of the Form $ax^2 + bx + c$

Objective A

To factor a trinomial of the form $ax^2 + bx + c$ by using trial factors

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Trinomials of the form $ax^2 + bx + c$, where a, b, and c are integers, are shown at the right.

$3x^2 - x + 4$; $a = 3, b = -1, c = 4$
$6x^2 + 2x - 3$; $a = 6, b = 2, c = -3$

These trinomials differ from those in the previous section in that the coefficient of x^2 is not 1. There are various methods of factoring these trinomials. The method described in this objective is factoring polynomials using trial factors.

To reduce the number of trial factors that must be considered, remember the following:

1. Use the signs of the constant term and the coefficient of x in the trinomial to determine the signs of the binomial factors. If the constant term is positive, the signs of the binomial factors will be the same as the sign of the coefficient of x in the trinomial. If the sign of the constant term is negative, the constant terms in the binomials have opposite signs.
2. If the terms of the trinomial do not have a common factor, then the terms of neither of the binomial factors will have a common factor.

Factor: $2x^2 - 7x + 3$

The terms have no common factor. The constant term is positive. The coefficient of x is negative. The binomial constants will be negative.

Positive Factors of 2 (coefficient of x^2)	*Negative Factors of 3* (constant term)
1, 2	$-1, -3$

Write trial factors. Use the **O**uter and **I**nner products of FOIL to determine the middle term, $-7x$, of the trinomial.

Trial Factors	*Middle Term*
$(x - 1)(2x - 3)$	$-3x - 2x = -5x$
$(x - 3)(2x - 1)$	$-x - 6x = -7x$

Write the factors of the trinomial. $2x^2 - 7x + 3 = (x - 3)(2x - 1)$

Factor: $3x^2 - 8x + 4$

The terms have no common factor. The constant term is positive. The coefficient of x is negative. The binomial constants will be negative.

Positive Factors of 3 (coefficient of x^2)	*Negative Factors of 4* (constant term)
1, 3	$-1, -4$ $-2, -2$

Write trial factors. Use the **O**uter and **I**nner products of FOIL to determine the middle term, $-8x$, of the trinomial.

Trial Factors	*Middle Term*
$(x - 1)(3x - 4)$	$-4x - 3x = -7x$
$(x - 4)(3x - 1)$	$-x - 12x = -13x$
$(x - 2)(3x - 2)$	$-2x - 6x = -8x$

Write the factors of the trinomial. $3x^2 - 8x + 4 = (x - 2)(3x - 2)$

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Factor: $6x^3 + 14x^2 - 12x$

Factor the GCF, $2x$, from the terms. $6x^3 + 14x^2 - 12x = 2x(3x^2 + 7x - 6)$

Factor the trinomial $3x^2 + 7x - 6$. The constant term is negative. The binomial constants will have opposite signs.

Positive Factors of 3	*Factors of −6*
1, 3	−1, 6
	1, −6
	−2, 3
	2, −3

TAKE NOTE

The binomial factor $3x + 6$ has a common factor of 3: $3x + 6 = 3(x + 2)$. Because $3x^2 + 7x - 6$ does not have a common factor, one of its binomial factors cannot have a common factor. For the examples and solutions in this text, all trial factors except those that have a common factor will be listed.

Write trial factors. Use the **O**uter and **I**nner products of FOIL to determine the middle term, $7x$, of the trinomial.

It is not necessary to test trial factors that have a common factor.

Trial Factors	*Middle Term*
$(x - 1)(3x + 6)$	Common factor
$(x + 6)(3x - 1)$	$-x + 18x = 17x$
$(x + 1)(3x - 6)$	Common factor
$(x - 6)(3x + 1)$	$x - 18x = -17x$
$(x - 2)(3x + 3)$	Common factor
$(x + 3)(3x - 2)$	$-2x + 9x = 7x$
$(x + 2)(3x - 3)$	Common factor
$(x - 3)(3x + 2)$	$2x - 9x = -7x$

Write the factors of the trinomial. $6x^3 + 14x^2 - 12x = 2x(x + 3)(3x - 2)$

For this example, all the trial factors were listed. Once the correct factors have been found, however, the remaining trial factors need not be checked.

Example 1

Factor: $3x^2 + x - 2$

Solution

Positive factors of 3: 1, 3

Factors of −2: 1, −2; −1, 2

Trial Factors	*Middle Term*
$(x + 1)(3x - 2)$	$-2x + 3x = x$
$(x - 2)(3x + 1)$	$x - 6x = -5x$
$(x - 1)(3x + 2)$	$2x - 3x = -x$
$(x + 2)(3x - 1)$	$-x + 6x = 5x$

$3x^2 + x - 2 = (x + 1)(3x - 2)$

You Try It 1

Factor: $2x^2 - x - 3$

Your solution

Example 2

Factor: $-12x^3 - 32x^2 + 12x$

Solution

The GCF is $-4x$.

$-12x^3 - 32x^2 + 12x = -4x(3x^2 + 8x - 3)$

Factor the trinomial.

Positive factors of 3: 1, 3

Factors of −3: 1, −3; −1, 3

Trial Factors	*Middle Term*
$(x - 3)(3x + 1)$	$x - 9x = -8x$
$(x + 3)(3x - 1)$	$-x + 9x = 8x$

$-12x^3 - 32x^2 + 12x = -4x(x + 3)(3x - 1)$

You Try It 2

Factor: $-45y^3 + 12y^2 + 12y$

Your solution

Solutions on pp. S20–S21

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Objective b

To factor a trinomial of the form $ax^2 + bx + c$ by grouping

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In the previous objective, trinomials of the form $ax^2 + bx + c$ were factored by using trial factors. In this objective, these trinomials will be factored by grouping.

To factor $ax^2 + bx + c$, first find two factors of $a \cdot c$ whose sum is b. Then use factoring by grouping to write the factorization of the trinomial.

➡ Factor: $2x^2 + 13x + 15$

Find two positive factors of 30 $(2 \cdot 15)$ whose sum is 13.

Positive Factors of 30	*Sum*
1, 30	31
2, 15	17
3, 10	13

- When the required sum has been found, the remaining factors need not be checked.

$$2x^2 + 13x + 15 = 2x^2 + 3x + 10x + 15$$

- Use the factors of 30 whose sum is 13 to write $13x$ as $3x + 10x$.

$$= (2x^2 + 3x) + (10x + 15)$$

- Factor by grouping.

$$= x(2x + 3) + 5(2x + 3)$$
$$= (2x + 3)(x + 5)$$

Check: $(2x + 3)(x + 5) = 2x^2 + 10x + 3x + 15$
$= 2x^2 + 13x + 15$

➡ Factor: $6x^2 - 11x - 10$

Find two factors of -60 $[6(-10)]$ whose sum is -11.

Factors of -60	*Sum*
$1, -60$	-59
$-1, 60$	59
$2, -30$	-28
$-2, 30$	28
$3, -20$	-17
$-3, 20$	17
$4, -15$	-11

$$6x^2 - 11x - 10 = 6x^2 + 4x - 15x - 10$$

- Use the factors of -60 whose sum is -11 to write $-11x$ as $4x - 15x$.

$$= (6x^2 + 4x) - (15x + 10)$$

- Factor by grouping. Recall that $-15x - 10 = -(15x + 10)$.

$$= 2x(3x + 2) - 5(3x + 2)$$
$$= (3x + 2)(2x - 5)$$

Check: $(3x + 2)(2x - 5) = 6x^2 - 15x + 4x - 10$
$= 6x^2 - 11x - 10$

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Factor: $3x^2 - 2x - 4$

Find two factors of -12 [$3(-4)$] whose sum is -2.

Factors of -12	*Sum*
$1, -12$	-11
$-1, 12$	11
$2, -6$	-4
$-2, 6$	4
$3, -4$	-1
$-3, 4$	1

Because no integer factors of -12 have a sum of -2, $3x^2 - 2x - 4$ is nonfactorable over the integers. $3x^2 - 2x - 4$ is a prime polynomial.

Example 3

Factor: $2x^2 + 19x - 10$

Solution

Factors of -20 [$2(-10)$]	*Sum*
$-1, 20$	19

$$\begin{aligned} 2x^2 + 19x - 10 &= 2x^2 - x + 20x - 10 \\ &= (2x^2 - x) + (20x - 10) \\ &= x(2x - 1) + 10(2x - 1) \\ &= (2x - 1)(x + 10) \end{aligned}$$

You Try It 3

Factor: $2a^2 + 13a - 7$

Your solution

Example 4

Factor: $24x^2y - 76xy + 40y$

Solution

The GCF is $4y$.

$$24x^2y - 76xy + 40y = 4y(6x^2 - 19x + 10)$$

Negative Factors of 60 [$6(10)$]	*Sum*
$-1, -60$	-61
$-2, -30$	-32
$-3, -20$	-23
$-4, -15$	-19

$$\begin{aligned} 6x^2 - 19x + 10 &= 6x^2 - 4x - 15x + 10 \\ &= (6x^2 - 4x) - (15x - 10) \\ &\quad 2x(3x - 2) \quad 5(3x \quad 2) \\ &\quad (3x - 2)(2x - 5) \end{aligned}$$

$$\begin{aligned} 24x^2y - 76xy + 40y &= 4y(6x^2 - 19x + 10) \\ &= 4y(3x - 2)(2x - 5) \end{aligned}$$

You Try It 4

Factor: $15x^3 + 40x^2 - 80x$

Your solution

Solutions on p. S21

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7.3 Exercises

Objective A

Factor by using trial factors.

1. $2x^2 + 3x + 1$ **2.** $5x^2 + 6x + 1$ **3.** $2y^2 + 7y + 3$ **4.** $3y^2 + 7y + 2$

5. $2a^2 - 3a + 1$ **6.** $3a^2 - 4a + 1$ **7.** $2b^2 - 11b + 5$ **8.** $3b^2 - 13b + 4$

9. $2x^2 + x - 1$ **10.** $4x^2 - 3x - 1$ **11.** $2x^2 - 5x - 3$ **12.** $3x^2 + 5x - 2$

13. $2t^2 - t - 10$ **14.** $2t^2 + 5t - 12$ **15.** $3p^2 - 16p + 5$ **16.** $6p^2 + 5p + 1$

17. $12y^2 - 7y + 1$ **18.** $6y^2 - 5y + 1$ **19.** $6z^2 - 7z + 3$ **20.** $9z^2 + 3z + 2$

21. $6t^2 - 11t + 4$ **22.** $10t^2 + 11t + 3$ **23.** $8x^2 + 33x + 4$ **24.** $7x^2 + 50x + 7$

25. $5x^2 - 62x - 7$ **26.** $9x^2 - 13x - 4$ **27.** $12y^2 + 19y + 5$ **28.** $5y^2 - 22y + 8$

29. $7a^2 + 47a - 14$ **30.** $11a^2 - 54a - 5$ **31.** $3b^2 - 16b + 16$ **32.** $6b^2 - 19b + 15$

33. $2z^2 - 27z - 14$ **34.** $4z^2 + 5z - 6$ **35.** $3p^2 + 22p - 16$ **36.** $7p^2 + 19p + 10$

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37. $4x^2 + 6x + 2$ **38.** $12x^2 + 33x - 9$ **39.** $15y^2 - 50y + 35$ **40.** $30y^2 + 10y - 20$

41. $2x^3 - 11x^2 + 5x$ **42.** $2x^3 - 3x^2 - 5x$ **43.** $3a^2b - 16ab + 16b$ **44.** $2a^2b - ab - 21b$

45. $3z^2 + 95z + 10$ **46.** $8z^2 - 36z + 1$ **47.** $36x - 3x^2 - 3x^3$ **48.** $-2x^3 + 2x^2 + 4x$

49. $80y^2 - 36y + 4$ **50.** $24y^2 - 24y - 18$ **51.** $8z^3 + 14z^2 + 3z$ **52.** $6z^3 - 23z^2 + 20z$

53. $6x^2y - 11xy - 10y$ **54.** $8x^2y - 27xy + 9y$ **55.** $10t^2 - 5t - 50$

56. $16t^2 + 40t - 96$ **57.** $3p^3 - 16p^2 + 5p$ **58.** $6p^3 + 5p^2 + p$

59. $26z^2 + 98z - 24$ **60.** $30z^2 - 87z + 30$ **61.** $10y^3 - 44y^2 + 16y$

62. $14y^3 + 94y^2 - 28y$ **63.** $4yz^3 + 5yz^2 - 6yz$ **64.** $12a^3 + 14a^2 - 48a$

65. $42a^3 + 45a^2 - 27a$ **66.** $36p^2 - 9p^3 - p^4$ **67.** $9x^2y - 30xy^2 + 25y^3$

68. $8x^2y - 38xy^2 + 35y^3$ **69.** $9x^3y - 24x^2y^2 + 16xy^3$ **70.** $9x^3y + 12x^2y + 4xy$

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Objective B

Factor by grouping.

71. $6x^2 - 17x + 12$

72. $15x^2 - 19x + 6$

73. $5b^2 + 33b - 14$

74. $8x^2 - 30x + 25$

75. $6a^2 + 7a - 24$

76. $14a^2 + 15a - 9$

77. $4z^2 + 11z + 6$

78. $6z^2 - 25z + 14$

79. $22p^2 + 51p - 10$

80. $14p^2 - 41p + 15$

81. $8y^2 + 17y + 9$

82. $12y^2 - 145y + 12$

83. $18t^2 - 9t - 5$

84. $12t^2 + 28t - 5$

85. $6b^2 + 71b - 12$

86. $8b^2 + 65b + 8$

87. $9x^2 + 12x + 4$

88. $25x^2 - 30x + 9$

89. $6b^2 - 13b + 6$

90. $20b^2 + 37b + 15$

91. $33b^2 + 34b - 35$

92. $15b^2 - 43b + 22$

93. $18y^2 - 39y + 20$

94. $24y^2 + 41y + 12$

95. $15a^2 + 26a - 21$

96. $6a^2 + 23a + 21$

97. $8y^2 - 26y + 15$

98. $18y^2 - 27y + 4$

99. $8z^2 + 2z - 15$

100. $10z^2 + 3z - 4$

101. $15x^2 - 82x + 24$

102. $13z^2 + 49z - 8$

103. $10z^2 - 29z + 10$

104. $15z^2 - 44z + 32$

105. $36z^2 + 72z + 35$

106. $16z^2 + 8z - 35$

107. $3x^2 + xy - 2y^2$

108. $6x^2 + 10xy + 4y^2$

109. $3a^2 + 5ab - 2b^2$

110. $2a^2 - 9ab + 9b^2$

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111. $4y^2 - 11yz + 6z^2$ **112.** $2y^2 + 7yz + 5z^2$ **113.** $28 + 3z - z^2$ **114.** $15 - 2z - z^2$

115. $8 - 7x - x^2$ **116.** $12 + 11x - x^2$ **117.** $9x^2 + 33x - 60$ **118.** $16x^2 - 16x - 12$

119. $24x^2 - 52x + 24$ **120.** $60x^2 + 95x + 20$ **121.** $35a^4 + 9a^3 - 2a^2$

122. $15a^4 + 26a^3 + 7a^2$ **123.** $15b^2 - 115b + 70$ **124.** $25b^2 + 35b - 30$

125. $3x^2 - 26xy + 35y^2$ **126.** $4x^2 + 16xy + 15y^2$ **127.** $216y^2 - 3y - 3$

128. $360y^2 + 4y - 4$ **129.** $21 - 20x - x^2$ **130.** $18 + 17x - x^2$

131. $15a^2 + 11ab - 14b^2$ **132.** $15a^2 - 31ab + 10b^2$ **133.** $33z - 8z^2 - z^3$

APPLYING THE CONCEPTS

Factor.

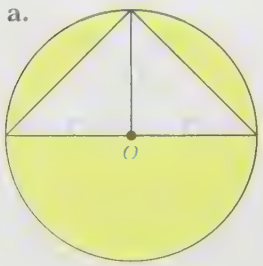
134. $2(y + 2)^2 - (y + 2) - 3$ **135.** $3(a + 2)^2 - (a + 2) - 4$ **136.** $4(y - 1)^2 - 7(y - 1) - 2$

Find all integers k such that the trinomial can be factored over the integers.

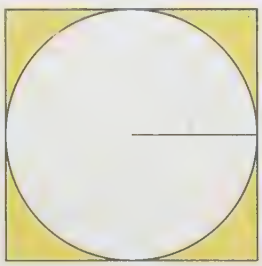
137. $2x^2 + kx + 3$ **138.** $2x^2 + kx - 3$ **139.** $3x^2 + kx + 2$

140. Write the area of the shaded portion of each diagram in factored form.

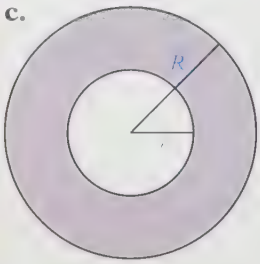
a.



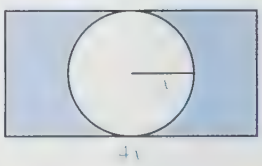
b.



c.



d.



141. In your own words, explain how the signs of the last terms of the two binomial factors of a trinomial are determined.

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7.4 Special Factoring

Objective A

To factor the difference of two perfect squares or a perfect-square trinomial

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TAKE NOTE
Here are some more examples of perfect squares:
$36, 49, x^8, y^{12}$.

The product of a term and itself is called a **perfect square**. The exponents on variables of perfect squares are always even numbers.

Term		*Perfect Square*
5	$5 \cdot 5 =$	25
x	$x \cdot x =$	x^2
$3y^4$	$3y^4 \cdot 3y^4 =$	$9y^8$
x^n	$x^n \cdot x^n =$	x^{2n}

The **square root of a perfect square** is one of the two equal factors of the perfect square. "$\sqrt{\ }$" is the symbol for square root. To find the exponent of the square root of a variable term, divide the exponent by 2.

$\sqrt{25} = 5$
$\sqrt{x^2} = x$
$\sqrt{9y^8} = 3y^4$
$\sqrt{x^{2n}} = x^n$

The difference of two perfect squares is the product of the sum and difference of two terms. The factors of the difference of two perfect squares are the sum and difference of the square roots of the perfect squares.

TAKE NOTE
$(a + b)$ is the sum of the two terms a and b.
$(a - b)$ is the difference of the two terms a and b. a and b are the square roots of a^2 and b^2.

Factors of the Difference of Two Perfect Squares

$a^2 - b^2 = (a + b)(a - b)$

The sum of two perfect squares, $a^2 + b^2$, is nonfactorable over the integers.

Factor: $4x^2 - 81y^2$

$4x^2 - 81y^2 = (2x)^2 - (9y)^2$
$= (2x + 9y)(2x - 9y)$

- Write the binomial as the difference of two perfect squares.
- The factors are the sum and difference of the square roots of the perfect squares.

A perfect-square trinomial is the square of a binomial.

Factors of a Perfect-Square Trinomial

$a^2 + 2ab + b^2 = (a + b)^2$
$a^2 - 2ab + b^2 = (a - b)^2$

In factoring a perfect-square trinomial, remember that the terms of the binomial are the square roots of the perfect squares of the trinomial. The sign in the binomial is the sign of the middle term of the trinomial.

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Factor: $4x^2 + 12x + 9$

Because $4x^2$ is a perfect square $[4x^2 = (2x)^2]$ and 9 is a perfect square $(9 = 3^2)$, try factoring $4x^2 + 12x + 9$ as the square of a binomial.

$4x^2 + 12x + 9 \stackrel{?}{=} (2x + 3)^2$

Check: $(2x + 3)^2 = (2x + 3)(2x + 3) = 4x^2 + 6x + 6x + 9 = 4x^2 + 12x + 9$

The check verifies that $4x^2 + 12x + 9 = (2x + 3)^2$.

It is important to check a proposed factorization as we did above. The next example illustrates the importance of this check.

Factor: $x^2 + 13x + 36$

Because x^2 is a perfect square and 36 is a perfect square, try factoring $x^2 + 13x + 36$ as the square of a binomial.

$x^2 + 13x + 36 \stackrel{?}{=} (x + 6)^2$

Check: $(x + 6)^2 = (x + 6)(x + 6) = x^2 + 6x + 6x + 36 = x^2 + 12x + 36$

In this case, the proposed factorization of $x^2 + 13x + 36$ does *not* check. Try another factorization. The numbers 4 and 9 are factors of 36 whose sum is 13.

$x^2 + 13x + 36 = (x + 4)(x + 9)$

Example 1

Factor: $25x^2 - 1$

Solution

$25x^2 - 1 = (5x)^2 - (1)^2$

$= (5x + 1)(5x - 1)$

You Try It 1

Factor: $x^2 - 36y^4$

Your solution

Example 2

Factor: $4x^2 - 20x + 25$

Solution

$4x^2 - 20x + 25 = (2x - 5)$

You Try It 2

Factor: $9x^2 + 12x + 4$

Your solution

Example 3

Factor: $(x + y)^2 - 4$

Solution

$(x + y)^2 - 4 = (x + y)^2 - (2)^2$

$= (x + y + 2)(x + y - 2)$

You Try It 3

Factor: $(a + b)^2 - (a - b)^2$

Your solution

Solutions on p. S21

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Objective E

To factor the sum or the difference of two cubes

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TAKE NOTE
Here are some more examples of perfect cubes:
64, 1000, b^9, z^{15}

The product of the same three factors is called a **perfect cube**. The exponents on variables of perfect cubes are always divisible by 3.

Term		*Perfect Cube*
2	$2 \cdot 2 \cdot 2 = 2^3 =$	8
$3y$	$3y \cdot 3y \cdot 3y = (3y)^3 =$	$27y^3$
y^2	$y^2 \cdot y^2 \cdot y^2 = (y^2)^3 =$	y^6

The **cube root** of a perfect cube is one of the three equal factors of the perfect cube. "$\sqrt[3]{\ }$" is the symbol for cube root. To find the exponent of the cube root of a variable term, divide the exponent by 3.

$\sqrt[3]{8} = 2$
$\sqrt[3]{27y^3} = 3y$
$\sqrt[3]{y^6} = y^2$

The following rules are used to factor the sum or difference of two perfect cubes.

Factors of the Sum or Difference of Two Cubes

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

➡ Factor: $8x^3 - 27$

Write the binomial as the difference of two perfect cubes.

$8x^3 - 27 = (2x)^3 - 3^3$

The terms of the binomial factor are the cube roots of the perfect cubes. The sign of the binomial factor is the same sign as in the given binomial. The trinomial factor is obtained from the binomial factor.

$= (2x - 3)(4x^2 + 6x + 9)$

Square of the first term
Opposite of the product of the two terms
Square of the last term

TAKE NOTE
You can always check a proposed factorization by multiplying the factors.

Check:

$$\begin{array}{r} 4x^2 + 6x + 9 \\ \hline 2x - 3 \\ \hline -12x^2 - 18x - 27 \\ 8x^3 + 12x^2 + 18x \\ \hline 8x^3 \qquad\quad - 27 \end{array}$$

• The original polynomial

➡ Factor: $a^3 + 64y^3$

$a^3 + 64y^3 = a^3 + (4y)^3$ • Write the binomial as the sum of two perfect cubes.

$= (a + 4y)(a^2 - 4ay + 16y^2)$ • Factor.

➡ Factor: $64y^4 - 125y$

$64y^4 - 125y = y(64y^3 - 125)$ • Factor out y, the GCF.

$= y[(4y)^3 - 5^3]$ • Write the binomial as the difference of two cubes.

$= y(4y - 5)(16y^2 + 20y + 25)$ • Factor.

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Example 4
Factor: $x^3y^3 - 1$

Solution
$x^3y^3 - 1 = (xy)^3 - 1^3$
$= (xy - 1)(x^2y^2 + xy + 1)$

You Try It 4
Factor: $8x^3 + y^3z^3$

Your solution

Example 5
Factor: $(x + y)^3 - x^3$

Solution
$(x + y)^3 - x^3$
$= [(x + y) - x][(x + y)^2 + x(x + y) + x^2]$
$= y(x^2 + 2xy + y^2 + x^2 + xy + x^2)$
$= y(3x^2 + 3xy + y^2)$

You Try It 5
Factor: $(x - y)^3 + (x + y)^3$

Your solution

Solutions on p. S21

To factor a trinomial that is quadratic in form

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Certain trinomials that are not quadratic can be expressed in the form $ax^2 + bx + c$ by making suitable variable substitutions. A trinomial is **quadratic in form** if it can be written as $au^2 + bu + c$.

As shown below, the trinomials $x^4 + 5x^2 + 6$ and $2x^2y^2 + 3xy - 9$ are quadratic in form.

	$x^4 + 5x^2 + 6$		$2x^2y^2 + 3xy - 9$
	$(x^2)^2 + 5(x^2) + 6$		$2(xy)^2 + 3(xy) - 9$
Let $u = x^2$.	$u^2 + 5u + 6$	Let $u = xy$.	$2u^2 + 3u - 9$

When we use this method to factor a trinomial that is quadratic in form, the variable part of the first term in each binomial will be u.

For example, $x^4 + 5x^2 + 6$ is quadratic in form by letting $u = x^2$. Thus,
$x^4 + 5x^2 + 6 = (x^2)^2 + 5(x^2) + 6 = u^2 + 5u + 6$.

Factor: $x^4 + 5x^2 + 6$

$x^4 + 5x^2 + 6 = u^2 + 5u + 6$ • Let $u = x^2$.
$= (u + 3)(u + 2)$ • Factor.
$= (x^2 + 3)(x^2 + 2)$ • Replace u by x^2.

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Example 6
Factor: $6x^2y^2 - xy - 12$

Solution
Let $u = xy$.

$$\begin{aligned} 6x^2y^2 - xy - 12 &= 6u^2 - u - 12 \\ &= (3u + 4)(2u - 3) \\ &= (3xy + 4)(2xy - 3) \end{aligned}$$

You Try It 6
Factor: $6x^2y^2 - 19xy + 10$

Your solution

Example 7
Factor: $2x^4 + 5x^2 - 12$

Solution
Let $u = x^2$.

$$\begin{aligned} 2x^4 + 5x^2 - 12 &= 2u^2 + 5u - 12 \\ &= (2u - 3)(u + 4) \\ &= (2x^2 - 3)(x^2 + 4) \end{aligned}$$

You Try It 7
Factor: $3x^4 + 4x^2 - 4$

Your solution

Example 8
Factor: $x^4y^4 + 3x^2y^2 - 10$

Solution
Let $u = x^2y^2$.

$$\begin{aligned} x^4y^4 + 3x^2y^2 - 10 &= u^2 + 3u - 10 \\ &= (u + 5)(u - 2) \\ &= (x^2y^2 + 5)(x^2y^2 - 2) \end{aligned}$$

You Try It 8
Factor: $a^4b^4 + 6a^2b^2 - 7$

Your solution

Solutions on p. S21

Objective D

To factor completely

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When factoring a polynomial completely, ask the following questions about the polynomial.

1. Is there a common factor? If so, factor out the GCF.
2. If the polynomial is a binomial, is it the difference of two perfect squares, the sum of two cubes, or the difference of two cubes? If so, factor.
3. If the polynomial is a trinomial, is it a perfect-square trinomial or the product of two binomials? If so, factor.
4. If the polynomial has four terms, can it be factored by grouping? If so, factor.
5. Is each factor nonfactorable over the integers? If not, factor.

TAKE NOTE
Remember that you may have to factor more than once in order to write the polynomial as a product of *prime* factors.

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Example 9

Factor: $6a^3 + 15a^2 - 36a$

Solution

$$6a^3 + 15a^2 - 36a = 3a(2a^2 + 5a - 12)$$
$$= 3a(2a - 3)(a + 4)$$

You Try It 9

Factor: $18x^3 - 6x^2 - 60x$

Your solution

Example 10

Factor: $x^2y + 2x^2 - y - 2$

Solution

$$x^2y + 2x^2 - y - 2 = (x^2y + 2x^2) - (y + 2)$$
$$= x^2(y + 2) - 1(y + 2)$$
$$= (y + 2)(x^2 - 1)$$
$$= (y + 2)(x + 1)(x - 1)$$

You Try It 10

Factor: $4x - 4y - x^3 + x^2y$

Your solution

Example 11

Factor: $x^{4n} - y^{4n}$

Solution

$$x^{4n} - y^{4n} = (x^{2n})^2 - (y^{2n})^2$$
$$= (x^{2n} + y^{2n})(x^{2n} - y^{2n})$$
$$= (x^{2n} + y^{2n})[(x^n)^2 - (y^n)^2]$$
$$= (x^{2n} + y^{2n})(x^n + y^n)(x^n - y^n)$$

You Try It 11

Factor: $x^{4n} - x^{2n}y^{2n}$

Your solution

Example 12

Factor: $x^{n+3} + x^ny^3$

Solution

$$x^{n+3} + x^ny^3 = x^n(x^3 + y^3)$$
$$= x^n(x + y)(x^2 - xy + y^2)$$

You Try It 12

Factor: $ax^5 - ax^2y^6$

Your solution

Solutions on pp. S21–S22

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7.4 Exercises

Objective 4

Which of the expressions are perfect squares?

1. $4;\ 8;\ 25x^6;\ 12y^{10};\ 100x^4y^4$

2. $9;\ 18;\ 15a^8;\ 49b^{12};\ 64a^{16}b^2$

Name the square root of the expression

3. $16z^8$

4. $36d^{10}$

5. $81a^4b^6$

6. $25m^2n^{12}$

Factor.

7. $x^2 - 16$

8. $y^2 - 49$

9. $4x^2 - 1$

10. $81x^2 - 4$

11. $16x^2 - 121$

12. $49y^2 - 36$

13. $1 - 9a^2$

14. $16 - 81y^2$

15. $x^2y^2 - 100$

16. $a^2b^2 - 25$

17. $x^2 + 4$

18. $a^2 + 16$

19. $25 - a^2b^2$

20. $64 - x^2y^2$

21. $a^{2n} - 1$

22. $b^{2n} - 16$

23. $x^2 - 12x + 36$

24. $y^2 - 6y + 9$

25. $b^2 - 2b + 1$

26. $a^2 + 14a + 49$

27. $16x^2 - 40x + 25$

28. $49x^2 + 28x + 4$

29. $4a^2 + 4a - 1$

30. $9x^2 + 12x - 4$

31. $b^2 + 7b + 14$

32. $y^2 - 5y + 25$

33. $x^2 + 6xy + 9y^2$

34. $4x^2y^2 + 12xy + 9$

35. $25a^2 - 40ab + 16b^2$

36. $4a^2 - 36ab + 81b^2$

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37. $x^{2n} + 6x^n + 9$ **38.** $y^{2n} - 16y^n + 64$ **39.** $(x - 4)^2 - 9$

40. $16 - (a - 3)^2$ **41.** $(x - y)^2 - (a + b)^2$ **42.** $(x - 2y)^2 - (x + y)^2$

Objective H

Which of the expressions are perfect cubes?

43. 4; 8; x^9; a^8b^8; $27c^{15}d^{18}$ **44.** 9; 27; y^{12}; m^3n^6; $64mn^9$

Name the cube root of the expression.

45. $8x^9$ **46.** $27y^{15}$ **47.** $64a^6b^{18}$ **48.** $125c^{12}d^3$

Factor.

49. $x^3 - 27$ **50.** $y^3 + 125$ **51.** $8x^3 - 1$

52. $64a^3 + 27$ **53.** $x^3 - y^3$ **54.** $x^3 - 8y^3$

55. $m^3 + n^3$ **56.** $27a^3 + b^3$ **57.** $64x^3 + 1$

58. $1 - 125b^3$ **59.** $27x^3 - 8y^3$ **60.** $64x^3 + 27y^3$

61. $x^3y^3 + 64$ **62.** $8x^3y^3 + 27$ **63.** $16x^3 - y^3$

64. $27x^3 - 8y^2$ **65.** $8x^3 - 9y^3$ **66.** $27a^3 - 16$

67. $(a - b)^3 - b^3$ **68.** $a^3 + (a + b)^3$ **69.** $x^{6n} + y^{3n}$

70. $x^{3n} + y^{3n}$ **71.** $x^{3n} + 8$ **72.** $a^{3n} + 64$

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Objective C

Factor.

73. $x^2y^2 - 8xy + 15$

74. $x^2y^2 - 8xy - 33$

75. $x^2y^2 - 17xy + 60$

76. $a^2b^2 + 10ab + 24$

77. $x^4 - 9x^2 + 18$

78. $y^4 - 6y^2 - 16$

79. $b^4 - 13b^2 - 90$

80. $a^4 + 14a^2 + 45$

81. $x^4y^4 - 8x^2y^2 + 12$

82. $a^4b^4 + 11a^2b^2 - 26$

83. $x^{2n} + 3x^n + 2$

84. $a^{2n} - a^n - 12$

85. $3x^2y^2 - 14xy + 15$

86. $5x^2y^2 - 59xy + 44$

87. $6a^2b^2 - 23ab + 21$

88. $10a^2b^2 + 3ab - 7$

89. $2x^4 - 13x^2 - 15$

90. $3x^4 + 20x^2 + 32$

91. $2x^{2n} - 7x^n + 3$

92. $4x^{2n} + 8x^n - 5$

93. $6a^{2n} + 19a^n + 10$

Objective D

Factor.

94. $5x^2 + 10x + 5$

95. $12x^2 - 36x + 27$

96. $3x^4 - 81x$

97. $27a^4 - a$

98. $7x^2 - 28$

99. $20x^2 - 5$

100. $y^4 - 10y^3 + 21y^2$

101. $y^5 + 6y^4 - 55y^3$

102. $x^4 - 16$

103. $16x^4 - 81$

104. $8x^5 - 98x^3$

105. $16a - 2a^4$

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106. $x^3y^3 - x^3$

107. $a^3b^6 - b^3$

108. $x^6y^6 - x^3y^3$

109. $8x^4 - 40x^3 + 50x^2$

110. $6x^5 + 74x^4 + 24x^3$

111. $x^4 - y^4$

112. $16a^4 - b^4$

113. $x^6 + y^6$

114. $x^4 - 5x^2 - 4$

115. $a^4 - 25a^2 - 144$

116. $3b^5 - 24b^2$

117. $16a^4 - 2a$

118. $16x^3y + 4x^2y^2 - 42xy^3$

119. $24a^2b^2 - 14ab^3 - 90b^4$

120. $x^3 - 2x^2 - x + 2$

121. $x^3 - 2x^2 - 4x + 8$

122. $4x^2y^2 - 4x^2 - 9y^2 + 9$

123. $4x^4 - x^2 - 4x^2y^2 + y^2$

124. $a^{2n+2} - 6a^{n+2} + 9a^2$

125. $x^{2n+1} + 2x^{n+1} + x$

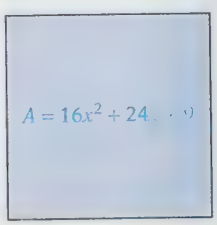
126. $2x^{n+2} - 7x^{n+1} + 3x^n$

127. $3b^{n+2} + 4b^{n+1} - 4b^n$

APPLYING THE CONCEPTS

128. Find all integers k such that the trinomial is a perfect square.
a. $x^2 + kx + 36$ **b.** $4x^2 - kx + 25$ **c.** $49x^2 + kxy + 64y^2$

129. The area of a square is $(16x^2 + 24x + 9)$ m^2. Find the dimensions of the square in terms of the variable x. Can $x = 0$? What are the possible values of x?



130. Can a third-degree polynomial have factors $(x - 1)$, $(x + 1)$, $(x - 3)$, and $(x + 4)$? Why or why not?

131. Given that $(x - 3)$ and $(x + 4)$ are factors of $x^3 + 6x^2 - 7x - 60$, explain how you can find a third *first-degree* factor of $x^3 + 6x^2 - 7x - 60$. Then find the factor.

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7.5 Solving Equations

Objective 1 To solve equations by factoring

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The Multiplication Property of Zero states that the product of a number and zero is zero. This property is stated below.

If a is a real number, then $a \cdot 0 = 0 \cdot a = 0$.

Now consider $x \cdot y = 0$. For this to be a true equation, then either $x = 0$ or $y = 0$.

Principle of Zero Products

If the product of two factors is zero, then at least one of the factors must be zero.

If $a \cdot b = 0$, then $a = 0$ or $b = 0$.

The Principle of Zero Products is used to solve some equations.

Solve: $(x - 2)(x - 3) = 0$

By the Principle of Zero Products, if $(x - 2)(x - 3) = 0$, then $x - 2 = 0$ or $x - 3 = 0$.

$(x - 2)(x - 3) = 0$

$x - 2 = 0 \qquad x - 3 = 0$ • Let each factor equal zero (the Principle of Zero Products).

$x = 2 \qquad x = 3$ • Solve each equation for x.

Check:

$$\begin{array}{r|l} (x-2)(x-3) = 0 & \\ \hline (2-2)(2-3) & 0 \\ 0(-1) & 0 \\ 0 = 0 & \text{True} \end{array} \qquad \begin{array}{r|l} (x-2)(x-3) = 0 & \\ \hline (3-2)(3-3) & 0 \\ (1)(0) & 0 \\ 0 = 0 & \text{True} \end{array}$$

The solutions are 2 and 3.

An equation of the form $ax^2 + bx + c = 0$, $a \neq 0$, is a **quadratic equation**. A quadratic equation is in **standard form** when the polynomial is in descending order and equal to zero. The quadratic equations at the right are in standard form.

$3x^2 + 2x + 1 = 0$

$4x^2 - 3x + 2 = 0$

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TAKE NOTE

Note the steps involved in solving a quadratic equation by factoring:

1. Write in standard form.
2. Factor.
3. Set each factor equal to 0.
4. Solve each equation.
5. Check the solutions.

Solve: $2x^2 + x = 6$

$$2x^2 + x = 6$$
$$2x^2 + x - 6 = 0$$
$$(2x - 3)(x + 2) = 0$$

- Write the equation in standard form.
- Factor.

$$2x - 3 = 0 \qquad x + 2 = 0$$
$$2x = 3 \qquad x = -2$$
$$x = \frac{3}{2}$$

- Use the Principle of Zero Products.
- Solve each equation for x.

$\frac{3}{2}$ and -2 check as solutions. The solutions are $\frac{3}{2}$ and -2.

Example 1

Solve: $x(x - 3) = 0$

Solution

$x(x - 3) = 0$

$x = 0 \qquad x - 3 = 0$
$\qquad\qquad x = 3$

The solutions are 0 and 3.

You Try It 1

Solve: $2x(x + 7) = 0$

Your solution

Example 2

Solve: $2x^2 - 50 = 0$

Solution

$2x^2 - 50 = 0$
$2(x^2 - 25) = 0$
$2(x + 5)(x - 5) = 0$

$x + 5 = 0 \qquad x - 5 = 0$
$x = -5 \qquad x = 5$

The solutions are -5 and 5.

You Try It 2

Solve: $4x^2 - 9 = 0$

Your solution

Example 3

Solve: $(x - 3)(x - 10) = -10$

Solution

$(x - 3)(x - 10) = -10$
$x^2 - 13x + 30 = -10$ • Multiply $(x - 3)(x - 10)$.
$x^2 - 13x + 40 = 0$ • Add 10 to each side of the equation. The equation is now in standard form.
$(x - 8)(x - 5) = 0$

$x - 8 = 0 \qquad x - 5 = 0$
$x = 8 \qquad x = 5$

The solutions are 8 and 5.

You Try It 3

Solve: $(x + 2)(x - 7) = 52$

Your solution

Solutions on p. S22

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Objective 6 To solve application problems









Recall that the integers are the numbers . . . , −3, −2, −1, 0, 1, 2, 3, ...

An **even integer** is an integer that is divisible by 2. Examples of even integers are −8, 0, and 22. An **odd integer** is an integer that is not divisible by 2. Examples of odd integers are −17, 1, and 39.

Consecutive integers are integers that follow one another in order. Examples of consecutive integers are shown at the right. (Assume that the variable n represents an integer.)

11, 12, 13
−8, −7, −6
$n, n + 1, n + 2$

Examples of **consecutive even integers** are shown at the right. (Assume that the variable n represents an even integer.)

24, 26, 28
−10, −8, −6
$n, n + 2, n + 4$

Examples of **consecutive odd integers** are shown at the right. (Assume that the variable n represents an odd integer.)

19, 21, 23
−1, 1, 3
$n, n + 2, n + 4$

Example 4

The sum of the squares of two consecutive positive even integers is equal to 100. Find the two integers.

Strategy

First positive even integer: n
Second positive even integer: $n + 2$

The sum of the square of the first positive even integer and the square of the second positive even integer is 100.

Solution

$$n^2 + (n + 2)^2 = 100$$
$$n^2 + n^2 + 4n + 4 = 100$$
$$2n^2 + 4n + 4 = 100$$
$$2n^2 + 4n - 96 = 0$$
$$2(n^2 + 2n - 48) = 0$$
$$n^2 + 2n - 48 = 0$$

• Divide each side of the equation by 2.

$$(n - 6)(n + 8) = 0$$

$$n - 6 = 0 \qquad n + 8 = 0$$
$$n = 6 \qquad n = -8$$

Because −8 is not a positive even integer, it is not a solution.

$$n = 6$$
$$n + 2 = 6 + 2 = 8$$

The two integers are 6 and 8.

You Try It 4

The sum of the squares of two positive consecutive integers is 61. Find the two integers.

Your strategy

Your solution

Solution on p. S22

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Example 5

A stone is thrown into a well with an initial speed of 4 ft/s. The well is 420 ft deep. How many seconds later will the stone hit the bottom of the well? Use the equation $d = vt + 16t^2$, where d is the distance in feet, v is the initial speed, and t is the time in seconds.

Strategy

To find the time for the stone to drop to the bottom of the well, replace the variables d and v by their given values and solve for t.

Solution

$$d = vt + 16t^2$$
$$420 = 4t + 16t^2$$
$$0 = -420 + 4t + 16t^2$$
$$16t^2 + 4t - 420 = 0$$
$$4(4t^2 + t - 105) = 0$$
$$4t^2 + t - 105 = 0$$

• Divide each side of the equation by 4.

$$(4t + 21)(t - 5) = 0$$

$$4t + 21 = 0 \qquad t - 5 = 0$$
$$4t = -21 \qquad t = 5$$
$$t = -\frac{21}{4}$$

Because the time cannot be a negative number, $-\frac{21}{4}$ is not a solution.

The time is 5 s.

You Try It 5

The length of a rectangle is 4 in. longer than twice the width. The area of the rectangle is 96 in^2. Find the length and width of the rectangle.

Your strategy

Your solution

Solution on p. S22

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7.5 Exercises

Objective A

1. In your own words, explain why it is possible to solve a quadratic equation using the Principle of Zero Products.

Solve.

2. $(y + 3)(y + 2) = 0$ **3.** $(y - 3)(y - 5) = 0$ **4.** $(z - 7)(z - 3) = 0$ **5.** $(z + 8)(z - 9) = 0$

6. $x(x - 5) = 0$ **7.** $x(x + 2) = 0$ **8.** $a(a - 9) = 0$ **9.** $a(a + 12) = 0$

10. $y(2y + 3) = 0$ **11.** $t(4t - 7) = 0$ **12.** $2a(3a - 2) = 0$ **13.** $4b(2b + 5) = 0$

14. $(b + 2)(b - 5) = 0$ **15.** $(b - 8)(b + 3) = 0$ **16.** $x^2 - 81 = 0$ **17.** $x^2 - 121 = 0$

18. $4x^2 - 49 = 0$ **19.** $16x^2 - 1 = 0$ **20.** $9x^2 - 1 = 0$ **21.** $16x^2 - 49 = 0$

22. $x^2 + 6x + 8 = 0$ **23.** $x^2 - 8x + 15 = 0$ **24.** $z^2 + 5z - 14 = 0$ **25.** $z^2 + z - 72 = 0$

26. $2a^2 - 9a - 5 = 0$ **27.** $3a^2 + 14a + 8 = 0$ **28.** $6z^2 + 5z + 1 = 0$ **29.** $6y^2 - 19y + 15 = 0$

30. $x^2 - 3x = 0$ **31.** $a^2 - 5a = 0$ **32.** $x^2 - 7x = 0$ **33.** $2a^2 - 8a = 0$

34. $a^2 + 5a = -4$ **35.** $a^2 - 5a = 24$ **36.** $y^2 - 5y = -6$ **37.** $y^2 - 7y = 8$

38. $2t^2 + 7t = 4$ **39.** $3t^2 + t = 10$ **40.** $3t^2 - 13t = -4$ **41.** $5t^2 - 16t = -12$

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42. $x(x - 12) = -27$ **43.** $x(x - 11) = 12$ **44.** $y(y - 7) = 18$ **45.** $y(y + 8) = -15$

46. $p(p + 3) = -2$ **47.** $p(p - 1) = 20$ **48.** $y(y + 4) = 45$ **49.** $y(y - 8) = -15$

50. $x(x + 3) = 28$ **51.** $p(p - 14) = 15$ **52.** $(x + 8)(x - 3) = -30$ **53.** $(x + 4)(x - 1) = 14$

54. $(z - 5)(z + 4) = 52$ **55.** $(z - 8)(z + 4) = -35$ **56.** $(z - 6)(z + 1) = -10$

57. $(a + 3)(a + 4) = 72$ **58.** $(a - 4)(a + 7) = -18$ **59.** $(2x + 5)(x + 1) = -1$

Objective 3 Application Problems

60. The square of a positive number is six more than five times the positive number. Find the number.

61. The square of a negative number is fifteen more than twice the negative number. Find the number.

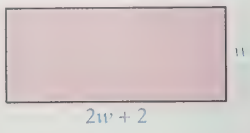
62. The sum of two numbers is six. The sum of the squares of the two numbers is twenty. Find the two numbers.

63. The sum of two numbers is eight. The sum of the squares of the two numbers is thirty-four. Find the two numbers.

64. The sum of the squares of two consecutive positive integers is forty-one. Find the two integers.

65. The sum of the squares of two consecutive positive even integers is one hundred sixty-four. Find the two integers.

66. The length of a rectangle is 2 ft more than twice the width. The area of the rectangle is 84 ft^2. Find the length and width of the rectangle.



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67. The length of a rectangle is 8 cm more than three times the width. The area of the rectangle is 380 cm^2. Find the length and width of the rectangle.

The formula $S = \frac{n^2 + n}{2}$ gives the sum, S, of the first n natural numbers. Use this formula for Exercises 68 and 69.

68. How many consecutive natural numbers beginning with 1 will give a sum of 78?

69. How many consecutive natural numbers beginning with 1 will give a sum of 171?

The formula $N = \frac{t^2 - t}{2}$ gives the number, N, of football games that must be scheduled in a league with t teams if each team is to play every other team once. Use this formula for Exercises 70 and 71.

70. How many teams are in a league that schedules 15 games in such a way that each team plays every other team once?

71. How many teams are in a league that schedules 45 games in such a way that each team plays every other team once?

The distance, s, in feet, that an object will fall (neglecting air resistance) in t seconds is given by $s = vt + 16t^2$, where v is the initial velocity of the object in feet per second. Use this formula for Exercises 72 and 73.

72. An object is released from the top of a building 192 ft high. The initial velocity is 16 ft/s, and air resistance is neglected. How many seconds later will the object hit the ground?

73. An object is released from the top of a building 320 ft high. The initial velocity is 16 ft/s, and air resistance is neglected. How many seconds later will the object hit the ground?

The height, h, in feet, an object will attain (neglecting air resistance) in t seconds is given by $h = vt - 16t^2$, where v is the initial velocity of the object in feet per second. Use this formula for Exercises 74 and 75.

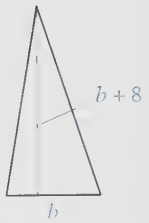


74. A golf ball is thrown onto a cement surface and rebounds straight up. The initial velocity of the rebound is 60 ft/s. How many seconds later will the golf ball return to the ground?

75. A foul ball leaves a bat and travels straight up with an initial velocity of 64 ft/s. How many seconds later will the ball be 64 ft above the ground?

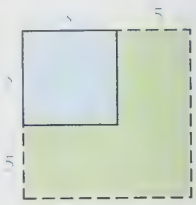
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76. The height of a triangle is 8 cm more than the length of the base. The area of the triangle is 64 cm^2. Find the base and height of the triangle.



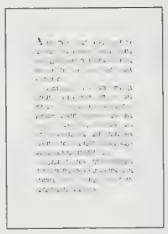
77. The height of a triangle is 4 m more than twice the length of the base. The area of the triangle is 35 m^2. Find the height of the triangle.

78. The length of each side of a square is extended 5 in. The area of the resulting square is 64 in^2. Find the length of a side of the original square.



79. A small garden measures 8 ft by 10 ft. A uniform border around the garden increases the total area to 143 ft^2. What is the width of the border?

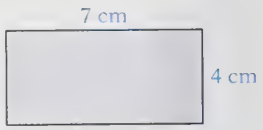
80. The page of a book measures 6 in. by 9 in. A uniform border around the page leaves 28 in^2 for type. What are the dimensions of the type area?



81. The radius of a circle is increased by 3 in.; this increases the area by 100 in^2. Find the radius of the original circle. Round to the nearest hundredth.

APPLYING THE CONCEPTS

82. The length of a rectangle is 7 cm, and the width is 4 cm. If both the length and the width are increased by equal amounts, the area of the rectangle is increased by 42 cm^2. Find the length and width of the larger rectangle.



83. A rectangular piece of cardboard is 10 in. longer than it is wide. Squares 2 in. on a side are to be cut from each corner, and then the sides will be folded up to make an open box with a volume of 192 in^3. Find the length and width of the piece of cardboard.



84. Solve: $p^3 = 9p^2$

85. Solve: $(x + 3)(2x - 1) = (3 - x)(5 - 3x)$

86. Find $3n^2$ if $n(n + 5) = -4$.

87. Explain the error made in solving the equation at the right. Solve the equation correctly.

$$(x + 2)(x - 3) = 6$$
$$x + 2 = 6 \quad x - 3 = 6$$
$$x = 4 \quad x = 9$$

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Focus on Problem Solving

Polya's Four-Step Process

Your success in mathematics and your success in the workplace are heavily dependent on your ability to solve problems. One of the foremost mathematicians to study problem solving was George Polya (1887–1985). The basic structure that Polya advocated for problem solving has four steps, as outlined below.



(For more information on Polya and his work, conduct a search on the Internet for "Polya." You can also search the Math Forum at **www.forum.swarthmore.edu**.)

Point of Interest

George Polya was born in Hungary and moved to the United States in 1940. He lived in Providence, Rhode Island, where he taught at Brown University until 1942, when he moved to California. There he taught at Stanford University until his retirement. While at Stanford, he published 10 books and a number of articles for mathematics journals. Of the books Polya published, *How To Solve It* (1945) is one of the best known. In this book, Polya outlines a strategy for solving problems. This strategy, although frequently applied to mathematics, can be used to solve problems from virtually any discipline.

1. Understand the Problem

You must have a clear understanding of the problem. To help you focus on understanding the problem, here are some questions to think about.

- Can you restate the problem in your own words?
- Can you determine what is known about these types of problems?
- Is there missing information that you need in order to solve the problem?
- Is there information given that is not needed?
- What is the goal?

2. Devise a Plan

Successful problem solvers use a variety of techniques when they attempt to solve a problem. Here are some frequently used strategies.

- Make a list of the known information.
- Make a list of information that is needed to solve the problem.
- Make a table or draw a diagram.
- Work backwards.
- Try to solve a similar but simpler problem.
- Research the problem to determine whether there are known techniques for solving problems of its kind.
- Try to determine whether some pattern exists.
- Write an equation.

3. Carry Out the Plan

Once you have devised a plan, you must carry it out.

- Work carefully.
- Keep an accurate and neat record of all your attempts.
- Realize that some of your initial plans will not work and that you may have to return to Step 2 and devise another plan or modify your existing plan.

4. Review Your Solution

Once you have found a solution, check the solution against the known facts.

- Ensure that the solution is consistent with the facts of the problem.
- Interpret the solution in the context of the problem.
- Ask yourself whether there are generalizations of the solution that could apply to other problems.
- Determine the strengths and weaknesses of your solution. For instance, is your solution only an approximation to the actual solution?
- Consider the possibility of alternative solutions.

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We will use Polya's four-step process to solve the following problem.



A large soft drink costs \$1.25 at a college cafeteria. The dimensions of the cup are shown at the left. Suppose you don't put any ice in the cup. Determine the cost per ounce for the soft drink.

1. *Understand the problem.* We must determine the cost per ounce for the soda. To do this, we need the dimensions of the cup (which are given), the cost of the drink (given), and a formula for the volume of the cup (unknown). Also, because the dimensions are given in inches, the volume will be in cubic inches; we need a conversion factor that will convert cubic inches to fluid ounces.
2. *Devise a plan.* Consult a resource book that gives the volume of the figure, which is called a **frustrum**. The formula for the volume is

$$V = \frac{\pi h}{3}(r^2 + rR + R^2)$$

where h is the height, r is the radius of the base, and R is the radius of the top. Also from a reference book, 1 $\text{in}^3 \approx 0.55$ fl oz. The general plan is to calculate the volume, convert the answer to fluid ounces, and then divide the cost by the number of fluid ounces.
3. *Carry out the plan.* Using the information from the drawing, evaluate the formula for the volume.

$$V = \frac{6\pi}{3}[1^2 + 1(1.5) + 1.5^2] = 9.5\pi \approx 29.8451 \text{ in}^3$$

$$V \approx 29.8451(0.55) \approx 16.4148 \text{ fl oz} \quad \bullet \text{ Convert to fluid ounces.}$$

$$\text{Cost per ounce} \approx \frac{1.25}{16.4148} \approx 0.07615 \quad \bullet \text{ Divide the cost by the volume.}$$

The cost of the soft drink is approximately 7.62 cents per ounce.

4. *Review the solution.* The cost of a 12-ounce can of soda from a vending machine is generally about 75¢. Therefore, the cost of canned soda is 75¢ ÷ 12 = 6.25¢ per ounce. This is consistent with our solution. This does not mean our solution is correct, but it does indicate that it is at least reasonable. Why might soda from a cafeteria be more expensive per ounce than soda from a vending machine?

 Is there an alternative way to obtain the solution? There are probably many, but one possibility is to get a measuring cup, pour the soft drink into it, and read the number of ounces. Name an advantage and a disadvantage of this method.

Use the four-step solution process for the following problems.

1. A cup dispenser next to a water cooler holds cups that have the shape of a right circular cone. The height of the cone is 4 in., and the radius of the circular top is 1.5 in. How many ounces of water can the cup hold?
2. Soft drink manufacturers do research into the preferences of consumers with regard to the look and feel and size of a soft drink can. Suppose that a manufacturer has determined that people want to have their hand reach around approximately 75% of the can. If this preference is to be achieved, how tall should the can be if it contains 12 oz of fluid? Assume the can is a right circular cylinder.

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Projects and Group Activities

Water Displacement

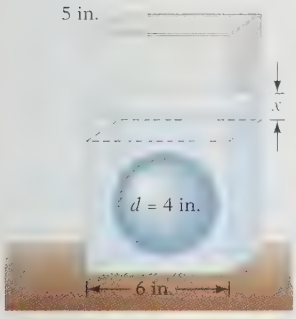
When an object is placed in water, the object displaces an amount of water that is equal to the volume of the object.

➡ A sphere with a diameter of 4 in. is placed in a rectangular tank of water that is 6 in. long and 5 in. wide. How much does the water level rise? Round to the nearest hundredth.

$V = \frac{4}{3}\pi r^3$ • Use the formula for the volume of a sphere.

$V = \frac{4}{3}\pi(2^3) = \frac{32}{3}\pi$ • $r = \frac{1}{2}d = \frac{1}{2}(4) = 2$

Let x represent the amount of the rise in water level. The volume of the sphere will equal the volume displaced by the water. As shown at the left, this volume is the rectangular solid with width 5 in., length 6 in., and height x in.



$V = LWH$ • Use the formula for the volume of a rectangular solid.

$\frac{32}{3}\pi = (6)(5)x$ • Substitute $\frac{32}{3}\pi$ for V, 5 for W, and 6 for L.

$\frac{32}{90}\pi = x$ • The exact height that the water will fill is $\frac{32}{90}\pi$.

$1.12 \approx x$ • Use a calculator to find an approximation.

The water will rise approximately 1.12 in.

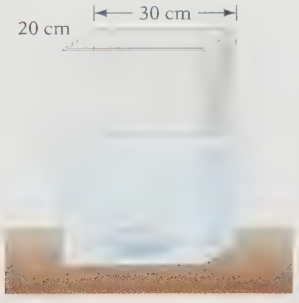


Figure 1

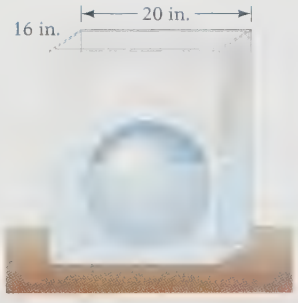


Figure 2

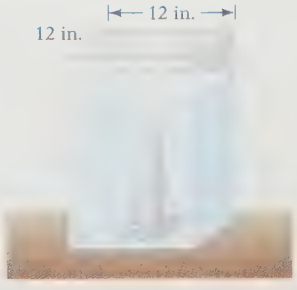


Figure 3

1. A cylinder with a 2-centimeter radius and a height of 10 cm is submerged in a tank of water that is 20 cm wide and 30 cm long (see Figure 1). How much does the water level rise? Round to the nearest hundredth.
2. A sphere with a radius of 6 in. is placed in a rectangular tank of water that is 16 in. wide and 20 in. long (see Figure 2). The sphere displaces water until two-thirds of the sphere is submerged. How much does the water level rise? Round to the nearest hundredth.
3. A chemist wants to know the density of a statue that weighs 15 lb. The statue is placed in a rectangular tank of water that is 12 in. long and 12 in. wide (see Figure 3). The water level rises 0.42 in. Find the density of the statue. Round to the nearest hundredth. (*Hint*: Density = weight ÷ volume)

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Chapter Summary

Key Words

The *greatest common factor* (GCF) of two or more monomials is the product of the GCF of the coefficients and the common variable factors. [p. 379]

To *factor* a polynomial means to write the polynomial as a product of other polynomials. [p. 379]

To *factor* a trinomial of the form $ax^2 + bx + c$ means to express the trinomial as the product of two binomials. [p. 385]

A polynomial is *nonfactorable over the integers* if it does not factor using only integers. Such a polynomial is called a *prime polynomial.* [p. 386]

A product of a term and itself is a *perfect square.* [p. 401]

The product of the same three factors is a *perfect cube.* [p. 403]

An equation of the form $ax^2 + bx + c = 0$, $a \neq 0$, is a *quadratic equation.* [p. 411]

A quadratic equation is in *standard form* when the polynomial is in descending order and equal to zero. The quadratic equation $ax^2 + bx + c = 0$ is in standard form. [p. 411]

Essential Rules

Factors of the Difference of Two Perfect Squares [p. 401]

$$a^2 - b^2 = (a + b)(a - b)$$

Factors of a Perfect-Square Trinomial [p. 401]

$$a^2 + 2ab + b^2 = (a + b)^2$$
$$a^2 - 2ab + b^2 = (a - b)^2$$

Factors of the Sum or Difference of Two Cubes [p. 403]

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Principle of Zero Products [p. 411]

If the product of two factors is zero, then at least one of the factors must be zero.

If $a \cdot b = 0$, then $a = 0$ or $b = 0$.

General Factoring Strategy [p. 405]

1. Is there a common factor? If so, factor out the GCF.
2. If the polynomial is a binomial, is it the difference of two perfect squares, the sum of two cubes, or the difference of two cubes? If so, factor.
3. If the polynomial is a trinomial, is it a perfect-square trinomial or the product of two binomials? If so, factor.
4. If the polynomial has four terms, can it be factored by grouping? If so, factor.
5. Is each factor nonfactorable over the integers? If not, factor.

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Chapter Review

1. Factor: $5x^3 + 10x^2 + 35x$

2. Factor: $12a^2b + 3ab^2$

3. Factor: $14y^9 - 49y^6 + 7y^3$

4. Factor: $4x(x - 3) - 5(3 - x)$

5. Factor: $10x^2 + 25x + 4xy + 10y$

6. Factor: $21ax - 35bx + 10by + 6ay$

7. Factor: $b^2 - 13b + 30$

8. Factor: $c^2 + 8c + 12$

9. Factor: $y^2 + 5y - 36$

10. Factor: $3a^2 - 15a - 42$

11. Factor: $4x^3 - 20x^2 - 24x$

12. Factor: $n^4 - 2n^3 - 3n^2$

13. Factor $6x^2 - 29x + 28$ by using trial factors.

14. Factor $12y^2 + 16y - 3$ by using trial factors.

15. Factor $2x^2 - 5x + 6$ by using trial factors.

16. Factor $3x^2 - 17x + 10$ by grouping.

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17. Factor $2a^2 - 19a - 60$ by grouping.

18. Factor $18a^2 - 3a - 10$ by grouping.

19. Factor: $x^2y^2 - 9$

20. Factor: $4x^2 + 12xy + 9y^2$

21. Factor: $x^{2n} - 12x^n + 36$

22. Factor: $64a^3 - 27b^3$

23. Factor: $15x^4 + x^2 - 6$

24. Factor: $21x^4y^4 + 23x^2y^2 + 6$

25. Factor: $3a^6 - 15a^4 - 18a^2$

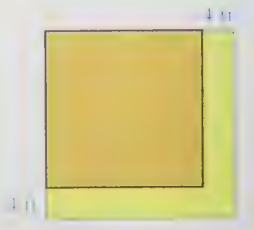
26. Solve: $4x^2 + 27x = 7$

27. Solve: $(x + 1)(x - 5) = 16$

28. The length of a hockey field is 20 yd less than twice the width of the hockey field. The area of the hockey field is 6000 yd^2. Find the length and width of the hockey field.

29. A rectangular photograph has dimensions 15 in. by 12 in. A picture frame around the photograph increases the total area to 270 in^2. What is the width of the frame?

30. The lengths of two sides of a square garden plot are extended 4 ft, as shown in the figure at the right. The area of the resulting square is 576 ft^2. Find the length of a side of the original garden plot.



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Chapter Test

1. Factor: $ab + 6a - 3b - 18$

2. Factor: $2y^4 - 14y^3 - 16y^2$

3. Factor $8x^2 + 20x - 48$ by grouping.

4. Factor $6x^2 + 19x + 8$ by using trial factors.

5. Factor: $a^2 - 19a + 48$

6. Factor: $6x^3 - 8x^2 + 10x$

7. Factor: $x^2 + 2x - 15$

8. Solve: $4x^2 - 1 = 0$

9. Factor: $5x^2 - 45x - 15$

10. Factor: $p^2 + 12p + 36$

11. Solve: $x(x - 8) = -15$

12. Factor: $3x^2 + 12xy + 12y^2$

13. Factor: $27x^3 - 8$

14. Factor $6x^2y^2 + 9xy^2 + 3y^2$ by grouping.

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15. Factor: $6a^4 - 13a^2 - 5$

16. Factor: $a(x - 2) + b(x - 2)$

17. Factor: $x(p + 1) - (p + 1)$

18. Factor: $3a^2 - 75$

19. Factor $2x^2 + 4x - 5$ by using trial factors.

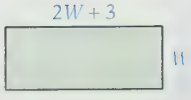
20. Factor: $x^2 - 9x - 36$

21. Factor: $4a^2 - 12ab + 9b^2$

22. Factor: $4x^2 - 49y^2$

23. Solve: $(2a - 3)(a + 7) = 0$

24. The length of a rectangle is 3 cm longer than twice its width. The area of the rectangle is 90 cm^2. Find the length and width of the rectangle.



25. The sum of two numbers is ten. The sum of the squares of the two numbers is fifty-eight. Find the two numbers.

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Cumulative Review

1. Subtract: $-2 - (-3) - 5 - (-11)$

2. Simplify: $(3 - 7)^2 \div (-2) \cdot 3(-4)$

3. Evaluate $-2a^2 \div (2b) - c$ when $a = -4$, $b = 2$, and $c = -1$.

4. Multiply: $-\frac{3}{4}(-20x^2)$

5. Simplify: $-2[4x - 2(3 - 2x) - 8x]$

6. Solve: $-\frac{5}{7}x = -\frac{10}{21}$

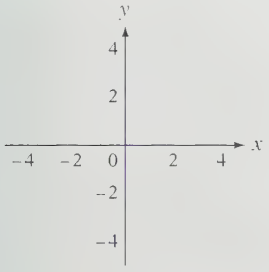
7. Solve: $3x - 2 = 12 - 5x$

8. Solve: $-2 + 4[3x - 2(4 - x) - 3] = 4x + 2$

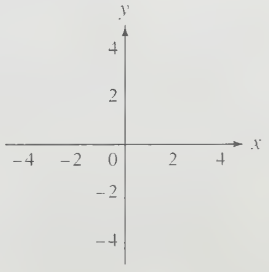
9. 120% of what number is 54?

10. Given $f(x) = -x^2 + 3x - 1$, find $f(2)$.

11. Graph $y = \frac{1}{4}x + 3$.



12. Graph $5x + 3y = 15$.



13. Find the equation of the line that contains the point $(-3, 4)$ and has slope $\frac{2}{3}$.

14. Solve by substitution: $8x - y = 2$
$y = 5x + 1$

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15. Solve by the addition method:
$5x + 2y = -9$
$12x - 7y = 2$

16. Simplify: $(-3a^3b^2)^2$

17. Multiply: $(x + 2)(x^2 - 5x + 4)$

18. Divide: $(8x^2 + 4x - 3) \div (2x - 3)$

19. Simplify: $(x^{-4}y^3)^2$

20. Factor: $3a - 3b - ax + bx$

21. Factor: $15xy^2 - 20xy^4$

22. Factor: $x^2 - 5xy - 14y^2$

23. Solve: $3x^2 + 19x - 14 = 0$

24. Solve: $6x^2 + 60 = 39x$

25. A triangle has a 31° angle and a right angle. Find the measure of the third angle.

26. A rectangular flower garden has a perimeter of 86 ft. The length of the garden is 28 ft. What is the width of the garden?

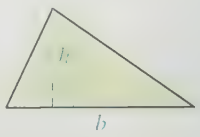


27. A board 10 ft long is cut into two pieces. Four times the length of the shorter piece is 2 ft less than three times the length of the longer piece. Find the length of each piece.

28. An investment of $4000 was made at an annual simple interest rate of 8%. How much more money was invested at an annual simple interest rate of 11% if the total interest earned in 1 year was $1035?

29. A family drove to a resort at an average speed of 42 mph and later returned over the same road at an average speed of 56 mph. Find the distance to the resort if the total driving time was 7 h.

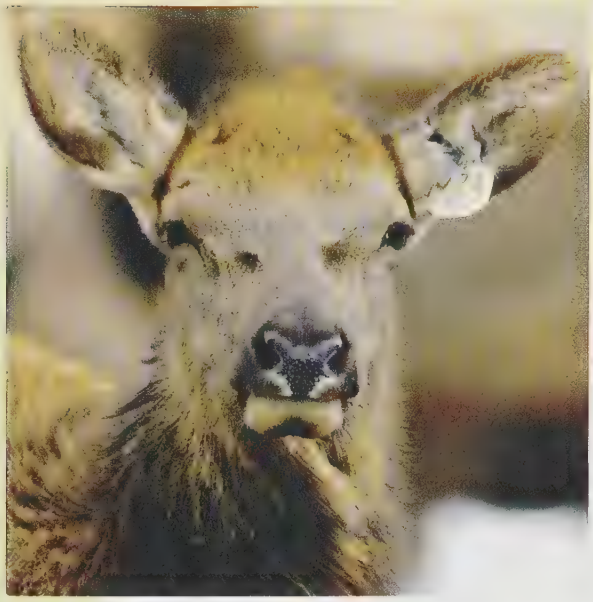
30. The length of the base of a triangle is three times the height. The area of the triangle is 24 in^2. Find the length of the base of the triangle.



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Chapter 8

Rational Expressions



In order to monitor species that are or are becoming endangered, scientists need to determine the present population of that species. Scientists catch and tag a certain number of the animals and then release them. Later, a group of the animals from that same habitat is caught and the number tagged is counted. A proportion is used to estimate the total population size in that region, as shown in **Exercise 70 on page 465.** Tracking the tagged animals also assists scientists in learning more about the habits of that species.

Objectives

Section 8.1

A To simplify a rational expression
B To multiply rational expressions
C To divide rational expressions

Section 8.2

A To find the least common multiple (LCM) of two or more polynomials
B To express two fractions in terms of the LCM of their denominators
C To add or subtract rational expressions with the same denominator
D To add or subtract rational expressions with different denominators

Section 8.3

A To simplify a complex fraction

Section 8.4

A To solve rational equations
B To solve proportions
C To solve problems involving similar triangles
D To solve application problems

Section 8.5

A To solve a literal equation for one of the variables

Section 8.6

A To solve work problems
B To solve uniform motion problems

Section 8.7

A To solve variation problems



Need help? For on-line student resources, such as section quizzes, visit this textbook's web site at **college.hmco.com/students.**

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Warm Ups

1. Find the LCM of 10 and 25.

For Exercises 2 to 5, add, subtract, multiply, or divide.

2. $-\frac{3}{8} \cdot \frac{4}{9}$

3. $-\frac{4}{5} \div \frac{8}{15}$

4. $-\frac{5}{6} + \frac{7}{8}$

5. $-\frac{3}{8} - \left(-\frac{7}{12}\right)$

6. Evaluate $\frac{2x - 3}{x^2 - x + 1}$ for $x = 2$.

7. Solve: $4(2x + 1) = 3(x - 2)$

8. Solve: $10\left(\frac{t}{2} + \frac{t}{5}\right) = 10(1)$

9. Two planes start from the same point and fly in opposite directions. The first plane is flying 20 mph slower than the second plane. In 2 h, the planes are 480 mi apart. Find the rate of each plane.

On Figure



If 6 machines can fill 12 boxes of cereal in 7 min, how many boxes of cereal can 14 machines fill in 12 min?

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8.1 Multiplication and Division of Rational Expressions

Objective A

To simplify a rational expression

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A fraction in which the numerator and denominator are polynomials is called a **rational expression**. Examples of rational expressions are shown at the right.

$$\frac{5}{z}, \quad \frac{x^2+1}{2x-1}, \quad \frac{y^2+y-1}{4y^2+1}$$

Care must be exercised with a rational expression to ensure that when the variables are replaced with numbers, the resulting denominator is not zero.

Consider the rational expression at the right. The value of x cannot be 3, because the denominator would then be zero.

$$\frac{4x^2-9}{2x-6}$$

$$\frac{4(3)^2-9}{2(3)-6}=\frac{27}{0}$$ Not a real number

A rational expression is in simplest form when the numerator and denominator have no common factors. The Multiplication Property of One is used to write a rational expression in simplest form.

Simplify: $\dfrac{x^2-4}{x^2-2x-8}$

$$\frac{x^2-4}{x^2-2x-8}=\frac{(x-2)(x+2)}{(x-4)(x+2)}$$

- Factor the numerator and denominator.

$$=\frac{x-2}{x-4}\cdot\frac{x+2}{x+2}=\frac{x-2}{x-4}\cdot 1$$

$$=\frac{x-2}{x-4}, x\neq -2, 4$$

- The restrictions $x \neq -2, 4$ are necessary to prevent division by zero.

This simplification is usually shown with slashes through the common factors. The last simplification would be shown as follows:

$$\frac{x^2-4}{x^2-2x-8}=\frac{(x-2)\overset{1}{\cancel{(x+2)}}}{(x-4)\underset{1}{\cancel{(x+2)}}}$$

- Factor the numerator and denominator.

$$=\frac{x-2}{x-4}, x\neq -2, 4$$

- Divide by the common factors.

Simplify: $\dfrac{10+3x-x^2}{x^2-4x-5}$

$$\frac{10+3x-x^2}{x^2-4x-5}=\frac{-(x^2-3x-10)}{x^2-4x-5}$$

- Factor -1 from the trinomial in the numerator.

$$=\frac{-(x+2)\overset{1}{\cancel{(x-5)}}}{(x+1)\underset{1}{\cancel{(x-5)}}}$$

- Factor the numerator and denominator. Divide by the common factors.

$$=-\frac{x+2}{x+1}, x\neq -1, 5$$

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For the remaining examples, we will omit the restrictions on the variables that prevent division by zero and assume that the values of the variables are such that division by zero is not possible.

Example 1

Simplify: $\dfrac{4x^3y^4}{6x^4y}$

Solution

$\dfrac{4x^3y^4}{6x^4y} = \dfrac{2y^3}{3x}$ • Use rules of exponents.

You Try It 1

Simplify: $\dfrac{6x^5y}{12x^2y^3}$

Your solution

Example 2

Simplify: $\dfrac{9 - x^2}{x^2 + x - 12}$

Solution

$$\frac{9 - x^2}{x^2 + x - 12} = \frac{-(x^2 - 9)}{x^2 + x - 12}$$

$$= \frac{-(x + 3)(\cancel{x - 3})}{(\cancel{x - 3})(x + 4)} = -\frac{x + 3}{x + 4}$$

You Try It 2

Simplify: $\dfrac{x^2 + 2x - 24}{16 - x^2}$

Your solution

Example 3

Simplify: $\dfrac{x^2 + 2x - 15}{x^2 - 7x + 12}$

Solution

$\dfrac{x^2 + 2x - 15}{x^2 - 7x + 12} = \dfrac{(x + 5)(\cancel{x - 3})}{(\cancel{x - 3})(x - 4)} = \dfrac{x + 5}{x - 4}$

You Try It 3

Simplify: $\dfrac{x^2 + 4x - 12}{x^2 - 3x + 2}$

Your solution

Solutions on pp. S22–S23

To multiply rational expressions

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The product of two fractions is a fraction whose numerator is the product of the numerators of the two fractions and whose denominator is the product of the denominators of the two fractions.

Rule for Multiplying Fractions

If $\dfrac{a}{b}$ and $\dfrac{c}{d}$ are fractions and $b \neq 0$, $d \neq 0$, then $\dfrac{a}{b} \cdot \dfrac{c}{d} = \dfrac{ac}{bd}$.

$\dfrac{2}{3} \cdot \dfrac{4}{5} = \dfrac{8}{15}$ $\qquad \dfrac{3x}{y} \cdot \dfrac{2}{z} = \dfrac{6x}{yz}$ $\qquad \dfrac{x + 2}{x} \cdot \dfrac{3}{x - 2} = \dfrac{3x + 6}{x^2 - 2x}$

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➡ Multiply: $\dfrac{x^2+3x}{x^2-3x-4}\cdot\dfrac{x^2-5x+4}{x^2+2x-3}$

$$\frac{x^2+3x}{x^2-3x-4}\cdot\frac{x^2-5x+4}{x^2+2x-3}$$

$$=\frac{x(x+3)}{(x-4)(x+1)}\cdot\frac{(x-4)(x-1)}{(x+3)(x-1)}$$

- Factor the numerator and denominator of each fraction.

$$=\frac{\overset{1}{\cancel{x(x+3)}}\overset{1}{\cancel{(x-4)}}\overset{1}{\cancel{(x-1)}}}{\underset{1}{\cancel{(x-4)}}(x+1)\underset{1}{\cancel{(x+3)}}\underset{1}{\cancel{(x-1)}}}$$

- Multiply.

$$=\frac{x}{x+1}$$

- Write the answer in simplest form.

Example 4

Multiply: $\dfrac{10x^2-15x}{12x-8}\cdot\dfrac{3x-2}{20x-25}$

Solution

$$\frac{10x^2-15x}{12x-8}\cdot\frac{3x-2}{20x-25}$$

$$=\frac{5x(2x-3)}{4(3x-2)}\cdot\frac{(3x-2)}{5(4x-5)}$$

$$=\frac{\overset{1}{5}x(2x-3)\overset{1}{\cancel{(3x-2)}}}{4\underset{1}{\cancel{(3x-2)}}\underset{1}{5}(4x-5)}=\frac{x(2x-3)}{4(4x-5)}$$

You Try It 4

Multiply: $\dfrac{12x^2+3x}{10x-15}\cdot\dfrac{8x-12}{9x+18}$

Your solution

Example 5

Multiply: $\dfrac{x^2+x-6}{x^2+7x+12}\cdot\dfrac{x^2+3x-4}{4-x^2}$

Solution

$$\frac{x^2+x-6}{x^2+7x+12}\cdot\frac{x^2+3x-4}{4-x^2}$$

$$=\frac{(x+3)(x-2)}{(x+3)(x+4)}\cdot\frac{(x+4)(x-1)}{(2-x)(2+x)}$$

$$=\frac{\overset{1}{\cancel{(x+3)}}\overset{-1}{\cancel{(x-2)}}\overset{1}{\cancel{(x+4)}}(x-1)}{\underset{1}{\cancel{(x+3)}}\underset{1}{\cancel{(x+4)}}\underset{1}{\cancel{(2-x)}}(2+x)}=-\frac{x-1}{x+2}$$

You Try It 5

Multiply: $\dfrac{x^2+2x-15}{9-x^2}\cdot\dfrac{x^2-3x-18}{x^2-7x+6}$

Your solution

Solutions on p. S23

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To divide rational expressions

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The reciprocal of a fraction is a fraction with the numerator and denominator interchanged.

Fraction	Reciprocal
$\frac{a}{b}$	$\frac{b}{a}$
$x^2 = \frac{x^2}{1}$	$\frac{1}{x^2}$
$\frac{x+2}{x}$	$\frac{x}{x+2}$

Rule for Dividing Fractions

Divide fractions by multiplying the dividend by the reciprocal of the divisor.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

$$\frac{4}{x} \div \frac{5}{x} = \frac{4}{x} \cdot \frac{x}{5} = \frac{20}{xy}$$

$$\frac{x+4}{x} \div \frac{x-2}{4} = \frac{x+4}{x} \cdot \frac{4}{x-2} = \frac{4(x+4)}{x(x-2)}$$

The basis for the division rule is shown at the right.

$$\frac{a}{b} \div \frac{c}{d} = \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{\frac{a}{b}}{\frac{c}{d}} \cdot \frac{\frac{d}{c}}{\frac{d}{c}} = \frac{\frac{a}{b} \cdot \frac{d}{c}}{1} = \frac{a}{b} \cdot \frac{d}{c}$$

Example 6

Divide: $\frac{xy^2 - 3x^2y}{z^2} \div \frac{6x^2 - 2xy}{z^3}$

Solution

$$\frac{xy^2 - 3x^2y}{z^2} \div \frac{6x^2 - 2xy}{z^3}$$

$$= \frac{xy^2 - 3x^2y}{z^2} \cdot \frac{z^3}{6x^2 - 2xy}$$

$$= \frac{xy(y - 3x) \cdot z^3}{z^2 \cdot 2x(3x - y)} = \frac{yz}{2}$$

You Try It 6

Divide: $\frac{a^2}{4bc^2 - 2b^2c} \div \frac{a}{6bc - 3b^2}$

Your solution

Example 7

Divide: $\frac{2x^2 + 5x + 2}{2x^2 + 3x - 2} \div \frac{3x^2 + 13x + 4}{2x^2 + 7x - 4}$

Solution

$$\frac{2x^2 + 5x + 2}{2x^2 + 3x - 2} \div \frac{3x^2 + 13x + 4}{2x^2 + 7x - 4}$$

$$= \frac{2x^2 + 5x + 2}{2x^2 + 3x - 2} \cdot \frac{2x^2 + 7x - 4}{3x^2 + 13x + 4}$$

$$= \frac{(2x + 1)(x + 2)(2x - 1)(x + 4)}{(2x - 1)(x + 2)(3x + 1)(x + 4)} = \frac{2x + 1}{3x + 1}$$

You Try It 7

Divide: $\frac{3x^2 + 26x + 16}{3x^2 - 7x - 6} \div \frac{2x^2 + 9x - 5}{x^2 + 2x - 15}$

Your solution

Solutions on p. S23

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8.1 Exercises

Objective 4

1. When is a rational expression in simplest form?

2. Are the rational expressions $\frac{x(x-2)}{2(x-2)}$ and $\frac{x}{2}$ equal for all values of x? Why or why not?

Simplify.

3. $\frac{9x^3}{12x^4}$

4. $\frac{16x^2y}{24xy^3}$

5. $\frac{(x+3)^2}{(x+3)^3}$

6. $\frac{(2x-1)^5}{(2x-1)^4}$

7. $\frac{3n-4}{4-3n}$

8. $\frac{5-2x}{2x-5}$

9. $\frac{6y(y+2)}{9y^2(y+2)}$

10. $\frac{12x^2(3-x)}{18x(3-x)}$

11. $\frac{6x(x-5)}{8x^2(5-x)}$

12. $\frac{14x^3(7-3x)}{21x(3x-7)}$

13. $\frac{a^2+4a}{ab+4b}$

14. $\frac{x^2-3x}{2x-6}$

15. $\frac{4-6x}{3x^2-2x}$

16. $\frac{5xy-3y}{9-15x}$

17. $\frac{y^2-3y+2}{y^2-4y+3}$

18. $\frac{x^2+5x+6}{x^2+8x+15}$

19. $\frac{x^2+3x-10}{x^2+2x-8}$

20. $\frac{a^2+7a-8}{a^2+6a-7}$

21. $\frac{x^2+x-12}{x^2-6x+9}$

22. $\frac{x^2+8x+16}{x^2-2x-24}$

23. $\frac{x^2-3x-10}{25-x^2}$

24. $\frac{4-y^2}{y^2-3y-10}$

25. $\frac{2x^3+2x^2-4x}{x^3+2x^2-3x}$

26. $\frac{3x^3-12x}{6x^3-24x^2+24x}$

27. $\frac{6x^2-7x+2}{6x^2+5x-6}$

28. $\frac{2n^2-9n+4}{2n^2-5n-12}$

29. $\frac{x^2+3x-28}{24-2x-x^2}$

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Multiply.

30. $\frac{8x^2}{9y^3} \cdot \frac{3y^2}{4x^3}$

31. $\frac{14a^2b^3}{15x^5y^2} \cdot \frac{25x^3y}{16ab}$

32. $\frac{12x^3y^4}{7a^2b^3} \cdot \frac{14a^3b^4}{9x^2y^2}$

33. $\frac{18a^4b^2}{25x^2y^3} \cdot \frac{50x^5y^6}{27a^6b^2}$

34. $\frac{3x - 6}{5x - 20} \cdot \frac{10x - 40}{27x - 54}$

35. $\frac{8x - 12}{14x + 7} \cdot \frac{42x + 21}{32x - 48}$

36. $\frac{3x^2 + 2x}{2xy - 3y} \cdot \frac{2xy^3 - 3y^3}{3x^3 + 2x^2}$

37. $\frac{4a^2x - 3a^2}{2by + 5b} \cdot \frac{2b^3y + 5b^3}{4ax - 3a}$

38. $\frac{x^2 + 5x + 4}{x^3y^2} \cdot \frac{x^2y^3}{x^2 + 2x + 1}$

39. $\frac{x^2 + x - 2}{xy^2} \cdot \frac{x^3y}{x^2 + 5x + 6}$

40. $\frac{x^4y^2}{x^2 + 3x - 28} \cdot \frac{x^2 - 49}{xy^4}$

41. $\frac{x^5y^3}{x^2 + 13x + 30} \cdot \frac{x^2 + 2x - 3}{x^7y^2}$

42. $\frac{2x^2 - 5x}{2xy + y} \cdot \frac{2xy^2 + y^2}{5x^2 - 2x^3}$

43. $\frac{3a^3 + 4a^2}{5ab - 3b} \cdot \frac{3b^3 - 5ab^3}{3a^2 + 4a}$

44. $\frac{x^2 - 2x - 24}{x^2 - 5x - 6} \cdot \frac{x^2 + 5x + 6}{x^2 + 6x + 8}$

45. $\frac{x^2 - 8x + 7}{x^2 + 3x - 4} \cdot \frac{x^2 + 3x - 10}{x^2 - 9x + 14}$

46. $\frac{x^2 + 2x - 35}{x^2 + 4x - 21} \cdot \frac{x^2 + 3x - 18}{x^2 + 9x + 18}$

47. $\frac{y^2 + y - 20}{y^2 + 2y - 15} \cdot \frac{y^2 + 4y - 21}{y^2 + 3y - 28}$

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48. $\dfrac{x^2 - 3x - 4}{x^2 + 6x + 5} \cdot \dfrac{x^2 + 5x + 6}{8 + 2x - x^2}$

49. $\dfrac{25 - n^2}{n^2 - 2n - 35} \cdot \dfrac{n^2 - 8n - 20}{n^2 - 3n - 10}$

50. $\dfrac{12x^2 - 6x}{x^2 + 6x + 5} \cdot \dfrac{2x^4 + 10x^3}{4x^2 - 1}$

51. $\dfrac{8x^3 + 4x^2}{x^2 - 3x + 2} \cdot \dfrac{x^2 - 4}{16x^2 + 8x}$

52. $\dfrac{16 + 6x - x^2}{x^2 - 10x - 24} \cdot \dfrac{x^2 - 6x - 27}{x^2 - 17x + 72}$

53. $\dfrac{x^2 - 11x + 28}{x^2 - 13x + 42} \cdot \dfrac{x^2 + 7x + 10}{20 - x - x^2}$

54. $\dfrac{2x^2 + 5x + 2}{2x^2 + 7x + 3} \cdot \dfrac{x^2 - 7x - 30}{x^2 - 6x - 40}$

55. $\dfrac{x^2 - 4x - 32}{x^2 - 8x - 48} \cdot \dfrac{3x^2 + 17x + 10}{3x^2 - 22x - 16}$

Objective 2

Divide.

56. $\dfrac{4x^2y^3}{15a^2b^3} \div \dfrac{6xy}{5a^3b^5}$

57. $\dfrac{9x^3y^4}{16a^4b^2} \div \dfrac{45x^4y^2}{14a^7b}$

58. $\dfrac{6x - 12}{8x + 32} \div \dfrac{18x - 36}{10x + 40}$

59. $\dfrac{28x + 14}{45x - 30} \div \dfrac{14x + 7}{30x - 20}$

60. $\dfrac{6x^3 + 7x^2}{12x - 3} \div \dfrac{6x^2 + 7x}{36x - 9}$

61. $\dfrac{5a^2y + 3a^2}{2x^3 + 5x^2} \div \dfrac{10ay + 6a}{6x^3 + 15x^2}$

62. $\dfrac{x^2 + 4x + 3}{x^2y} \div \dfrac{x^2 + 2x + 1}{xy^2}$

63. $\dfrac{x^3y^2}{x^2 - 3x - 10} \div \dfrac{xy^4}{x^2 - x - 20}$

64. $\dfrac{x^2 - 49}{x^4y^3} \div \dfrac{x^2 - 14x + 49}{x^4y^3}$

65. $\dfrac{x^2y^5}{x^2 - 11x + 30} \div \dfrac{xy^6}{x^2 - 7x + 10}$

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66. $\dfrac{4ax - 8a}{c^2} \div \dfrac{2y - xy}{c^3}$

67. $\dfrac{3x^2y - 9xy}{a^2b} \div \dfrac{3x^2 - x^3}{ab^2}$

68. $\dfrac{x^2 - 5x + 6}{x^2 - 9x + 18} \div \dfrac{x^2 - 6x + 8}{x^2 - 9x + 20}$

69. $\dfrac{x^2 + 3x - 40}{x^2 + 2x - 35} \div \dfrac{x^2 + 2x - 48}{x^2 + 3x - 18}$

70. $\dfrac{x^2 + 2x - 15}{x^2 - 4x - 45} \div \dfrac{x^2 + x - 12}{x^2 - 5x - 36}$

71. $\dfrac{y^2 - y - 56}{y^2 + 8y + 7} \div \dfrac{y^2 - 13y + 40}{y^2 - 4y - 5}$

72. $\dfrac{8 + 2x - x^2}{x^2 + 7x + 10} \div \dfrac{x^2 - 11x + 28}{x^2 - x - 42}$

73. $\dfrac{x^2 - x - 2}{x^2 - 7x + 10} \div \dfrac{x^2 - 3x - 4}{40 - 3x - x^2}$

74. $\dfrac{2x^2 - 3x - 20}{2x^2 - 7x - 30} \div \dfrac{2x^2 - 5x - 12}{4x^2 + 12x + 9}$

75. $\dfrac{6n^2 + 13n + 6}{4n^2 - 9} \div \dfrac{6n^2 + n - 2}{4n^2 - 1}$

APPLYING THE CONCEPTS

76. Given the expression $\dfrac{9}{x^2 + 1}$, choose some values of x and evaluate the expression for those values. Is it possible to choose a value of x for which the value of the expression is greater than 10? If so, give such a value. If not, explain why it is not possible.

77. Given the expression $\dfrac{1}{y - 3}$, choose some values of y and evaluate the expression for those values. Is it possible to choose a value of y for which the value of the expression is greater than 10,000,000? If so, give such a value. If not, explain why it is not possible.

Complete each simplification.

78. $\dfrac{8x}{9y} \div \underline{\quad ? \quad} = \dfrac{10y}{9}$

79. $\dfrac{n}{n + 3} \div \underline{\quad ? \quad} = \dfrac{n}{n - 2}$

Simplify.

80. $\dfrac{xy}{3} \cdot \dfrac{x}{y^2} \div \dfrac{x}{4}$

81. $\left(\dfrac{y}{3}\right) \div \left(\dfrac{y}{2} \cdot \dfrac{y}{4}\right)$

82. $\left(\dfrac{x - 4}{y^2}\right)^3 \cdot \left(\dfrac{y}{4 - x}\right)^3$

83. $\dfrac{x - 2}{x + 5} \div \dfrac{x - 3}{x + 5} \cdot \dfrac{x - 3}{x - 2}$

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8.2 Addition and Subtraction of Rational Expressions

Objective A

To find the least common multiple (LCM) of two or more polynomials

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The **least common multiple (LCM)** of two or more numbers is the smallest number that contains the prime factorization of each number.

TAKE NOTE
The LCM of 12 and 18 is 36 because 36 is the smallest number that both 12 and 18 divide evenly into.

The LCM of 12 and 18 is 36 because 36 contains the prime factors of 12 and the prime factors of 18.

$12 = 2 \cdot 2 \cdot 3$
$18 = 2 \cdot 3 \cdot 3$

$\text{LCM} = 36 = \underbrace{2 \cdot 2 \cdot 3}_{\text{Factors of 12}} \cdot \underbrace{3}_{\text{Factors of 18}}$

The least common multiple of two or more polynomials is the polynomial of least degree that contains the factors of each polynomial.

To find the LCM of two or more polynomials, first factor each polynomial completely. The LCM is the product of each factor the greatest number of times it occurs in any one factorization.

Find the LCM of $4x^2 + 4x$ and $x^2 + 2x + 1$.

TAKE NOTE
The LCM must contain the factors of each polynomial. As shown with the braces at the right, the LCM contains the factors of $4x^2 + 4x$ and the factors of $x^2 + 2x + 1$.

The LCM of the polynomials is the product of the LCM of the numerical coefficients and each variable factor the greatest number of times it occurs in any one factorization.

$4x^2 + 4x = 4x(x + 1) = 2 \cdot 2 \cdot x(x + 1)$
$x^2 + 2x + 1 = (x + 1)(x + 1)$

$\text{LCM} = \underbrace{2 \cdot 2 \cdot x(x + 1)}_{\text{Factors of } 4x^2 + 4x}(x + 1) = 4x(x + 1)(x + 1)$

Factors of $x^2 + 2x + 1$

Example 1
Find the LCM of $4x^2y$ and $6xy^2$.

Solution
$4x^2y = 2 \cdot 2 \cdot x \cdot x \cdot y$
$6xy^2 = 2 \cdot 3 \cdot x \cdot y \cdot y$
$\text{LCM} = 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot y \cdot y = 12x^2y^2$

You Try It 1
Find the LCM of $8uv^2$ and $12uw$.

Your solution

Example 2
Find the LCM of $x^2 - x - 6$ and $9 - x^2$.

Solution
$x^2 - x - 6 = (x - 3)(x + 2)$
$9 - x^2 = -(x^2 - 9) = -(x + 3)(x - 3)$
$\text{LCM} = (x - 3)(x + 2)(x + 3)$

You Try It 2
Find the LCM of $m^2 - 6m + 9$ and $m^2 - 2m - 3$.

Your solution

Solutions on p. S23

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To express two fractions in terms of the LCM of their denominators

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When adding and subtracting fractions, it is frequently necessary to express two or more fractions in terms of a common denominator. This common denominator is the LCM of the denominators of the fractions.

➡ Write the fractions $\frac{x+1}{4x^2}$ and $\frac{x-3}{6x^2-12x}$ in terms of the LCM of the denominators.

Find the LCM of the denominators.

The LCM is $12x^2(x-2)$.

For each fraction, multiply the numerator and denominator by the factors whose product with the denominator is the LCM.

$$\frac{x+1}{4x^2} = \frac{x+1}{4x^2} \cdot \frac{3(x-2)}{3(x-2)} = \frac{3x^2-3x-6}{12x^2(x-2)}$$

$$\frac{x-3}{6x^2-12x} = \frac{x-3}{6x(x-2)} \cdot \frac{2x}{2x} = \frac{2x^2-6x}{12x^2(x-2)}$$

LCM

TAKE NOTE

$\frac{3(x-2)}{3(x-2)} = 1$ and $\frac{2x}{2x} = 1$. We are multiplying each fraction by 1, so we are not changing the value of either fraction.

Example 3

Write the fractions $\frac{x+2}{3x^2}$ and $\frac{x-1}{8xy}$ in terms of the LCM of the denominators.

Solution

The LCM is $24x^2y$.

$$\frac{x+2}{3x^2} = \frac{x+2}{3x^2} \cdot \frac{8y}{8y} = \frac{8xy+16y}{24x^2y}$$

$$\frac{x-1}{8xy} = \frac{x-1}{8xy} \cdot \frac{3x}{3x} = \frac{3x^2-3x}{24x^2y}$$

You Try It 3

Write the fractions $\frac{x-3}{4xy^2}$ and $\frac{2x+1}{9y^2z}$ in terms of the LCM of the denominators.

Your solution

Example 4

Write the fractions $\frac{2x-1}{2x-x^2}$ and $\frac{x}{x^2+x-6}$ in terms of the LCM of the denominators.

Solution

$$\frac{2x-1}{2x-x^2} = \frac{2x-1}{-(x^2-2x)} = -\frac{2x-1}{x^2-2x}$$

The LCM is $x(x-2)(x+3)$.

$$\frac{2x-1}{2x-x^2} = -\frac{2x-1}{x(x-2)} \cdot \frac{x+3}{x+3} = -\frac{2x^2+5x-3}{x(x-2)(x+3)}$$

$$\frac{x}{x^2+x-6} = \frac{x}{(x-2)(x+3)} \cdot \frac{x}{x} = \frac{x^2}{x(x-2)(x+3)}$$

You Try It 4

Write the fractions $\frac{x+4}{x^2-3x-10}$ and $\frac{2x}{25-x^2}$ in terms of the LCM of the denominators.

Your solution

Solutions on p. S23

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Objective 5

To add or subtract rational expressions with the same denominator

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When adding rational expressions in which the denominators are the same, add the numerators. The denominator of the sum is the common denominator.

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$$

$$\frac{5x}{18} + \frac{7x}{18} = \frac{5x + 7x}{18} = \frac{12x}{18} = \frac{2x}{3}$$

$$\frac{x}{x^2 - 1} + \frac{1}{x^2 - 1} = \frac{x + 1}{x^2 - 1} = \frac{\cancel{(x+1)}^{1}}{(x-1)\cancel{(x+1)}_{1}} = \frac{1}{x - 1}$$

Note that the sum is written in simplest form.

When subtracting rational expressions in which the denominators are the same, subtract the numerators. The denominator of the difference is the common denominator. Write the answer in simplest form.

TAKE NOTE

Be careful with signs when subtracting algebraic fractions. Note that we must subtract the *entire* numerator $2x + 3$.
$(3x - 1) - (2x + 3) = 3x - 1 - 2x - 3$.

$$\frac{2x}{x - 2} - \frac{4}{x - 2} = \frac{2x - 4}{x - 2} = \frac{2\cancel{(x-2)}^{1}}{\cancel{x-2}_{1}} = 2$$

$$\frac{3x - 1}{x^2 - 5x + 4} - \frac{2x + 3}{x^2 - 5x + 4} = \frac{(3x - 1) - (2x + 3)}{x^2 - 5x + 4} = \frac{3x - 1 - 2x - 3}{x^2 - 5x + 4}$$

$$= \frac{x - 4}{x^2 - 5x + 4} = \frac{\cancel{(x-4)}^{1}}{\cancel{(x-4)}_{1}(x - 1)} = \frac{1}{x - 1}$$

Example 5

Add: $\frac{7}{x^2} + \frac{9}{x^2}$

Solution

$$\frac{7}{x^2} + \frac{9}{x^2} = \frac{7 + 9}{x^2} = \frac{16}{x^2}$$

You Try It 5

Add: $\frac{3}{xy} + \frac{12}{xy}$

Your solution

Example 6

Subtract: $\frac{3x^2}{x^2 - 1} - \frac{x + 4}{x^2 - 1}$

Solution

$$\frac{3x^2}{x^2 - 1} - \frac{x + 4}{x^2 - 1} = \frac{3x^2 - (x + 4)}{x^2 - 1}$$

$$= \frac{3x^2 - x - 4}{x^2 - 1}$$

$$= \frac{(3x - 4)\cancel{(x+1)}^{1}}{(x - 1)\cancel{(x+1)}_{1}} = \frac{3x - 4}{x - 1}$$

You Try It 6

Subtract: $\frac{2x^2}{x^2 - x - 12} - \frac{7x + 4}{x^2 - x - 12}$

Your solution

Solutions on p. S23

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Example 7

Simplify:

$$\frac{2x^2+5}{x^2+2x-3}-\frac{x^2-3x}{x^2+2x-3}+\frac{x-2}{x^2+2x-3}$$

Solution

$$\frac{2x^2+5}{x^2+2x-3}-\frac{x^2-3x}{x^2+2x-3}+\frac{x-2}{x^2+2x-3}$$

$$=\frac{(2x^2+5)-(x^2-3x)+(x-2)}{x^2+2x-3}$$

$$=\frac{2x^2+5-x^2+3x+x-2}{x^2+2x-3}$$

$$=\frac{x^2+4x+3}{x^2+2x-3}=\frac{\cancel{(x+3)}^{1}(x+1)}{\cancel{(x+3)}_{1}(x-1)}=\frac{x+1}{x-1}$$

You Try It 7

Simplify:

$$\frac{x^2-1}{x^2-8x+12}-\frac{2x+1}{x^2-8x+12}+\frac{x}{x^2-8x+12}$$

Your solution

Solution on p. S23

To add or subtract rational expressions with different denominators

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TAKE NOTE

This objective requires using the skills learned in Objective 8.2B (to express two fractions in terms of the LCM of their denominators) and 8.2C (to add or subtract algebraic fractions with the same denominator).

Note the steps involved in adding or subtracting rational expressions:

1. Find the LCM of the denominators.
2. Rewrite each fraction in terms of the common denominator.
3. Add or subtract the rational expressions.
4. Simplify the resulting sum or difference.

Before two fractions with unlike denominators can be added or subtracted, each fraction must be expressed in terms of a common denominator. This common denominator is the LCM of the denominators of the fractions.

Add: $\frac{x-3}{x^2-2x}+\frac{6}{x^2-4}$

$x^2-2x=x(x-2)$; $x^2-4=(x+2)(x-2)$ The LCM is $x(x-2)(x+2)$.

$$\frac{x-3}{x^2-2x}+\frac{6}{x^2-4}=\frac{x-3}{x(x-2)}\cdot\frac{x+2}{x+2}+\frac{6}{(x-2)(x+2)}\cdot\frac{x}{x}$$

- Rewrite each fraction with $x(x-2)(x+2)$ as the denominator.

$$=\frac{x^2-x-6}{x(x-2)(x+2)}+\frac{6x}{x(x-2)(x+2)}$$

$$=\frac{(x^2-x-6)+6x}{x(x-2)(x+2)}$$

- Add the fractions.

$$=\frac{x^2+5x-6}{x(x-2)(x+2)}$$

$$=\frac{(x+6)(x-1)}{x(x-2)(x+2)}$$

The last step is to factor the numerator to determine whether there are common factors in the numerator and denominator. For this example there are no common factors, so the answer is in simplest form.

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Example 8

Simplify: $\frac{y}{x} - \frac{4y}{3x} + \frac{3y}{4x}$

Solution

The LCM of the denominators is $12x$.

$$\frac{y}{x} - \frac{4y}{3x} + \frac{3y}{4x} = \frac{y}{x} \cdot \frac{12}{12} - \frac{4y}{3x} \cdot \frac{4}{4} + \frac{3y}{4x} \cdot \frac{3}{3}$$

$$= \frac{12y}{12x} - \frac{16y}{12x} + \frac{9y}{12x}$$

$$= \frac{12y - 16y + 9y}{12x} = \frac{5y}{12x}$$

You Try It 8

Simplify: $\frac{z}{8y} - \frac{4z}{3y} + \frac{5z}{4y}$

Your solution

Example 9

Subtract: $\frac{2x}{x-3} - \frac{5}{3-x}$

Solution

Remember: $3 - x = -(x - 3)$.

Therefore, $\frac{5}{3-x} = \frac{5}{-(x-3)} = \frac{-5}{x-3}$.

$$\frac{2x}{x-3} - \frac{5}{3-x} = \frac{2x}{x-3} - \frac{-5}{x-3}$$

$$= \frac{2x - (-5)}{x-3} = \frac{2x+5}{x-3}$$

You Try It 9

Subtract: $\frac{5x}{x-2} - \frac{3}{2-x}$

Your solution

Example 10

Subtract: $\frac{2x}{2x-3} - \frac{1}{x+1}$

Solution

The LCM is $(2x - 3)(x + 1)$.

$$\frac{2x}{2x-3} - \frac{1}{x+1}$$

$$= \frac{2x}{2x-3} \cdot \frac{x+1}{x+1} - \frac{1}{x+1} \cdot \frac{2x-3}{2x-3}$$

$$= \frac{2x^2 + 2x}{(2x-3)(x+1)} - \frac{2x-3}{(2x-3)(x+1)}$$

$$= \frac{(2x^2 + 2x) - (2x - 3)}{(2x-3)(x+1)} = \frac{2x^2 + 3}{(2x-3)(x+1)}$$

You Try It 10

Subtract: $\frac{4x}{3x-1} - \frac{9}{x+4}$

Your solution

Solutions on pp. S23–S24

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Example 11

Add: $1 + \frac{3}{x^2}$

Solution

The LCM is x^2.

$$1 + \frac{3}{x^2} = 1 \cdot \frac{x^2}{x^2} + \frac{3}{x^2}$$

$$= \frac{x^2}{x^2} + \frac{3}{x^2} = \frac{x^2 + 3}{x^2}$$

You Try It 11

Subtract: $2 - \frac{1}{x - 3}$

Your solution

Example 12

Subtract: $\frac{x}{2x - 4} - \frac{4 - x}{x^2 - 2x}$

Solution

$2x - 4 = 2(x - 2)$; $x^2 - 2x = x(x - 2)$

The LCM is $2x(x - 2)$.

$$\frac{x}{2x - 4} - \frac{4 - x}{x^2 - 2x} = \frac{x}{2(x - 2)} \cdot \frac{x}{x} - \frac{4 - x}{x(x - 2)} \cdot \frac{2}{2}$$

$$= \frac{x^2 - (4 - x)2}{2x(x - 2)}$$

$$= \frac{x^2 - (8 - 2x)}{2x(x - 2)} = \frac{x^2 + 2x - 8}{2x(x - 2)}$$

$$= \frac{(x + 4)\cancel{(x - 2)}^{1}}{2x\cancel{(x - 2)}_{1}} = \frac{x + 4}{2x}$$

You Try It 12

Add: $\frac{a - 3}{a^2 - 5a} + \frac{a - 9}{a^2 - 25}$

Your solution

Example 13

Simplify: $\frac{3x + 2}{2x^2 - x - 1} - \frac{3}{2x + 1} + \frac{4}{x - 1}$

Solution

The LCM is $(2x + 1)(x - 1)$.

$$\frac{3x + 2}{2x^2 - x - 1} - \frac{3}{2x + 1} + \frac{4}{x - 1}$$

$$= \frac{3x + 2}{(2x + 1)(x - 1)} - \frac{3}{2x + 1} \cdot \frac{x - 1}{x - 1} + \frac{4}{x - 1} \cdot \frac{2x + 1}{2x + 1}$$

$$= \frac{3x + 2}{(2x + 1)(x - 1)} - \frac{3x - 3}{(2x + 1)(x - 1)} + \frac{8x + 4}{(2x + 1)(x - 1)}$$

$$= \frac{(3x + 2) - (3x - 3) + (8x + 4)}{(2x + 1)(x - 1)}$$

$$= \frac{3x + 2 - 3x + 3 + 8x + 4}{(2x + 1)(x - 1)} = \frac{8x + 9}{(2x + 1)(x - 1)}$$

You Try It 13

Simplify: $\frac{2x - 3}{3x^2 - x - 2} + \frac{5}{3x + 2} - \frac{1}{x - 1}$

Your solution

Solutions on p. S24

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8.2 Exercises

Objective A

Find the LCM of the expressions.

1. $8x^3y$, $12xy^2$

2. $6ab^2$, $18ab^3$

3. $10x^4y^2$, $15x^3y$

4. $12a^2b$, $18ab^3$

5. $8x^2$, $4x^2 + 8x$

6. $6y^2$, $4y + 12$

7. $2x^2y$, $3x^2 + 12x$

8. $4xy^2$, $6xy^2 + 12y^2$

9. $9x(x + 2)$, $12(x + 2)^2$

10. $8x^2(x - 1)^2$, $10x^3(x - 1)$

11. $3x + 3$, $2x^2 + 4x + 2$

12. $4x - 12$, $2x^2 - 12x + 18$

13. $(x - 1)(x + 2)$, $(x - 1)(x + 3)$

14. $(2x - 1)(x + 4)$, $(2x + 1)(x + 4)$

15. $(2x + 3)^2$, $(2x + 3)(x - 5)$

16. $(x - 7)(x + 2)$, $(x - 7)^2$

17. $x - 1$, $x - 2$, $(x - 1)(x - 2)$

18. $(x + 4)(x - 3)$, $x + 4$, $x - 3$

19. $x^2 - x - 6$, $x^2 + x - 12$

20. $x^2 + 3x - 10$, $x^2 + 5x - 14$

21. $x^2 + 5x + 4$, $x^2 - 3x - 28$

22. $x^2 - 10x + 21$, $x^2 - 8x + 15$

23. $x^2 - 2x - 24$, $x^2 - 36$

24. $x^2 + 7x + 10$, $x^2 - 25$

25. $x^2 - 7x - 30$, $x^2 - 5x - 24$

26. $2x^2 - 7x + 3$, $2x^2 + x - 1$

27. $3x^2 - 11x + 6$, $3x^2 + 4x - 4$

28. $2x^2 - 9x + 10$, $2x^2 + x - 15$

29. $6 + x - x^2$, $x + 2$, $x - 3$

30. $15 + 2x - x^2$, $x - 5$, $x + 3$

31. $5 + 4x - x^2$, $x - 5$, $x + 1$

32. $x^2 + 3x - 18$, $3 - x$, $x + 6$

33. $x^2 - 5x + 6$, $1 - x$, $x - 6$

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Write each fraction in terms of the LCM of the denominators.

34. $\frac{4}{x}, \frac{3}{x^2}$

35. $\frac{5}{ab^2}, \frac{6}{ab}$

36. $\frac{x}{3y^2}, \frac{z}{4y}$

37. $\frac{5y}{6x^2}, \frac{7}{9xy}$

38. $\frac{y}{x(x-3)}, \frac{6}{x^2}$

39. $\frac{a}{y^2}, \frac{6}{y(y+5)}$

40. $\frac{9}{(x-1)^2}, \frac{6}{x(x-1)}$

41. $\frac{a^2}{y(y+7)}, \frac{a}{(y+7)^2}$

42. $\frac{3}{x-3}, \frac{5}{x(3-x)}$

43. $\frac{b}{y(y-4)}, \frac{b^2}{4-y}$

44. $\frac{3}{(x-5)^2}, \frac{2}{5-x}$

45. $\frac{3}{7-y}, \frac{2}{(y-7)^2}$

46. $\frac{3}{x^2+2x}, \frac{4}{x^2}$

47. $\frac{2}{y-3}, \frac{3}{y^3-3y^2}$

48. $\frac{x-2}{x+3}, \frac{x}{x-4}$

49. $\frac{x^2}{2x-1}, \frac{x+1}{x+4}$

50. $\frac{3}{x^2+x-2}, \frac{x}{x+2}$

51. $\frac{3x}{x-5}, \frac{4}{x^2-25}$

52. $\frac{5}{2x^2-9x+10}, \frac{x-1}{2x-5}$

53. $\frac{x-3}{3x^2+4x-4}, \frac{2}{x+2}$

54. $\frac{x}{x^2+x-6}, \frac{2x}{x^2-9}$

55. $\frac{x-1}{x^2+2x-15}, \frac{x}{x^2+6x+5}$

56. Explain the procedure for subtracting rational expressions with the same denominator.

Add or subtract.

57. $\frac{3}{y^2}+\frac{8}{y^2}$

58. $\frac{6}{ab}-\frac{2}{ab}$

59. $\frac{3}{x+4}-\frac{10}{x+4}$

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60. $\frac{x}{x+6} - \frac{2}{x+6}$

61. $\frac{3x}{2x+3} + \frac{5x}{2x+3}$

62. $\frac{6y}{4y+1} - \frac{11y}{4y+1}$

63. $\frac{2x+1}{x-3} + \frac{3x+6}{x-3}$

64. $\frac{4x+3}{2x-7} + \frac{3x-8}{2x-7}$

65. $\frac{5x-1}{x+9} - \frac{3x+4}{x+9}$

66. $\frac{6x-5}{x-10} - \frac{3x-4}{x-10}$

67. $\frac{x-7}{2x+7} - \frac{4x-3}{2x+7}$

68. $\frac{2n}{3n+4} - \frac{5n-3}{3n+4}$

69. $\frac{x}{x^2+2x-15} - \frac{3}{x^2+2x-15}$

70. $\frac{3x}{x^2+3x-10} - \frac{6}{x^2+3x-10}$

71. $\frac{2x+3}{x^2-x-30} - \frac{x-2}{x^2-x-30}$

72. $\frac{3x-1}{x^2+5x-6} - \frac{2x-7}{x^2+5x-6}$

73. $\frac{4y+7}{2y^2+7y-4} - \frac{y-5}{2y^2+7y-4}$

74. $\frac{x+1}{2x^2-5x-12} + \frac{x+2}{2x^2-5x-12}$

75. $\frac{2x^2+3x}{x^2-9x+20} + \frac{2x^2-3}{x^2-9x+20} - \frac{4x^2+2x+1}{x^2-9x+20}$

76. $\frac{2x^2+3x}{x^2-2x-63} - \frac{x^2-3x+21}{x^2-2x-63} - \frac{x-7}{x^2-2x-63}$

Objective 3

77. Explain the procedure for adding rational expressions with different denominators.

Add or subtract.

78. $\frac{4}{x} + \frac{5}{y}$

79. $\frac{7}{a} + \frac{5}{b}$

80. $\frac{12}{x} - \frac{5}{2x}$

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81. $\frac{5}{3a} - \frac{3}{4a}$

82. $\frac{1}{2x} - \frac{5}{4x} + \frac{7}{6x}$

83. $\frac{7}{4y} + \frac{11}{6y} - \frac{8}{3y}$

84. $\frac{5}{3x} - \frac{2}{x^2} + \frac{3}{2x}$

85. $\frac{6}{y^2} + \frac{3}{4y} - \frac{2}{5y}$

86. $\frac{2}{x} - \frac{3}{2y} + \frac{3}{5x} - \frac{1}{4y}$

87. $\frac{5}{2a} + \frac{7}{3b} - \frac{2}{b} - \frac{3}{4a}$

88. $\frac{2x+1}{3x} + \frac{x-1}{5x}$

89. $\frac{4x-3}{6x} + \frac{2x+3}{4x}$

90. $\frac{x-3}{6x} + \frac{x+4}{8x}$

91. $\frac{2x-3}{2x} + \frac{x+3}{3x}$

92. $\frac{2x+9}{9x} - \frac{x-5}{5x}$

93. $\frac{3y-2}{12y} - \frac{y-3}{18y}$

94. $\frac{x+4}{2x} - \frac{x-1}{x^2}$

95. $\frac{x-2}{3x^2} - \frac{x+4}{x}$

96. $\frac{x-10}{4x^2} + \frac{x+1}{2x}$

97. $\frac{x+5}{3x^2} + \frac{2x+1}{2x}$

98. $\frac{4}{x+4} - x$

99. $2x + \frac{1}{x}$

100. $5 - \frac{x-2}{x+1}$

101. $3 + \frac{x-1}{x+1}$

102. $\frac{x+3}{6x} - \frac{x-3}{8x^2}$

103. $\frac{x+2}{xy} - \frac{3x-2}{x^2y}$

104. $\frac{3x-1}{xy^2} - \frac{2x+3}{xy}$

105. $\frac{4x-3}{3x^2y} + \frac{2x+1}{4xy^2}$

106. $\frac{5x+7}{6xy^2} - \frac{4x-3}{8x^2y}$

107. $\frac{x-2}{8x^2} \cdot \frac{x+7}{12xy}$

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108. $\dfrac{3x-1}{6y^2}-\dfrac{x+5}{9xy}$

109. $\dfrac{4}{x-2}+\dfrac{5}{x+3}$

110. $\dfrac{2}{x-3}+\dfrac{5}{x-4}$

111. $\dfrac{6}{x-7}-\dfrac{4}{x+3}$

112. $\dfrac{3}{y+6}-\dfrac{4}{y-3}$

113. $\dfrac{2x}{x+1}+\dfrac{1}{x-3}$

114. $\dfrac{3x}{x-4}+\dfrac{2}{x+6}$

115. $\dfrac{4x}{2x-1}-\dfrac{5}{x-6}$

116. $\dfrac{6x}{x+5}-\dfrac{3}{2x+3}$

117. $\dfrac{2a}{a-7}+\dfrac{5}{7-a}$

118. $\dfrac{4x}{6-x}+\dfrac{5}{x-6}$

119. $\dfrac{x}{x^2-9}+\dfrac{3}{x-3}$

120. $\dfrac{y}{y^2-16}+\dfrac{1}{y-4}$

121. $\dfrac{2x}{x^2-x-6}-\dfrac{3}{x+2}$

122. $\dfrac{(x-1)^2}{(x+1)^2}-1$

123. $1-\dfrac{(y-2)^2}{(y+2)^2}$

124. $\dfrac{x}{1-x^2}-1+\dfrac{x}{1+x}$

125. $\dfrac{y}{x-y}+2-\dfrac{x}{y-x}$

126. $\dfrac{3x-1}{x^2-10x+25}-\dfrac{3}{x-5}$

127. $\dfrac{2a+3}{a^2-7a+12}-\dfrac{2}{a-3}$

128. $\dfrac{x+4}{x^2-x-42}+\dfrac{3}{7-x}$

129. $\dfrac{x+3}{x^2-3x-10}+\dfrac{2}{5-x}$

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130. $\dfrac{1}{x+1}+\dfrac{x}{x-6}-\dfrac{5x-2}{x^2-5x-6}$

131. $\dfrac{x}{x-4}+\dfrac{5}{x+5}-\dfrac{11x-8}{x^2+x-20}$

132. $\dfrac{3x+1}{x-1}-\dfrac{x-1}{x-3}+\dfrac{x+1}{x^2-4x+3}$

133. $\dfrac{4x+1}{x-8}-\dfrac{3x+2}{x+4}-\dfrac{49x+4}{x^2-4x-32}$

134. $\dfrac{2x+9}{3-x}+\dfrac{x+5}{x+7}-\dfrac{2x^2+3x-3}{x^2+4x-21}$

135. $\dfrac{3x+5}{x+5}-\dfrac{x+1}{2-x}-\dfrac{4x^2-3x-1}{x^2+3x-10}$

APPLYING THE CONCEPTS

136. Simplify.

a. $\left(\dfrac{b}{6}-\dfrac{6}{b}\right)\div\left(\dfrac{6}{b}-4+\dfrac{b}{2}\right)$

b. $\dfrac{x^2+x-6}{x^2+2x-8}\cdot\dfrac{x^2+5x+4}{x^2+2x-3}-\dfrac{2}{x-1}$

137. Rewrite the fraction as the sum of two fractions in simplest form.

a. $\dfrac{3x+6y}{xy}$

b. $\dfrac{4a^2+3ab}{a^2b^2}$

c. $\dfrac{3m^2n+2mn^2}{12m^3n^2}$

138. Let $f(x)=\dfrac{x}{x+2}$, $g(x)=\dfrac{4}{x-3}$, and $S(x)=\dfrac{x^2+x+8}{x^2-x-6}$. Evaluate $f(4)$, $g(4)$, and $S(4)$. Does $f(4)+g(4)=S(4)$? Let a be a real number ($a\neq -2, a\neq 3$). Express $S(a)$ in terms of $f(a)$ and $g(a)$.

139. Suppose you drive about 12,000 mi per year and that the cost of gasoline averages \$1.60 per gallon.

a. Let x represent the number of miles per gallon your car gets. Write a variable expression for the amount you spend on gasoline for 1 year.

b. Write and simplify a variable expression for the amount of money you will save each year if you can increase your gas mileage by 5 mi per gallon.

c. If you currently get 25 mi per gallon and you increase your gas mileage by 5 mi per gallon, how much will you save in 1 year?

140. When is the LCM of two expressions equal to their product?

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8.3 Complex Fractions

Objective A To simplify a complex fraction

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Point of Interest

There are many instances of complex fractions in application problems. The fraction $\dfrac{1}{\frac{1}{R_1}+\frac{1}{R_2}}$ is used to determine the total resistance in certain electric circuits.

A **complex fraction** is a fraction whose numerator or denominator contains one or more fractions. Examples of complex fractions are shown at the right.

$$\frac{3}{2-\frac{1}{2}}, \qquad \frac{4+\frac{1}{x}}{3+\frac{2}{x}}, \qquad \frac{\frac{1}{x-1}+x+3}{x-3+\frac{1}{x+4}}$$

➡ Simplify: $\dfrac{1-\frac{4}{x^2}}{1+\frac{2}{x}}$

Find the LCM of the denominators of the fractions in the numerator and denominator. The LCM of x and x^2 is x^2.

$$\frac{1-\frac{4}{x^2}}{1+\frac{2}{x}} = \frac{1-\frac{4}{x^2}}{1+\frac{2}{x}} \cdot \frac{x^2}{x^2}$$

- Multiply the numerator and denominator by the LCM.

$$= \frac{1 \cdot x^2 - \frac{4}{x^2} \cdot x^2}{1 \cdot x^2 + \frac{2}{x} \cdot x^2}$$

- Simplify.

$$= \frac{x^2-4}{x^2+2x} = \frac{(x-2)(\cancel{x+2})}{x(\cancel{x+2})}$$

$$= \frac{x-2}{x}$$

TAKE NOTE

First of all, we are multiplying the complex fraction by $\frac{x^2}{x^2}$, which equals 1, so we are not changing the value of the fraction.

Second, we are using the Distributive Property to multiply $\left(1-\frac{4}{x^2}\right)x^2$ and $\left(1+\frac{2}{x}\right)x^2$.

The method shown above of simplifying a complex fraction by multiplying the numerator and denominator by the LCM of the denominators is used in the examples on the next page. However, a different approach is to rewrite the numerator and denominator of the complex fraction as single fractions and then divide the numerator by the denominator. The example shown above is simplified below using this alternative method.

$$\frac{1-\frac{4}{x^2}}{1+\frac{2}{x}} = \frac{1 \cdot \frac{x^2}{x^2} - \frac{4}{x^2}}{1 \cdot \frac{x}{x} + \frac{2}{x}} = \frac{\frac{x^2}{x^2} - \frac{4}{x^2}}{\frac{x}{x} + \frac{2}{x}} = \frac{\frac{x^2-4}{x^2}}{\frac{x+2}{x}}$$

- Rewrite the numerator and denominator of the complex fraction as single fractions.

$$= \frac{x^2-4}{x^2} \div \frac{x+2}{x} = \frac{x^2-4}{x^2} \cdot \frac{x}{x+2}$$

- Divide the numerator of the complex fraction by the denominator.

$$= \frac{(x^2-4)x}{x^2(x+2)} = \frac{(x+2)(x-2)x}{x^2(x+2)}$$

- Multiply the fractions. Factor the numerator.

$$= \frac{x-2}{x}$$

- Simplify.

Note that this is the same result as shown above.

TAKE NOTE

Recall that the fraction bar can be read "divided by."

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Example 1

Simplify: $\dfrac{\frac{1}{x}+\frac{1}{2}}{\frac{1}{x^2}-\frac{1}{4}}$

Solution

The LCM of x, 2, x^2, and 4 is $4x^2$.

$$\frac{\frac{1}{x}+\frac{1}{2}}{\frac{1}{x^2}-\frac{1}{4}} = \frac{\frac{1}{x}+\frac{1}{2}}{\frac{1}{x^2}-\frac{1}{4}} \cdot \frac{4x^2}{4x^2} = \frac{\frac{1}{x}\cdot 4x^2+\frac{1}{2}\cdot 4x^2}{\frac{1}{x^2}\cdot 4x^2-\frac{1}{4}\cdot 4x^2}$$

$$= \frac{4x+2x^2}{4-x^2} = \frac{2x(2+x)}{(2-x)(2+x)} = \frac{2x}{2-x}$$

You Try It 1

Simplify: $\dfrac{\frac{1}{3}-\frac{1}{x}}{\frac{1}{9}-\frac{1}{x^2}}$

Your solution

Example 2

Simplify: $\dfrac{x-8+\frac{20}{x+4}}{x-10+\frac{24}{x+4}}$

Solution

The LCM is $x+4$.

$$\frac{x-8+\frac{20}{x+4}}{x-10+\frac{24}{x+4}}$$

$$= \frac{x-8+\frac{20}{x+4}}{x-10+\frac{24}{x+4}} \cdot \frac{x+4}{x+4}$$

$$= \frac{x(x+4)-8(x+4)+\frac{20}{x+4}\cdot(x+4)}{x(x+4)-10(x+4)+\frac{24}{x+4}\cdot(x+4)}$$

$$= \frac{x^2+4x-8x-32+20}{x^2+4x-10x-40+24} = \frac{x^2-4x-12}{x^2-6x-16}$$

$$= \frac{(x-6)(x+2)}{(x-8)(x+2)} = \frac{x-6}{x-8}$$

You Try It 2

Simplify: $\dfrac{x+3-\frac{20}{x-5}}{x+8+\frac{30}{x-5}}$

Your solution

Solutions on p. S24

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8.3 Exercises

Objective A

Simplify.

1. $\dfrac{1+\dfrac{3}{x}}{1-\dfrac{9}{x^2}}$

2. $\dfrac{1+\dfrac{4}{x}}{1-\dfrac{16}{x^2}}$

3. $\dfrac{2-\dfrac{8}{x+4}}{3-\dfrac{12}{x+4}}$

4. $\dfrac{5-\dfrac{25}{x+5}}{1-\dfrac{3}{x+5}}$

5. $\dfrac{1+\dfrac{5}{y-2}}{1-\dfrac{2}{y-2}}$

6. $\dfrac{2-\dfrac{11}{2x-1}}{3-\dfrac{17}{2x-1}}$

7. $\dfrac{4-\dfrac{2}{x+7}}{5+\dfrac{1}{x+7}}$

8. $\dfrac{5+\dfrac{3}{x-8}}{2-\dfrac{1}{x-8}}$

9. $\dfrac{1-\dfrac{1}{x}-\dfrac{6}{x^2}}{1-\dfrac{9}{x^2}}$

10. $\dfrac{1+\dfrac{4}{x}+\dfrac{4}{x^2}}{1-\dfrac{2}{x}-\dfrac{8}{x^2}}$

11. $\dfrac{1-\dfrac{5}{x}-\dfrac{6}{x^2}}{1+\dfrac{6}{x}+\dfrac{5}{x^2}}$

12. $\dfrac{1-\dfrac{7}{a}+\dfrac{12}{a^2}}{1+\dfrac{1}{a}-\dfrac{20}{a^2}}$

13. $\dfrac{1-\dfrac{6}{x}+\dfrac{8}{x^2}}{\dfrac{4}{x^2}+\dfrac{3}{x}-1}$

14. $\dfrac{1+\dfrac{3}{x}-\dfrac{18}{x^2}}{\dfrac{21}{x^2}-\dfrac{4}{x}-1}$

15. $\dfrac{x-\dfrac{4}{x+3}}{1+\dfrac{1}{x+3}}$

16. $\dfrac{y+\dfrac{1}{y-2}}{1+\dfrac{1}{y-2}}$

17. $\dfrac{1-\dfrac{x}{2x+1}}{x-\dfrac{1}{2x+1}}$

18. $\dfrac{1-\dfrac{2x-2}{3x-1}}{x-\dfrac{4}{3x-1}}$

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19. $\dfrac{x-5+\dfrac{14}{x+4}}{x+3-\dfrac{2}{x+4}}$

20. $\dfrac{a+4+\dfrac{5}{a-2}}{a+6+\dfrac{15}{a-2}}$

21. $\dfrac{x+3-\dfrac{10}{x-6}}{x+2-\dfrac{20}{x-6}}$

22. $\dfrac{x-7+\dfrac{5}{x-1}}{x-3+\dfrac{1}{x-1}}$

23. $\dfrac{y-6+\dfrac{22}{2y+3}}{y-5+\dfrac{11}{2y+3}}$

24. $\dfrac{x+2-\dfrac{12}{2x-1}}{x+1-\dfrac{9}{2x-1}}$

25. $\dfrac{x-\dfrac{2}{2x-3}}{2x-1-\dfrac{8}{2x-3}}$

26. $\dfrac{x+3-\dfrac{18}{2x+1}}{x-\dfrac{6}{2x+1}}$

27. $\dfrac{\dfrac{1}{x}-\dfrac{2}{x-1}}{\dfrac{3}{x}+\dfrac{1}{x-1}}$

28. $\dfrac{\dfrac{3}{n+1}+\dfrac{1}{n}}{\dfrac{2}{n+1}+\dfrac{3}{n}}$

29. $\dfrac{\dfrac{3}{2x-1}-\dfrac{1}{x}}{\dfrac{4}{x}+\dfrac{2}{2x-1}}$

30. $\dfrac{\dfrac{4}{3x+1}+\dfrac{3}{x}}{\dfrac{6}{x}-\dfrac{2}{3x+1}}$

APPLYING THE CONCEPTS

Simplify.

31. $1+\dfrac{1}{1+\dfrac{1}{2}}$

32. $1+\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{2}}}$

33. $1-\dfrac{1}{1-\dfrac{1}{x}}$

34. $\dfrac{a^{-1}-b^{-1}}{a^{-2}-b^{-2}}$

35. $\left(\dfrac{y}{4}-\dfrac{4}{y}\right)\div\left(\dfrac{4}{y}-3+\dfrac{y}{2}\right)$

36. $\dfrac{1+x^{-1}}{1-x^{-1}}$

37. How would you explain to a classmate why we multiply the numerator and denominator of a complex fraction by the LCM of the denominators of the fractions in the numerator and denominator?

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8.4 Rational Equations

Objective A To solve rational equations

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To solve an equation containing fractions, **clear denominators** by multiplying each side of the equation by the LCM of the denominators. Then solve for the variable.

➡ Solve: $\frac{3x-1}{4x}+\frac{2}{3x}=\frac{7}{6x}$

TAKE NOTE
Note that we are now solving *equations*, not operating on *expressions*. We are not writing each fraction in terms of the LCM of the denominators; we are multiplying both sides of the equation by the LCM of the denominators.

$$\frac{3x-1}{4x}+\frac{2}{3x}=\frac{7}{6x}$$

- The LCM of $4x$, $3x$, and $6x$ is $12x$.

$$12x\left(\frac{3x-1}{4x}+\frac{2}{3x}\right)=12x\left(\frac{7}{6x}\right)$$

- Multiply each side of the equation by the LCM of the denominators.

$$12x\left(\frac{3x-1}{4x}\right)+12x\left(\frac{2}{3x}\right)=12x\left(\frac{7}{6x}\right)$$

- Simplify using the Distributive Property.

$$\frac{12x}{1}\left(\frac{3x-1}{4x}\right)+\frac{12x}{1}\left(\frac{2}{3x}\right)=\frac{12x}{1}\left(\frac{7}{6x}\right)$$

$$3(3x-1)+4(2)=2(7)$$

- Solve for x.

$$9x-3+8=14$$
$$9x+5=14$$
$$9x=9$$
$$x=1$$

1 checks as a solution. The solution is 1.

Occasionally, a value of the variable that appears to be a solution of an equation will make one of the denominators zero. In this case, the equation has no solution for that value of the variable.

➡ Solve: $\frac{2x}{x-2}=1+\frac{4}{x-2}$

$$\frac{2x}{x-2}=1+\frac{4}{x-2}$$

- The LCM is $x-2$. Multiply each side of the equation by the LCM.

$$(x-2)\frac{2x}{x-2}=(x-2)\left(1+\frac{4}{x-2}\right)$$

- Simplify using the Distributive Property and the Properties of Fractions.

$$(x-2)\frac{2x}{x-2}=(x-2)\cdot 1+(x-2)\frac{4}{x-2}$$

$$\frac{\cancel{(x-2)}^{1}}{1}\cdot\frac{2x}{\cancel{x-2}_{1}}=(x-2)\cdot 1+\frac{\cancel{(x-2)}^{1}}{1}\cdot\frac{4}{\cancel{x-2}_{1}}$$

$$2x=x-2+4$$

- Solve for x.

$$2x=x+2$$
$$x=2$$

When x is replaced by 2, the denominators of $\frac{2x}{x-2}$ and $\frac{4}{x-2}$ are zero. Therefore, the equation has no solution.

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Example 1

Solve: $\frac{x}{x+4} = \frac{2}{x}$

Solution

The LCM is $x(x + 4)$.

$$\frac{x}{x+4} = \frac{2}{x}$$

$$x(x+4)\left(\frac{x}{x+4}\right) = x(x+4)\left(\frac{2}{x}\right)$$

$$\frac{\overset{1}{\cancel{x(x+4)}}}{1} \cdot \frac{x}{\underset{1}{\cancel{x+4}}} = \frac{\overset{1}{\cancel{x}}(x+4)}{1} \cdot \frac{2}{\underset{1}{\cancel{x}}}$$

$$x^2 = (x+4)2$$

$$x^2 = 2x + 8$$

Solve the quadratic equation by factoring.

$$x^2 - 2x - 8 = 0$$

$$(x-4)(x+2) = 0$$

$$x - 4 = 0 \qquad x + 2 = 0$$

$$x = 4 \qquad x = -2$$

Both 4 and -2 check as solutions.
The solutions are 4 and -2.

You Try It 1

Solve: $\frac{x}{x+6} = \frac{3}{x}$

Your solution

Example 2

Solve: $\frac{3x}{x-4} = 5 + \frac{12}{x-4}$

Solution

The LCM is $x - 4$.

$$\frac{3x}{x-4} = 5 + \frac{12}{x-4}$$

$$(x-4)\left(\frac{3x}{x-4}\right) = (x-4)\left(5 + \frac{12}{x-4}\right)$$

$$\frac{\overset{1}{\cancel{(x-4)}}}{1} \cdot \frac{3x}{\underset{1}{\cancel{x-4}}} = (x-4)5 + \frac{\overset{1}{\cancel{(x-4)}}}{1} \cdot \frac{12}{\underset{1}{\cancel{x-4}}}$$

$$3x = (x-4)5 + 12$$

$$3x = 5x - 20 + 12$$

$$x = 5x - 8$$

$$-2x = -8$$

$$x = 4$$

4 does not check as a solution.
The equation has no solution.

You Try It 2

Solve: $\frac{5x}{x+2} = 3 - \frac{10}{x+2}$

Your solution

Solutions on p. S25

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Objective B To solve proportions

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Quantities such as 4 meters, 15 seconds, and 8 gallons are number quantities written with units. In these examples, the units are meters, seconds, and gallons.

A **ratio** is the quotient of two quantities that have the same unit.

The length of a living room is 16 ft and the width is 12 ft. The ratio of the length to the width is written

$$\frac{16 \text{ ft}}{12 \text{ ft}} = \frac{16}{12} = \frac{4}{3}$$

A ratio is in simplest form when the two numbers do not have a common factor. Note that the units are not written.

A **rate** is the quotient of two quantities that have different units.

There are 2 lb of salt in 8 gal of water. The salt-to-water rate is

$$\frac{2 \text{ lb}}{8 \text{ gal}} = \frac{1 \text{ lb}}{4 \text{ gal}}$$

A rate is in simplest form when the two numbers do not have a common factor. The units are written as part of the rate.

A **proportion** is an equation that states the equality of two ratios or rates. Examples of proportions are shown at the right.

$$\frac{30 \text{ mi}}{4 \text{ h}} = \frac{15 \text{ mi}}{2 \text{ h}}$$

$$\frac{4}{6} = \frac{8}{12}$$

$$\frac{3}{4} = \frac{x}{8}$$

Solve: $\frac{4}{x} = \frac{2}{3}$

$$\frac{4}{x} = \frac{2}{3}$$ • The LCM of x and 3 is $3x$.

$$3x\left(\frac{4}{x}\right) = 3x\left(\frac{2}{3}\right)$$ • Multiply each side of the proportion by $3x$.

$$12 = 2x$$ • Solve the equation.

$$6 = x$$

The solution is 6.

Example 3

Solve: $\frac{8}{x+3} = \frac{4}{x}$

Solution

$$\frac{8}{x+3} = \frac{4}{x}$$

$$x(x+3)\frac{8}{x+3} = x(x+3)\frac{4}{x}$$

$$8x = 4(x+3)$$

$$8x = 4x + 12$$

$$4x = 12$$

$$x = 3$$

The solution is 3.

You Try It 3

Solve: $\frac{2}{x+3} = \frac{6}{5x+5}$

Your solution

Solution on p. S25

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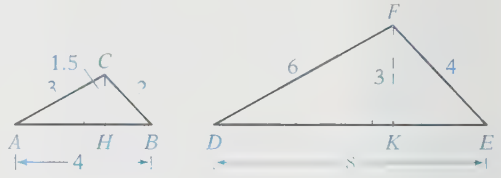
To solve problems involving similar triangles

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Similar objects have the same shape but not necessarily the same size. A tennis ball is similar to a basketball. A model ship is similar to an actual ship.

Similar objects have corresponding parts; for example, the rudder on the model ship corresponds to the rudder on the actual ship. The relationship between the sizes of each of the corresponding parts can be written as a ratio, and each ratio will be the same. If the rudder on the model ship is $\frac{1}{100}$ the size of the rudder on the actual ship, then the model wheelhouse is $\frac{1}{100}$ the size of the actual wheelhouse, the width of the model is $\frac{1}{100}$ the width of the actual ship, and so on.

The two triangles ABC and DEF shown at the right are similar. Side $\overline{AB}$ corresponds to $\overline{DE}$, side BC corresponds to $\overline{EF}$, and side $\overline{AC}$ corresponds to $\overline{DF}$. The height $\overline{CH}$ corresponds to the height $\overline{FK}$. The ratios of corresponding parts are equal.



$$\frac{AB}{DE} = \frac{4}{8} = \frac{1}{2}, \quad \frac{AC}{DF} = \frac{3}{6} = \frac{1}{2}, \quad \frac{BC}{EF} = \frac{2}{4} = \frac{1}{2}, \quad \text{and} \quad \frac{CH}{FK} = \frac{1.5}{3} = \frac{1}{2}$$

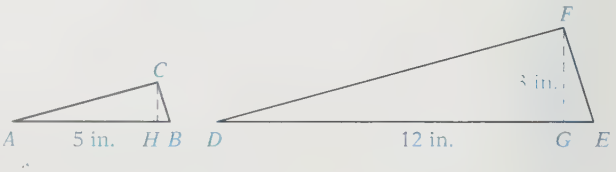
Since the ratios of corresponding parts are equal, three proportions can be formed using the sides of the triangles.

$$\frac{AB}{DE} = \frac{AC}{DF}, \quad \frac{AB}{DE} = \frac{BC}{EF}, \quad \text{and} \quad \frac{AC}{DF} = \frac{BC}{EF}$$

Three proportions can also be formed by using the sides and height of the triangles.

$$\frac{AB}{DE} = \frac{CH}{FK}, \quad \frac{AC}{DF} = \frac{CH}{FK}, \quad \text{and} \quad \frac{BC}{EF} = \frac{CH}{FK}$$

➡ Triangles ABC and DEF at the right are similar. Find the height of triangle ABC.



$$\frac{AB}{DE} = \frac{CH}{FG}$$

• Solve a proportion to find the height of triangle *ABC*.

$$\frac{5}{12} = \frac{CH}{3}$$

$$12 \cdot \frac{5}{12} = 12 \cdot \frac{CH}{3}$$

$$5 = 4(CH)$$

$$1.25 = CH$$

The height of triangle ABC is 1.25 in.

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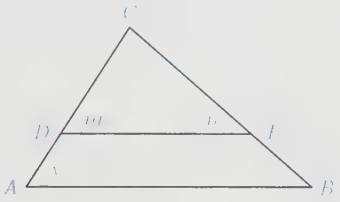
The corresponding angles in similar triangles are equal. Therefore, for the triangles in the last example,

$$\angle A = \angle D, \qquad \angle B = \angle E, \qquad \text{and} \qquad \angle C = \angle F$$

It is also true that if the three angles of one triangle are equal respectively to the three angles of another triangle, then the two triangles are similar.

TAKE NOTE
Vertical angles of intersecting lines, parallel lines, and angles of a triangle are discussed in the chapter titled "Geometry."

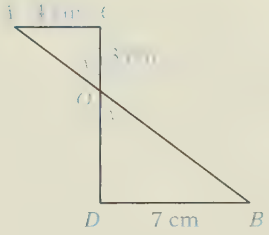
A line segment $\overline{DE}$ is drawn parallel to the base AB in the triangle at the right. $\angle x = \angle m$ and $\angle y = \angle n$ because corresponding angles are equal. $\angle C = \angle C$, and thus the three angles of triangle DEC are equal respectively to the three angles of triangle ABC. The triangle DEC is similar to the triangle ABC.



The sum of the three angles of a triangle is 180°. If two angles of one triangle are equal to two angles of another triangle, then the third angles must be equal. Thus we can say that if two angles of one triangle are equal to two angles of another triangle, then the two triangles are similar. This fact is used in Example 4.

Example 4

In the figure at the right, $\overline{AC}$ is parallel to $\overline{BD}$ and angles C and D are right angles. Find the length of DO.



Strategy

$\angle C = \angle D$ because they are right angles.
$\angle x = \angle y$ because they are vertical angles.
Therefore, triangle AOC is similar to triangle BOD because two angles of one triangle are equal to two angles of the other. Use a proportion to find the length of DO.

Solution

$$\frac{AC}{DB} = \frac{CO}{DO}$$

$$\frac{4}{7} = \frac{3}{DO}$$

$$7(DO)\frac{4}{7} = 7(DO)\frac{3}{DO}$$

$$4(DO) = 7(3)$$

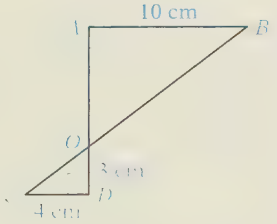
$$4(DO) = 21$$

$$DO = 5.25$$

The length of DO is 5.25 cm.

You Try It 4

In the figure at the right, $\overline{AB}$ is parallel to $\overline{DC}$ and angles A and D are right angles. Find the area of triangle AOB.



Your strategy

Your solution

Solution on p. S25

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To solve application problems

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Example 5

The monthly loan payment for a car is $28.35 for each $1000 borrowed. At this rate, find the monthly payment for a $6000 car loan.

Strategy

To find the monthly payment, write and solve a proportion, using P to represent the monthly car payment.

Solution

$$\frac{28.35}{1000} = \frac{P}{6000}$$

$$6000\left(\frac{28.35}{1000}\right) = 6000\left(\frac{P}{6000}\right)$$

$$170.10 = P$$

The monthly payment is $170.10.

You Try It 5

Sixteen ceramic tiles are needed to tile an area of 9 ft^2. At this rate, how many square feet can be tiled using 256 ceramic tiles?

Your strategy

Your solution

Example 6

An investment of $500 earns $60 each year. At the same rate, how much additional money must be invested to earn $90 each year?

Strategy

To find the additional amount of money that must be invested, write and solve a proportion, using x to represent the additional money. Then $500 + x$ is the total amount invested.

Solution

$$\frac{60}{500} = \frac{90}{500 + x}$$

$$\frac{3}{25} = \frac{90}{500 + x}$$

$$25(500 + x)\left(\frac{3}{25}\right) = 25(500 + x)\left(\frac{90}{500 + x}\right)$$

$$(500 + x)3 = 25(90)$$

$$1500 + 3x = 2250$$

$$3x = 750$$

$$x = 250$$

An additional $250 must be invested.

You Try It 6

Three ounces of a certain medication are required for a 150-pound adult. At the same rate, how many additional ounces of this medication are required for a 200-pound adult?

Your strategy

Your solution

Solutions on pp. S25–S26

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8.4 Exercises

Objective A

Solve.

1. $\frac{2x}{3} - \frac{5}{2} = -\frac{1}{2}$

2. $\frac{x}{3} - \frac{1}{4} = \frac{1}{12}$

3. $\frac{x}{3} - \frac{1}{4} = \frac{x}{4} - \frac{1}{6}$

4. $\frac{2y}{9} - \frac{1}{6} = \frac{y}{9} + \frac{1}{6}$

5. $\frac{2x - 5}{8} + \frac{1}{4} = \frac{x}{8} + \frac{3}{4}$

6. $\frac{3x + 4}{12} - \frac{1}{3} = \frac{5x + 2}{12} - \frac{1}{2}$

7. $\frac{6}{2a + 1} = 2$

8. $\frac{12}{3x - 2} = 3$

9. $\frac{9}{2x - 5} = -2$

10. $\frac{6}{4 - 3x} = 3$

11. $2 + \frac{5}{x} = 7$

12. $3 + \frac{8}{n} = 5$

13. $1 - \frac{9}{x} = 4$

14. $3 - \frac{12}{x} = 7$

15. $\frac{2}{y} + 5 = 9$

16. $\frac{6}{x} + 3 = 11$

17. $\frac{3}{x - 2} = \frac{4}{x}$

18. $\frac{5}{x + 3} = \frac{3}{x - 1}$

19. $\frac{2}{3x - 1} = \frac{3}{4x + 1}$

20. $\frac{5}{3x - 4} = \frac{-3}{1 - 2x}$

21. $\frac{-3}{2x + 5} = \frac{2}{x - 1}$

22. $\frac{4}{5y - 1} = \frac{2}{2y - 1}$

23. $\frac{4x}{x - 4} + 5 = \frac{5x}{x - 4}$

24. $\frac{2x}{x + 2} - 5 = \frac{7x}{x + 2}$

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25. $2 + \frac{3}{a-3} = \frac{a}{a-3}$

26. $\frac{x}{x+4} = 3 - \frac{4}{x+4}$

27. $\frac{x}{x-1} = \frac{8}{x+2}$

28. $\frac{x}{x+12} - \frac{1}{x+5}$

29. $\frac{2x}{x+4} = \frac{3}{x-1}$

30. $\frac{5}{3n-8} = \frac{n}{n+2}$

31. $x + \frac{6}{x-2} = \frac{3x}{x-2}$

32. $x - \frac{6}{x-3} = \frac{2x}{x-3}$

33. $\frac{8}{y} = \frac{2}{y-2} + 1$

34. How does a ratio differ from a rate?

35. What is a proportion?

Solve.

36. $\frac{x}{12} = \frac{3}{4}$

37. $\frac{6}{x} = \frac{2}{3}$

38. $\frac{4}{9} = \frac{x}{27}$

39. $\frac{16}{9} = \frac{64}{x}$

40. $\frac{x+3}{12} = \frac{5}{6}$

41. $\frac{3}{5} = \frac{x-4}{10}$

42. $\frac{18}{x+4} = \frac{9}{5}$

43. $\frac{2}{11} = \frac{20}{x-3}$

44. $\frac{2}{x} = \frac{4}{x+1}$

45. $\frac{16}{x-2} = \frac{8}{x}$

46. $\frac{x+3}{4} = \frac{x}{8}$

47. $\frac{x-6}{3} = \frac{x}{5}$

48. $\frac{2}{x-1} = \frac{6}{2x+1}$

49. $\frac{9}{x+2} = \frac{3}{x-2}$

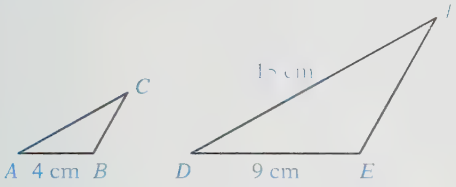
50. $\frac{2x}{7} = \frac{x-2}{14}$

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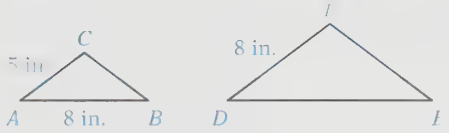
Objective C

Triangles *ABC* and *DEF* in Exercises 51 to 58 are similar. Round answers to the nearest tenth.

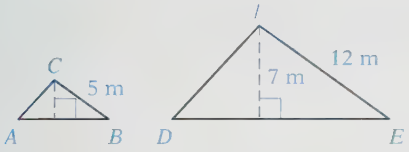
51. Find side *AC*.



52. Find side *DE*.



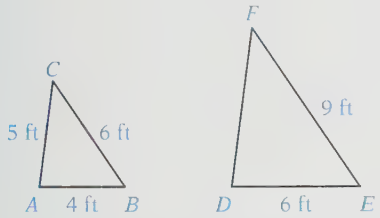
53. Find the height of triangle *ABC*.



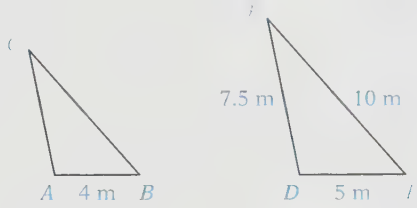
54. Find the height of triangle *DEF*.



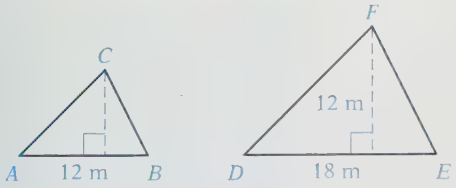
55. Find the perimeter of triangle *DEF*.



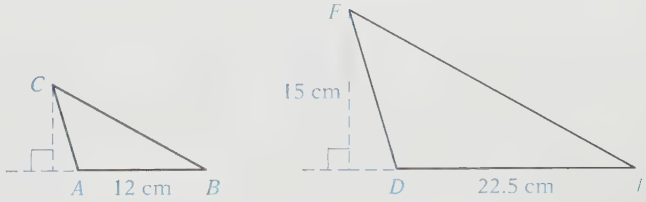
56. Find the perimeter of triangle *ABC*.



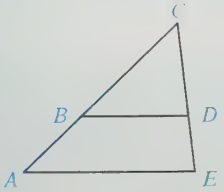
57. Find the area of triangle *ABC*.



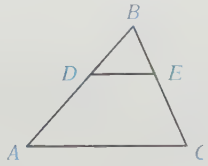
58. Find the area of triangle *ABC*.



59. Given $\overline{BD} \parallel \overline{AE}$, *BD* measures 5 cm, *AE* measures 8 cm, and *AC* measures 10 cm, find the length of *BC*.

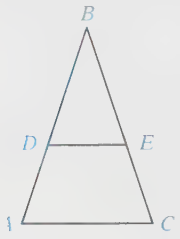


60. Given $\overline{AC} \parallel \overline{DE}$, *BD* measures 8 m, *AD* measures 12 m, and *BE* measures 6 m, find the length of *BC*.

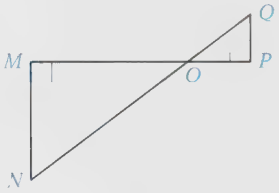


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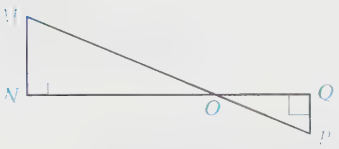
61. Given $\overline{DE} \parallel \overline{AC}$, DE measures 6 in., AC measures 10 in., and AB measures 15 in., find the length of DA.



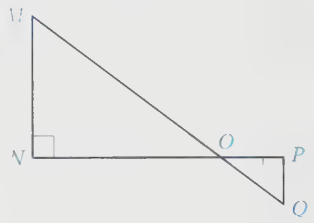
62. Given $\overline{MP}$ and $\overline{NQ}$ intersect at O, NO measures 25 ft, MO measures 20 ft, and PO measures 8 ft, find the length of QO.



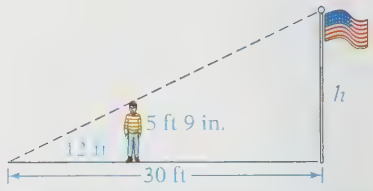
63. Given $\overline{MP}$ and $\overline{NQ}$ intersect at O, NO measures 24 cm, MN measures 10 cm, MP measures 39 cm, and QO measures 12 cm, find the length of OP.



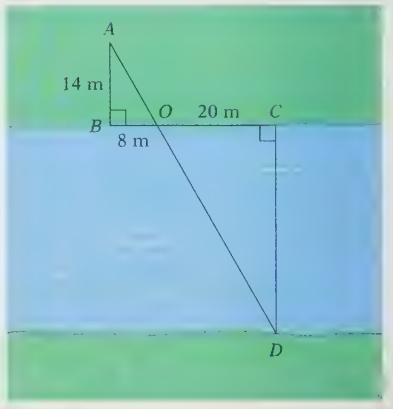
64. Given $\overline{MQ}$ and $\overline{NP}$ intersect at O, NO measures 12 m, MN measures 9 m, PQ measures 3 m, and MQ measures 20 m, find the perimeter of triangle OPQ.



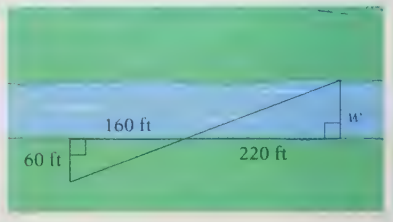
65. The sun's rays cast a shadow as shown in the diagram at the right. Find the height of the flagpole. Write the answer in terms of feet.



66. Similar triangles can be used as an indirect way to measure inaccessible distances. The diagram at the right represents a river of width DC. The triangles AOB and DOC are similar. The distances AB, BO, and OC can be measured. Find the width of the river.

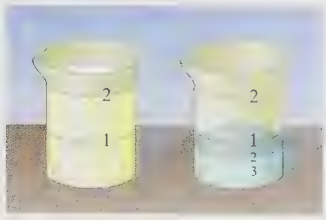


67. The diagram at the right shows how surveyors laid out similar triangles along a ravine. Find the width, w, of the ravine.



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Objective D Application Problems



68. Simple syrup used in making some desserts requires 2 cups (c) of sugar for every $\frac{2}{3}$ c of boiling water. At this rate, how many cups of sugar are required for 2 c boiling water?

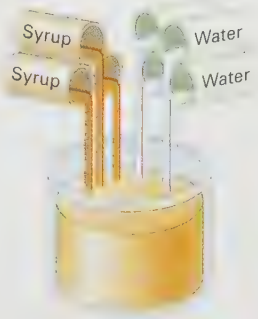
69. An exit poll survey showed that 4 out of every 7 voters cast a ballot in favor of an amendment to a city charter. At this rate, how many voters voted in favor of the amendment if 35,000 people voted?

70. As part of a conservation effort for a lake, 40 fish are caught, tagged, and then released. Later 80 fish are caught. Four of the 80 fish are found to have tags. Estimate the number of fish in the lake.



71. On a map, two cities are $2\frac{5}{8}$ in. apart. If $\frac{3}{8}$ in. on the map represents 25 mi, find the number of miles between the two cities.

72. The lighting for some billboards is provided by using solar energy. If 3 small solar energy panels can generate 10 W of power, how many panels are necessary to provide 600 W of power?



73. A soft drink is made by mixing 4 parts carbonated water with every 3 parts syrup. How many milliliters of water are in 280 ml of soft drink?

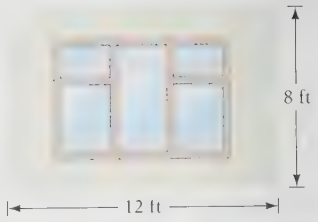
74. An air conditioning specialist recommends 2 air vents for each 300 ft^2 of floor space. At this rate, how many air vents are required for a 21,000-square-foot office building?

75. A laser printer is rated by the number of pages per minute it can print. An inexpensive laser printer can print 5 pages every 2 min. At this rate, how long would it take to print a document 45 pages long?

76. A company decides to accept a large shipment of 10,000 computer chips if there are 2 or fewer defects in a sample of 100 randomly chosen chips. Assuming that there are 300 defective chips in the shipment and that the rate of defective chips in the sample is the same as the rate in the shipment, will the shipment be accepted?

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77. To conserve energy and still allow for as much natural lighting as possible, an architect suggests that the ratio of the area of a window to the area of the total wall surface be 5 to 12. Using this ratio, determine the recommended area of a window to be installed in a wall that measures 8 ft by 12 ft.

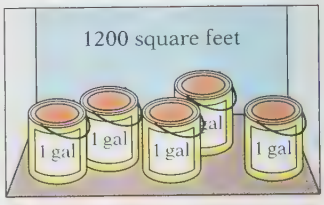


78. Leonardo da Vinci measured various distances on the human body in order to make accurate drawings. He determined that generally the ratio of the kneeling height of a person to the standing height of that person is $\frac{3}{4}$. Using this ratio, determine the height of a person who has a kneeling height of 48 in.



79. In one of Leonardo da Vinci's notebooks he wrote that ". . . from the top to the bottom of the chin is the sixth part of a face, and it is the fifty-fourth part of the man." Suppose the distance from the top to the bottom of the chin of a person is 1.25 in. Using da Vinci's measurements, determine the height of this person.

80. A painter estimates that 5 gal of paint will cover 1200 ft^2 of wall space. At this rate, how many additional gallons will be necessary to cover 1680 ft^2?



APPLYING THE CONCEPTS

81. Three people put their money together to buy lottery tickets. The first person put in \$25, the second person put in \$30, and the third person put in \$35. One of their tickets was a winning ticket. If they won \$4.5 million, what was the first person's share of the winnings?

82. No one belongs to both the Math Club and the Photography Club, but the two clubs join to hold a car wash. Ten members of the Math Club and 6 members of the Photography Club participate. The profits from the car wash are \$120. If each club's profits are proportional to the number of members participating, what share of the profits does the Math Club receive?



83. A basketball player has made 5 out of every 6 foul shots attempted in one year of play. If 42 foul shots were missed that year, how many shots did the basketball player make?

84. Explain the procedure for solving an equation that contains fractions. Include in your discussion how the LCM of the denominators is used to eliminate fractions in the equation.

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8.5 Literal Equations

Objective A

To solve a literal equation for one of the variables



A **literal equation** is an equation that contains more than one variable. Examples of literal equations are shown at the right.

$2x + 3y = 6$
$4w - 2x + z = 0$

Formulas are used to express a relationship among physical quantities. A **formula** is a literal equation that states rules about measurements. Examples of formulas are shown at the right.

$\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R}$ (Physics)
$s = a + (n - 1)d$ (Mathematics)
$A = P + Prt$ (Business)

The Addition and Multiplication Properties can be used to solve a literal equation for one of the variables. The goal is to rewrite the equation so that the variable being solved for is alone on one side of the equation and all the other numbers and variables are on the other side.

➡ Solve $A = P(1 + i)$ for i.

The goal is to rewrite the equation so that i is on one side of the equation and all other variables are on the other side.

$A = P(1 + i)$
$A = P + Pi$ • Use the Distributive Property to remove parentheses.
$A - P = P - P + Pi$ • Subtract P from each side of the equation.
$A - P = Pi$
$\frac{A - P}{P} = \frac{Pi}{P}$ • Divide each side of the equation by P.
$\frac{A - P}{P} = i$

Example 1

Solve $s = a + (n - 1)d$ for d.

Solution

$s = a + (n - 1)d$
$s - a = a - a + (n - 1)d$
$s - a = (n - 1)d$
$\frac{s - a}{n - 1} = \frac{(n - 1)d}{n - 1}$
$\frac{s - a}{n - 1} = d$

You Try It 1

Solve $A = P + Prt$ for r.

Your solution

Solution on p. S26

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Example 2

Solve $I = \frac{E}{R + r}$ for R.

Solution

$$I = \frac{E}{R + r}$$

$$(R + r)I = (R + r)\frac{E}{R + r}$$

$$RI + rI = E$$

$$RI + rI - rI = E - rI$$

$$RI = E - rI$$

$$\frac{RI}{I} = \frac{E - rI}{I}$$

$$R = \frac{E - rI}{I}$$

You Try It 2

Solve $s = \frac{A + L}{2}$ for L.

Your solution

Example 3

Solve $L = a(1 + ct)$ for c.

Solution

$$L = a(1 + ct)$$

$$L = a + act$$

$$L - a = a - a + act$$

$$L - a = act$$

$$\frac{L - a}{at} = \frac{act}{at}$$

$$\frac{L - a}{at} = c$$

You Try It 3

Solve $s = a + (n - 1)d$ for n.

Your solution

Example 4

Solve $M = C - rC$ for C.

Solution

$$M = C - rC$$

$$M = (1 - r)C$$

$$\frac{M}{1 - r} = \frac{(1 - r)C}{1 - r}$$

$$\frac{M}{1 - r} = C$$

You Try It 4

Solve $S = C + rC$ for C.

Your solution

Solutions on p. S26

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8.5 Exercises

Objective A

Solve the formula for the given variable.

1. $A = \frac{1}{2}bh; h$ (Geometry)

2. $P = a + b + c; b$ (Geometry)

3. $d = rt; t$ (Physics)

4. $E = IR; R$ (Physics)

5. $PV = nRT; T$ (Chemistry)

6. $A = bh; h$ (Geometry)

7. $P = 2L + 2W; L$ (Geometry)

8. $F = \frac{9}{5}C + 32; C$ (Temperature conversion)

9. $A = \frac{1}{2}h(b_1 + b_2); b_1$ (Geometry)

10. $C = \frac{5}{9}(F - 32); F$ (Temperature conversion)

11. $V = \frac{1}{3}Ah; h$ (Geometry)

12. $P = R - C; C$ (Business)

13. $R = \frac{C - S}{t}; S$ (Business)

14. $P = \frac{R - C}{n}; R$ (Business)

15. $A = P + Prt; P$ (Business)

16. $T = fm - gm; m$ (Engineering)

17. $A = Sw + w; w$ (Physics)

18. $a = S - Sr; S$ (Mathematics)

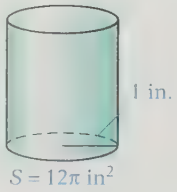
APPLYING THE CONCEPTS

The surface area of a right circular cylinder is given by the formula $S = 2\pi rh + 2\pi r^2$, where r is the radius of the base, and h is the height of the cylinder.

19. **a.** Solve the formula $S = 2\pi rh + 2\pi r^2$ for h.

b. Use your answer to part **a** to find the height of a right circular cylinder when the surface area is 12π in^2 and the radius is 1 in.

c. Use your answer to part **a** to find the height of a right circular cylinder when the surface area is 24π in^2 and the radius is 2 in.



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When markup is based on selling price, the selling price of a product is given by the formula $S = C + rC$, where C is the cost of the product, and r is the markup rate.

20. **a.** Solve the formula $S = C + rC$ for r.

b. Use your answer to part **a** to find the markup rate on a tennis racket when the cost is \$112 and the selling price is \$180.

c. Use your answer to part **a** to find the markup rate on a radio when the cost is \$120 and the selling price is \$172.

Break-even analysis is a method used to determine the sales volume required for a company to break even, or experience neither a profit nor a loss on the sale of a product. The **break-even point** represents the number of units that must be made and sold for income from sales to equal the cost of the product. The break-even point can be calculated using the formula $B = \dfrac{F}{S - V}$, where F is the fixed costs, S is the selling price per unit, and V is the variable costs per unit.

21. **a.** Solve the formula $B = \dfrac{F}{S - V}$ for S.

b. Use your answer to part **a** to find the required selling price per desk for a company to break even. The fixed costs are \$20,000, the variable costs per desk are \$80, and the company plans to make and sell 200 desks.

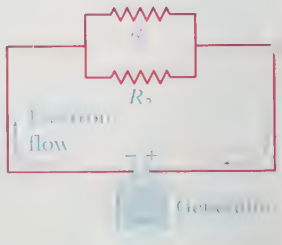
c. Use your answer to part **a** to find the required selling price per camera for a company to break even. The fixed costs are \$15,000, the variable costs per camera are \$50, and the company plans to make and sell 600 cameras.

Resistors are used to control the flow of current. The total resistance of two resistors in a circuit can be given by the formula $R = \dfrac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}}$, where R_1 and R_2 are the two resistors in the circuit. Resistance is measured in ohms.

22. **a.** Solve the formula $R = \dfrac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}}$ for R_1.

b. Use your answer to part **a** to find the resistance in R_1 if the resistance in R_2 is 30 ohms and the total resistance is 12 ohms.

c. Use your answer to part **a** to find the resistance in R_1 if the resistance in R_2 is 15 ohms and the total resistance is 6 ohms.



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8.6 Work and Uniform Motion Problems

Objective A

To solve work problems

VIDEO & DVD CD TUTOR WEB SSM

Point of Interest

The following problem was recorded in the *Jiuzhang*, a Chinese text that dates to the Han dynasty (about 200 B.C. to A.D. 200). "A reservoir has 5 channels bringing water to it. The first can fill the reservoir in $\frac{1}{3}$ day, the second in 1 day, the third in $2\frac{1}{2}$ days, the fourth in 3 days, and the fifth in 5 days. If all channels are open, how long does it take to fill the reservoir?" This problem is the earliest known work problem.

If a painter can paint a room in 4 h, then in 1 h the painter can paint $\frac{1}{4}$ of the room. The painter's rate of work is $\frac{1}{4}$ of the room each hour. The **rate of work** is the part of a task that is completed in one unit of time.

A pipe can fill a tank in 30 min. This pipe can fill $\frac{1}{30}$ of the tank in 1 min. The rate of work is $\frac{1}{30}$ of the tank each minute. If a second pipe can fill the tank in x min, the rate of work for the second pipe is $\frac{1}{x}$ of the tank each minute.

In solving a work problem, the goal is to determine the time it takes to complete a task. The basic equation that is used to solve work problems is

Rate of work × time worked = part of task completed

For example, if a faucet can fill a sink in 6 min, then in 5 min the faucet can fill $\frac{1}{6} \times 5 = \frac{5}{6}$ of the sink. In 5 min the faucet completes $\frac{5}{6}$ of the task.

➡ A painter can paint a wall in 20 min. The painter's apprentice can paint the same wall in 30 min. How long will it take them to paint the wall when they work together?

Strategy for Solving a Work Problem

1. For each person or machine, write a numerical or variable expression for the rate of work, the time worked, and the part of the task completed. The results can be recorded in a table.

TAKE NOTE

Use the information given in the problem to fill in the "Rate" and "Time" columns of the table. Fill in the "Part Completed" column by multiplying the two expressions you wrote in each row.

Unknown time to paint the wall working together: t

	Rate of Work	·	*Time Worked*	=	*Part of Task Completed*
Painter	$\frac{1}{20}$	·	t	=	$\frac{t}{20}$
Apprentice	$\frac{1}{30}$	·	t	=	$\frac{t}{30}$

2. Determine how the parts of the task completed are related. Use the fact that the sum of the parts of the task completed must equal 1; the complete task.

$$\frac{t}{20} + \frac{t}{30} = 1$$

- The sum of the part of the task completed by the painter and the part of the task completed by the apprentice is 1.

$$60\left(\frac{t}{20} + \frac{t}{30}\right) = 60 \cdot 1$$
$$3t + 2t = 60$$
$$5t = 60$$
$$t = 12$$

- Multiply by the LCM of 20 and 30.

Working together, they will paint the wall in 12 min.

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Example 1

A small water pipe takes three times longer to fill a tank than does a large water pipe. With both pipes open, it takes 4 h to fill the tank. Find the time it would take the small pipe, working alone, to fill the tank.

Strategy

- Time for large pipe to fill the tank: t
 Time for small pipe to fill the tank: $3t$

Fills tank in $3t$ hours



Fills $\frac{4}{3t}$ of the tank in 4 hours

Fills tank in t hours



Fills $\frac{4}{t}$ of the tank in 4 hours

	Rate	*Time*	*Part*
Small pipe	$\frac{1}{3t}$	4	$\frac{4}{3t}$
Large pipe	$\frac{1}{t}$	4	$\frac{4}{t}$

- The sum of the parts of the task completed by each pipe must equal 1.

Solution

$$\frac{4}{3t} + \frac{4}{t} = 1$$

$$3t\left(\frac{4}{3t} + \frac{4}{t}\right) = 3t \cdot 1$$

$$4 + 12 = 3t$$

$$16 = 3t$$

$$\frac{16}{3} = t$$

$$3t = 3\left(\frac{16}{3}\right) = 16$$

The small pipe working alone takes 16 h to fill the tank.

You Try It 1

Two computer printers that work at the same rate are working together to print the payroll checks for a large corporation. After they work together for 2 h, one of the printers quits. The second requires 3 h more to complete the payroll checks. Find the time it would take one printer, working alone, to print the payroll.

Your strategy

Your solution

Solution on p. S26

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Objective **To solve uniform motion problems**



A car that travels constantly in a straight line at 30 mph is in uniform motion. **Uniform motion** means that the speed or direction of an object does not change.

The basic equation used to solve uniform motion problems is

$$\textbf{Distance} = \textbf{rate} \times \textbf{time}$$

An alternative form of this equation can be written by solving the equation for time.

$$\frac{\textbf{Distance}}{\textbf{Rate}} = \textbf{time}$$

This form of the equation is useful when the total time of travel for two objects or the time of travel between two points is known.

➡ The speed of a boat in still water is 20 mph. The boat traveled 75 mi down a river in the same amount of time it took to travel 45 mi up the river. Find the rate of the river's current.

Strategy for Solving a Uniform Motion Problem

1. For each object, write a numerical or variable expression for the distance, rate, and time. The results can be recorded in a table.

TAKE NOTE
Use the information given in the problem to fill in the "Distance" and "Rate" columns of the table. Fill in the "Time" column by dividing the two expressions you wrote in each row.

The unknown rate of the river's current: r

	Distance	÷	*Rate*	=	*Time*
Down river	75	÷	$20 + r$	=	$\frac{75}{20 + r}$
Up river	45	÷	$20 - r$	=	$\frac{45}{20 - r}$

2. Determine how the times traveled by each object are related. For example, it may be known that the times are equal, or the total time may be known.

$$\frac{75}{20 + r} = \frac{45}{20 - r}$$

- The time down the river is equal to the time up the river.

$$(20 + r)(20 - r)\frac{75}{20 + r} = (20 + r)(20 - r)\frac{45}{20 - r}$$

- Multiply by the LCM of the denominators.

$$\begin{aligned} (20 - r)75 &= (20 + r)45 \\ 1500 - 75r &= 900 + 45r \\ -120r &= -600 \\ r &= 5 \end{aligned}$$

The rate of the river's current is 5 mph.

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Example 2

A cyclist rode the first 20 mi of a trip at a constant rate. For the next 16 mi, the cyclist reduced the speed by 2 mph. The total time for the 36 mi was 4 h. Find the rate of the cyclist for each leg of the trip.

Strategy

- Rate for the first 20 mi: r
 Rate for the next 16 mi: $r - 2$



	Distance	*Rate*	*Time*
First 20 mi	20	r	$\frac{20}{r}$
Next 16 mi	16	$r - 2$	$\frac{16}{r-2}$

- The total time for the trip was 4 h.

Solution

$$\frac{20}{r} + \frac{16}{r-2} = 4$$

$$r(r-2)\left[\frac{20}{r} + \frac{16}{r-2}\right] = r(r-2) \cdot 4$$

$$(r-2)20 + 16r = (r^2 - 2r) \cdot 4$$

$$20r - 40 + 16r = 4r^2 - 8r$$

$$36r - 40 = 4r^2 - 8r$$

Solve the quadratic equation by factoring.

$$0 = 4r^2 - 44r + 40$$

$$0 = 4(r^2 - 11r + 10)$$

$$0 = 4(r-10)(r-1)$$

$$r - 10 = 0 \qquad r - 1 = 0$$

$$r = 10 \qquad r = 1$$

The solution $r = 1$ mph is not possible, because the rate on the last 16 mi would then be -1 mph.

10 mph was the rate for the first 20 mi.
8 mph was the rate for the next 16 mi.

You Try It 2

The total time it took for a sailboat to sail back and forth across a lake 6 km wide was 2 h. The rate sailing back was three times the rate sailing across. Find the rate sailing out across the lake.

Your strategy

Your solution

Solution on p. S26

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8.6 Exercises

Application Problems

1. A park has two sprinklers that are used to fill a fountain. One sprinkler can fill the fountain in 3 h, whereas the second sprinkler can fill the fountain in 6 h. How long will it take to fill the fountain with both sprinklers operating?

	Rate	*Time*	*Part*
First	$\frac{1}{3}$	t	$\frac{t}{3}$
Second	$\frac{1}{6}$	t	$\frac{t}{6}$

2. One grocery clerk can stock a shelf in 20 min, whereas a second clerk requires 30 min to stock the same shelf. How long would it take to stock the shelf if the two clerks worked together?

3. One person with a skiploader requires 12 h to remove a large quantity of earth. A second, larger skiploader can remove the same amount of earth in 4 h. How long would it take to remove the earth with both skiploaders working together?

4. An experienced painter can paint a fence twice as fast as an inexperienced painter. Working together, the painters require 4 h to paint the fence. How long would it take the experienced painter working alone to paint the fence?

5. One computer can solve a complex prime factorization problem in 75 h. A second computer can solve the same problem in 50 h. How long would it take both computers, working together, to solve the problem?

6. A new machine can make 10,000 aluminum cans three times as fast as an older machine. With both machines working, 10,000 cans can be made in 9 h. How long would it take the new machine, working alone, to make the 10,000 cans?

7. A small air conditioner can cool a room 5° in 75 min. A larger air conditioner can cool the room 5° in 50 min. How long would it take to cool the room 5° with both air conditioners working?

8. One printing press can print the first edition of a book in 55 min, whereas a second printing press requires 66 min to print the same number of copies. How long would it take to print the first edition with both presses operating?

9. Two oil pipelines can fill a small tank in 30 min. Using one of the pipelines would require 45 min to fill the tank. How long would it take the second pipeline alone to fill the tank?

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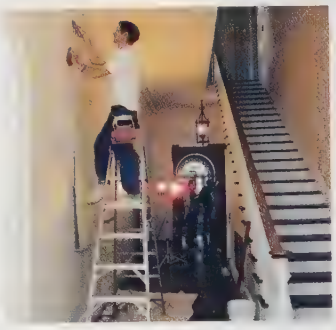
10. Working together, two dock workers can load a crate in 6 min. One dock worker, working alone, can load the crate in 15 min. How long would it take the second dock worker, working alone, to load the crate?

11. A mason can construct a retaining wall in 10 h. With the mason's apprentice assisting, the task would take 6 h. How long would it take the apprentice working alone to construct the wall?

12. A mechanic requires 2 h to repair a transmission, whereas an apprentice requires 6 h to make the same repairs. The mechanic worked alone for 1 h and then stopped. How long will it take the apprentice, working alone, to complete the repairs?

13. One computer technician can wire a modem in 4 h, whereas it takes 6 h for a second technician to do the same job. After working alone for 2 h, the first technician quit. How long will it take the second technician to complete the wiring?

14. A wallpaper hanger requires 2 h to hang the wallpaper on one wall of a room. A second wallpaper hanger requires 4 h to hang the same amount of paper. The first wallpaper hanger worked alone for 1 h and then quit. How long will it take the second wallpaper hanger, working alone, to complete the wall?



15. Two welders who work at the same rate are welding the girders of a building. After they work together for 10 h, one of the welders quits. The second welder requires 20 more hours to complete the welds. Find the time it would have taken one of the welders, working alone, to complete the welds.

16. A large and a small heating unit are being used to heat the water of a pool. The larger unit, working alone, requires 8 h to heat the pool. After both units have been operating for 2 h, the larger unit is turned off. The small unit requires 9 h more to heat the pool. How long would it take the small unit, working alone, to heat the pool?

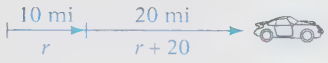
17. Two machines that fill cereal boxes work at the same rate. After they work together for 7 h, one machine breaks down. The second machine requires 14 h more to finish filling the boxes. How long would it have taken one of the machines, working alone, to fill the boxes?

18. A large and a small drain are opened to drain a pool. The large drain can empty the pool in 6 h. After both drains have been open for 1 h, the large drain becomes clogged and is closed. The smaller drain remains open and requires 9 h more to empty the pool. How long would it have taken the small drain, working alone, to empty the pool?

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Objective B *Application Problems*

19. Commuting from work to home, a lab technician traveled 10 mi at a constant rate through congested traffic. On reaching the expressway, the technician increased the speed by 20 mph. An additional 20 mi was traveled at the increased speed. The total time for the trip was 1 h. Find the rate of travel through the congested traffic.



20. The president of a company traveled 1800 mi by jet and 300 mi on a prop plane. The rate of the jet was four times the rate of the prop plane. The entire trip took a total of 5 h. Find the rate of the jet plane.

21. As part of a conditioning program, a jogger ran 8 mi in the same time a cyclist rode 20 mi. The rate of the cyclist was 12 mph faster than the rate of the jogger. Find the rate of the jogger and that of the cyclist.



22. An express train travels 600 mi in the same amount of time it takes a freight train to travel 360 mi. The rate of the express train is 20 mph faster than that of the freight train. Find the rate of each train.

23. To assess the damage done by a fire, a forest ranger traveled 1080 mi by jet and then an additional 180 mi by helicopter. The rate of the jet was 4 times the rate of the helicopter. The entire trip took a total of 5 h. Find the rate of the jet.

24. A twin-engine plane can fly 800 mi in the same time that it takes a single-engine plane to fly 600 mi. The rate of the twin-engine plane is 50 mph faster than that of the single-engine plane. Find the rate of the twin-engine plane.

25. The rate of a bicyclist is 7 mph greater than the rate of a long-distance runner. The bicyclist travels 30 mi in the same amount of time as it takes the runner to travel 16 mi. Find the rate of the runner.

26. A car and a bus leave a town at 1 P.M. and head for a town 300 mi away. The rate of the car is twice the rate of the bus. The car arrives 5 h ahead of the bus. Find the rate of the car.

27. A car is traveling at a rate that is 36 mph greater than the rate of a cyclist. The car travels 384 mi in the same time it takes the cyclist to travel 96 mi. Find the rate of the car.

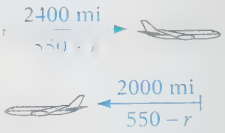


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28. A backpacker hiking into a wilderness area walked 9 mi at a constant rate and then reduced this rate by 1 mph. Another 4 mi was hiked at this reduced rate. The time required to hike the 4 mi was 1 h less than the time required to walk the 9 mi. Find the rate at which the hiker walked the first 9 mi.

29. A plane can fly 180 mph in calm air. Flying with the wind, the plane can fly 600 mi in the same amount of time it takes to fly 480 mi against the wind. Find the rate of the wind.

30. A commercial jet can fly 550 mph in calm air. Traveling with the jet stream, the plane flew 2400 mi in the same amount of time it takes to fly 2000 mi against the jet stream. Find the rate of the jet stream.



31. A cruise ship can sail at 28 mph in calm water. Sailing with the gulf current, the ship can sail 170 mi in the same amount of time that it can sail 110 mi against the gulf current. Find the rate of the gulf current.

32. Rowing with the current of a river, a rowing team can row 25 mi in the same amount of time it takes to row 15 mi against the current. The rate of the rowing team in calm water is 20 mph. Find the rate of the current.

33. On a recent trip, a trucker traveled 330 mi at a constant rate. Because of road construction, the trucker then had to reduce the speed by 25 mph. An additional 30 mi was traveled at the reduced rate. The total time for the entire trip was 7 h. Find the rate of the trucker for the first 330 mi.

APPLYING THE CONCEPTS

34. One pipe can fill a tank in 2 h, a second pipe can fill the tank in 4 h, and a third pipe can fill the tank in 5 h. How long will it take to fill the tank with all three pipes working?

35. A mason can construct a retaining wall in 10 h. The mason's more-experienced apprentice can do the same job in 15 h. How long would it take the mason's less-experienced apprentice to do the job if, working together, all three can complete the job in 5 h?

36. A surveyor traveled 32 mi by canoe and then hiked 4 mi. The rate by boat was four times the rate on foot. If the time spent walking was 1 h less than the time spent canoeing, find the amount of time spent traveling by canoe.

37. Because of bad weather, a bus driver reduced the usual speed along a 150-mile bus route by 10 mph. The bus arrived only 30 min later than its usual arrival time. How fast does the bus usually travel?

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8.7 Variation

Objective A To solve variation problems

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Direct variation is a special function that can be expressed as the equation $y = kx$, where k is a constant. The equation $y = kx$ is read "y varies directly as x" or "y is directly proportional to x." The constant k is called the **constant of variation** or the **constant of proportionality**.

The circumference (C) of a circle varies directly as the diameter (d). The direct variation equation is written $C = \pi d$. The constant of variation is π.

A nurse makes \$25 per hour. The total wage (w) of the nurse is directly proportional to the number of hours (h) worked. The equation of variation is $w = 25h$. The constant of proportionality is 25.

A direct variation equation can be written in the form $y = kx^n$, where n is a positive number. For example, the equation $y = kx^2$ is read "y varies directly as the square of x."

The area (A) of a circle varies directly as the square of the radius (r) of the circle. The direct variation equation is $A = \pi r^2$. The constant of variation is π.

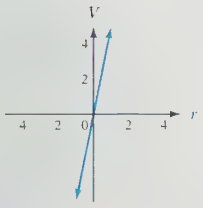
➡ Given that V varies directly as r and that $V = 20$ when $r = 4$, find the constant of variation and the equation of variation.

$V = kr$ • Write the basic direct variation equation.

$20 = k \cdot 4$ • Replace V and r by the given values. Then solve for k.

$5 = k$ • This is the constant of variation.

$V = 5r$ • Write the direct variation equation by substituting the value of k into the basic direct variation equation.



The Graph of $V = 5r$

➡ The tension (T) in a spring varies directly as the distance (x) it is stretched. If $T = 8$ lb when $x = 2$ in., find T when $x = 4$ in.

$T = kx$ • Write the basic direct variation equation.

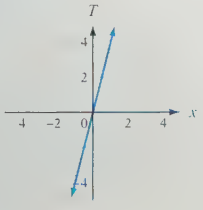
$8 = k \cdot 2$ • Replace T and x by the given values.

$4 = k$ • Solve for the constant of variation.

$T = 4x$ • Write the direct variation equation.

$T = 4x = 4 \cdot 4 = 16$ • To find T when $x = 4$, substitute 4 for x in the equation and solve for T.

The tension is 16 lb.

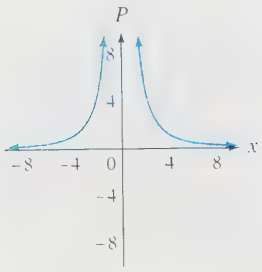


The Graph of $T = 4x$

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Inverse variation is a function that can be expressed as the equation $y = \frac{k}{x}$, where k is a constant. The equation $y = \frac{k}{x}$ is read "y varies inversely as x" or "y is inversely proportional to x."

In general, an inverse variation equation can be written $y = \frac{k}{x^n}$, where n is a positive number. For example, the equation $y = \frac{k}{x^2}$ is read "y varies inversely as the square of x."



The Graph of $P = \frac{20}{x^2}$

➡ Given that P varies inversely as the square of x and that $P = 5$ when $x = 2$, find the constant of variation and the equation of variation.

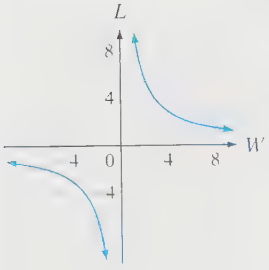
$P = \frac{k}{x^2}$ • Write the basic inverse variation equation.

$5 = \frac{k}{2^2}$ • Replace P and x by the given values. Then solve for k.

$5 = \frac{k}{4}$

$20 = k$ • This is the constant of variation.

$P = \frac{20}{x^2}$ • Write the inverse variation equation by substituting the value of k into the basic inverse variation equation.



The Graph of $L = \frac{12}{W}$

➡ The length (L) of a rectangle with fixed area is inversely proportional to the width (W). If $L = 6$ ft when $W = 2$ ft, find L when $W = 3$ ft.

$L = \frac{k}{W}$ • Write the basic inverse variation equation.

$6 = \frac{k}{2}$ • Replace L and W by the given values.

$12 = k$ • Solve for the constant of variation.

$L = \frac{12}{W}$ • Write the inverse variation equation.

$L = \frac{12}{W} = \frac{12}{3} = 4$ • To find L when $W = 3$ ft, substitute 3 for W in the equation and solve for L.

The length is 4 ft.

Joint variation is a variation in which a variable varies directly as the product of two or more other variables. A joint variation can be expressed as the equation $z = kxy$, where k is a constant. The equation $z = kxy$ is read "z varies jointly as x and y."

The area (A) of a triangle varies jointly as the base (b) and the height (h). The joint variation equation is written $A = \frac{1}{2}bh$. The constant of variation is $\frac{1}{2}$.

A **combined variation** is a variation in which two or more types of variation occur at the same time. For example, in physics, the volume (V) of a gas varies directly as the temperature (T) and inversely as the pressure (P). This combined variation is written $V = \frac{kT}{P}$.

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A ball is being twirled on the end of a string. The tension (T) in the string is directly proportional to the square of the speed (v) of the ball and inversely proportional to the length (r) of the string. The tension is 96 lb when the length of the string is 0.5 ft and the speed is 4 ft/s. Find the tension when the length of the string is 1 ft and the speed is 5 ft/s.

$$T = \frac{kv^2}{r}$$ • Write the basic combined variation equation.

$$96 = \frac{k \cdot 4^2}{0.5}$$ • Replace T, v, and r by the given values.

$$96 = \frac{k \cdot 16}{0.5}$$

$$96 = k \cdot 32$$ • Solve for the constant of variation.

$$3 = k$$

$$T = \frac{3v^2}{r}$$ • Write the combined variation equation.

$$T = \frac{3v^2}{r} = \frac{3 \cdot 5^2}{1} = 3 \cdot 25 = 75$$ • To find T when $r = 1$ ft and $v = 5$ ft/s, substitute 1 for r and 5 for v and solve for T.

The tension is 75 lb.

Example 1

The amount (A) of medication prescribed for a person is directly related to the person's weight (W). For a 50-kilogram person, 2 ml of medication are prescribed. How many milliliters of medication are required for a person who weighs 75 kg?

Strategy

To find the required amount of medication:

- Write the basic direct variation equation, replace the variables by the given values, and solve for k.
- Write the direct variation equation, replacing k by its value. Substitute 75 for W and solve for A.

Solution

$$A = kW$$

$$2 = k \cdot 50$$

$$\frac{1}{25} = k$$

$$A = \frac{1}{25}W = \frac{1}{25} \cdot 75 = 3$$

The required amount of medication is 3 ml.

You Try It 1

The distance (s) a body falls from rest varies directly as the square of the time (t) of the fall. An object falls 64 ft in 2 s. How far will it fall in 5 s?

Your strategy

Your solution

Solution on pp. S26–S27

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Example 2

A company that produces personal computers has determined that the number of computers it can sell (s) is inversely proportional to the price (P) of the computer. Two thousand computers can be sold when the price is \$2500. How many computers can be sold when the price of a computer is \$2000?

Strategy

To find the number of computers:

- Write the basic inverse variation equation, replace the variables by the given values, and solve for k.
- Write the inverse variation equation, replacing k by its value. Substitute 2000 for P and solve for s.

Solution

$$s = \frac{k}{P}$$

$$2000 = \frac{k}{2500}$$

$$5{,}000{,}000 = k$$

$$s = \frac{5{,}000{,}000}{P} = \frac{5{,}000{,}000}{2000} = 2500$$

At \$2000 each, 2500 computers can be sold.

You Try It 2

The resistance (R) to the flow of electric current in a wire of fixed length is inversely proportional to the square of the diameter (d) of a wire. If a wire of diameter 0.01 cm has a resistance of 0.5 ohm, what is the resistance in a wire that is 0.02 cm in diameter?

Your strategy

Your solution

Example 3

The pressure (P) of a gas varies directly as the temperature (T) and inversely as the volume (V). When $T = 50°$ and $V = 275$ in^3, $P = 20$ lb/in^2. Find the pressure of a gas when $T = 60°$ and $V = 250$ in^3.

Strategy

To find the pressure:

- Write the basic combined variation equation, replace the variables by the given values, and solve for k.
- Write the combined variation equation, replacing k by its value. Substitute 60 for T and 250 for V, and solve for P.

Solution

$$P = \frac{kT}{V}$$

$$20 = \frac{k \cdot 50}{275}$$

$$110 = k$$

$$P = \frac{110T}{V} = \frac{110 \cdot 60}{250} = 26.4$$

The pressure is 26.4 lb/in^2.

You Try It 3

The strength (s) of a rectangular beam varies jointly as its width (w) and the square of its depth (d) and inversely as its length (L). If the strength of a beam 2 in. wide, 12 in. deep, and 12 ft long is 1200 lb, find the strength of a beam that is 4 in. wide, 8 in. deep, and 16 ft long.

Your strategy

Your solution

Solutions on p. S27

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8.7 Exercises

Objective A

1. Which of the following represent(s) direct variation? Why?

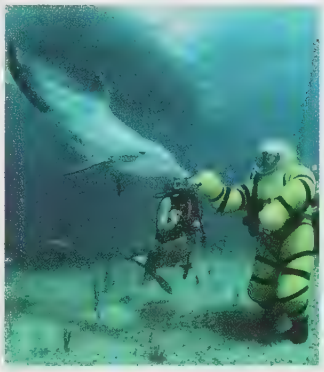
a. $y = kx$ **b.** $y = \frac{k}{x}$ **c.** $y = k + x$ **d.** $y = \frac{k}{x^2}$

2. Which of the following equations represent(s) "d varies directly as t"? Explain your answer.

a. $d = 400t$ **b.** $d = \frac{16}{t}$ **c.** $d = 25t$ **d.** $d = t - 50$

3. The profit (P) realized by a company varies directly as the number of products it sells (s). If a company makes a profit of \$4000 on the sale of 250 products, what is the profit when the company sells 5000 products?

4. The income (I) of a computer analyst varies directly as the number of hours (h) worked. If the analyst earns \$672 for working 8 h, how much will the analyst earn by working 36 h?



5. The pressure (p) on a diver in the water varies directly as the depth (d). If the pressure is 4.5 lb/in^2 when the depth is 10 ft, what is the pressure when the depth is 15 ft?

6. The distance (d) a spring will stretch varies directly as the force (f) applied to the spring. If a force of 6 lb is required to stretch a spring 3 in., what force is required to stretch the spring 4 in.?

7. The distance (d) an object will fall is directly proportional to the square of the time (t) of the fall. If an object falls 144 ft in 3 s, how far will the object fall in 10 s?

8. The period (p) of a pendulum, or the time it takes the pendulum to make one complete swing, varies directly as the square root of the length (L) of the pendulum. If the period of a pendulum is 1.5 s when the length is 2 ft, find the period when the length is 5 ft. Round to the nearest hundredth.

9. The distance (s) a ball will roll down an inclined plane is directly proportional to the square of the time (t). If the ball rolls 6 ft in 1 s, how far will it roll in 3 s?

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10. The stopping distance (s) of a car varies directly as the square of its speed (v). If a car traveling 30 mph requires 63 ft to stop, find the stopping distance for a car traveling 55 mph.

11. The time (t) for a car to travel between two cities is inversely proportional to the rate (r) of travel. If it takes 5 h to travel between the cities at a rate of 55 mph, find the time to travel between the two cities at a rate of 65 mph. Round to the nearest tenth.

12. The speed (v) of a gear varies inversely as the number of teeth (t). If a gear that has 45 teeth makes 24 revolutions per minute, how many revolutions per minute will a gear that has 36 teeth make?

13. The pressure (p) of a liquid varies directly as the product of the depth (d) and the density (D) of the liquid. If the pressure is 150 lb/in^2 when the depth is 100 in. and the density is 1.2, find the pressure when the density remains the same and the depth is 75 in.

14. The current (I) in a wire varies directly as the voltage (V) and inversely as the resistance (R). If the current is 10 amps when the voltage is 110 volts and the resistance is 11 ohms, find the current when the voltage is 180 volts and the resistance is 24 ohms.

15. The repulsive force (f) between the north poles of two magnets is inversely proportional to the square of the distance (d) between them. If the repulsive force is 20 lb when the distance is 4 in., find the repulsive force when the distance is 2 in.

APPLYING THE CONCEPTS

16. In the inverse variation equation $y = \frac{k}{x}$, what is the effect on x if y doubles?

17. In the direct variation equation $y = kx$, what is the effect on y when x doubles?

Complete using the word *directly* or *inversely*.

18. If a varies directly as b and inversely as c, then c varies _____ as b and _____ as a.

19. If a varies _____ as b and c, then abc is constant.

20. If the length of a rectangle is held constant, the area of the rectangle varies _____ as the width.

21. If the area of a rectangle is held constant, the length of the rectangle varies _____ as the width.

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Focus on Problem Solving

Find a Pattern

Polya's four recommended problem-solving steps are stated below.

1. Understand the problem.
2. Devise a plan.
3. Carry out the plan.
4. Review your solution.

Point of Interest

Karl Friedrich Gauss (1777–1855) has been called the "Prince of Mathematicians" by some historians. He applied his genius to many areas of mathematics and science. A unit of magnetism, the *gauss*, is named in his honor. Some electronic equipment (the television, for instance) contains a *degausser* that controls magnetic fields.

One of the several ways of devising a plan is first to try to find a pattern. Karl Friedrich Gauss supposedly used this method to solve a problem that was given to his math class when he was in elementary school. As the story goes, his teacher wanted to grade some papers while the class worked on a math problem. The problem given to the class was to find the sum

$$1 + 2 + 3 + 4 + \cdots + 100$$

Gauss quickly solved the problem by seeing a pattern. Here is what he saw.

$$\underbrace{1 + 2 + 3 + 4 + \cdots + 97 + 98 + 99 + 100}_{101,\ 101,\ 101,\ 101}$$

Note that

$$1 + 100 = 101$$
$$2 + 99 = 101$$
$$3 + 98 = 101$$
$$4 + 97 = 101$$

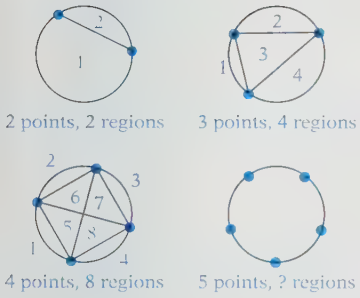
Gauss noted that there were 50 sums of 101. Therefore, the sum of the first 100 natural numbers is

$$1 + 2 + 3 + 4 + \cdots + 97 + 98 + 99 + 100 = 50(101) = 5050$$

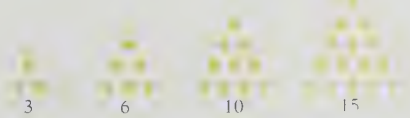
Try to solve the following problems by finding a pattern.

1. Find the sum $2 + 4 + 6 + \cdots + 96 + 98 + 100$.
2. Find the sum $1 + 3 + 5 + \cdots + 97 + 99 + 101$.
3. Find another method of finding the sum $1 + 3 + 5 + \cdots + 97 + 99 + 101$ given in the previous exercise.
4. Find the sum $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{49 \cdot 50}$.

 Hint: $\frac{1}{1 \cdot 2} = \frac{1}{2}$, $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} = \frac{2}{3}$, $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} = \frac{3}{4}$



5. The following problem shows that checking a few cases does not always result in a conjecture that is true for *all* cases. Select any two points on a circle and draw a *chord,* the line connecting the points (see the drawing in the left margin). The chord divides the circle into 2 regions. Now select 3 different points and draw chords connecting each of the three points with every other point. The chords divide the circle into 4 regions. Now select 4 points and connect each of the points with every other point. Make a conjecture about the relationship between the number of regions and the number of points on the circle. Does your conjecture work for 5 points? 6 points?
6. A *polygonal number* is a number that can be represented by arranging that number of dots in rows to form a geometric figure such as a triangle, square, pentagon, or hexagon. For instance, the first four *triangular numbers,* 3, 6, 10, and 15, are given (and shown) below. What are the next two triangular numbers?



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Projects and Group Activities

Continued Fractions

The following complex fraction is called a **continued fraction**.

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cdots}}}}$$

The dots indicate that the pattern continues to repeat forever.

A **convergent** of a continued fraction is an approximation of the repeated pattern. For instance,

$$c_2 = 1 + \cfrac{1}{1 + \cfrac{1}{1 + 1}} \qquad c_3 = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + 1}}} \qquad c_4 = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + 1}}}}$$

1. Calculate c_5 for the continued fraction above.

This particular continued fraction is related to the golden rectangle, which has been used in architectural designs as diverse as the Parthenon in Athens, built around 440 B.C., and the United Nations building. A golden rectangle is one for which

$$\frac{\text{length}}{\text{width}} = \frac{\text{length} + \text{width}}{\text{length}}$$

An example of a golden rectangle is shown at the right.



Here is another continued fraction that was discovered by Leonhard Euler (1707–1793). Calculating the convergents of this continued fraction yields approximations that are closer and closer to π.

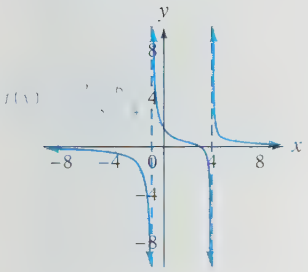
$$\pi = 3 + \cfrac{1^2}{6 + \cfrac{3^2}{6 + \cfrac{5^2}{6 + \cfrac{7^2}{6 + \cdots}}}}$$

2. Calculate $c_5 = 3 + \cfrac{1^2}{6 + \cfrac{3^2}{6 + \cfrac{5^2}{6 + \cfrac{7^2}{6 + \cfrac{9^2}{6 + 11^2}}}}}$

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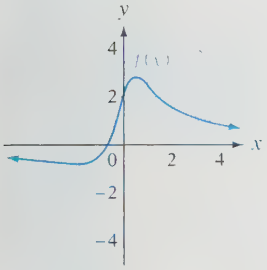
Graphing Rational Functions

The domain of a function is the set of the first coordinates of all the ordered pairs of the function. When a function is given by an equation, the domain of the function is all real numbers for which the function evaluates to a real number. For rational functions, we must exclude from the domain all those values of the variable for which the denominator of the rational function is zero. The graphing calculator is a useful tool to show the graphs of functions with excluded values in the domain of the function.



The graph of the function $f(x) = \dfrac{2x - 6}{x^2 - 3x - 4}$ is shown at the left. Note that the graph never intersects the lines $x = -1$ and $x = 4$ (shown as dashed lines). These are the two values excluded from the domain of f. The graph of the function f gets closer to the dashed line $x = 4$ as x gets closer to 4. The graph of the function f also gets closer to $x = -1$ as x gets closer to -1. The lines that are "approached" by the function are called **asymptotes**.

The domain is $\{x | x \neq -1, x \neq 4\}$.



The graph of the function $f(x) = \dfrac{3x + 2}{x^2 + 1}$ is shown at the left. The domain must exclude values of x for which $x^2 + 1 = 0$. It is not possible that $x^2 + 1 = 0$, because $x^2 \geq 0$, and a positive number added to a number equal to or greater than zero cannot equal zero. Therefore, there are no real numbers that must be excluded from the domain of f.

The domain is $\{x | x \in \text{real numbers}\}$.

Use a graphing calculator to find the domain of the rational function.

1. $h(x) = \dfrac{5x}{3x + 9}$ **2.** $f(x) = \dfrac{-2x}{6 - 2x}$ **3.** $q(x) = \dfrac{2x - 1}{x^2 + x - 6}$

4. $G(x) = \dfrac{3 - 4x}{x^2 + 4x - 5}$ **5.** $H(x) = \dfrac{x + 3}{x^2 + 2}$ **6.** $g(x) = \dfrac{x^2 + 1}{x^2}$

Graphing Variation Equations

1. Graph $y = kx$ when $k = 2$.
What kind of function does the graph represent?

2. Graph $y = kx$ when $k = \dfrac{1}{2}$.
What kind of function does the graph represent?

3. Graph $y = \dfrac{k}{x}$ when $k = 2$ and $x > 0$.
Is this the graph of a function?

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Chapter Summary

Key Words

A *rational expression* is a fraction in which the numerator and denominator are polynomials. A rational expression is in *simplest form* when the numerator and denominator have no common factors. [p. 431]

The *least common multiple* (LCM) of two or more polynomials is the simplest polynomial that contains the factors of each polynomial. [p. 439]

The *reciprocal* of a rational expression is the rational expression with the numerator and denominator interchanged. [p. 434]

A *complex fraction* is a fraction whose numerator or denominator contains one or more fractions. [p. 451]

A *ratio* is the quotient of two quantities that have the same unit. [p. 457]

A *rate* is the quotient of two quantities that have different units. [p. 457]

A *proportion* is an equation that states the equality of two ratios or rates. [p. 457]

A *literal equation* is an equation that contains more than one variable. [p. 467]

Direct variation is a special function that can be expressed as the equation $y = kx$, where k is a constant called the *constant of variation* or the *constant of proportionality.* [p. 479]

Inverse variation is a function that can be expressed as the equation $y = \frac{k}{x}$, where k is a constant. [p. 480]

Joint variation is a variation in which a variable varies directly as the product of two or more variables. A joint variation can be expressed as the equation $z = kxy$, where k is a constant. [p. 480]

Combined variation is a variation in which two or more types of variation occur at the same time. [p. 480]

Essential Rules

To Multiply Fractions [p. 432]

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

To Divide Fractions [p. 434]

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

To Add Fractions [p. 441]

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

To Subtract Fractions [p. 441]

$$\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$$

Equation for Work Problems [p. 471]

$$\text{Rate of work} \times \text{time worked} = \text{part of task completed}$$

Uniform Motion Equation [p. 473]

$$\text{Distance} = \text{rate} \times \text{time}$$

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Chapter Review

1. Divide: $\frac{6a^2b^7}{25x^3y} \div \frac{12a^3b^4}{5x^2y^2}$

2. Add: $\frac{x+7}{15x} + \frac{x-2}{20x}$

3. Simplify: $\dfrac{x - \frac{16}{5x-2}}{3x - 4 - \frac{88}{5x-2}}$

4. Simplify: $\frac{x^2 + x - 30}{15 + 2x - x^2}$

5. Simplify: $\frac{16x^5y^3}{24xy^{10}}$

6. Solve: $\frac{20}{x+2} = \frac{5}{16}$

7. Divide: $\frac{10 - 23y + 12y^2}{6y^2 - y - 5} \div \frac{4y^2 - 13y + 10}{18y^2 + 3y - 10}$

8. Multiply: $\frac{8ab^2}{15x^3y} \cdot \frac{5xy^4}{16a^2b}$

9. Simplify: $\dfrac{1 - \frac{1}{x}}{1 - \frac{8x-7}{x^2}}$

10. Write each fraction in terms of the LCM of the denominators.

$$\frac{x}{12x^2 + 16x - 3}, \frac{4x^2}{6x^2 + 7x - 3}$$

11. Solve $T = 2(ab + bc + ca)$ for a.

12. Solve: $\frac{5}{7} + \frac{x}{2} = 2 - \frac{x}{7}$

13. Solve $i = \frac{100m}{c}$ for c.

14. Solve: $\frac{x+8}{x+4} = 1 + \frac{5}{x+4}$

15. Divide: $\frac{20x^2 - 45x}{6x^3 + 4x^2} \div \frac{40x^3 - 90x^2}{12x^2 + 8x}$

16. Add: $\frac{2y}{5y - 7} + \frac{3}{7 - 5y}$

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17. Subtract: $\dfrac{5x+3}{2x^2+5x-3} - \dfrac{3x+4}{2x^2+5x-3}$

18. Find the LCM of $10x^2 - 11x + 3$ and $20x^2 - 17x + 3$.

19. Solve $4x + 9y = 18$ for y.

20. Multiply: $\dfrac{24x^2-94x+15}{12x^2-49x+15} \cdot \dfrac{24x^2+7x-5}{4-27x+18x^2}$

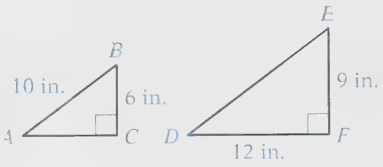
21. Solve: $\dfrac{20}{2x+3} = \dfrac{17x}{2x+3} - 5$

22. Add: $\dfrac{x-1}{x+2} + \dfrac{3x-2}{5-x} + \dfrac{5x^2+15x-11}{x^2-3x-10}$

23. Solve: $\dfrac{6}{x-7} = \dfrac{8}{x-6}$

24. Solve: $\dfrac{3}{20} = \dfrac{x}{80}$

25. Triangles ABC and DEF are similar. Find the perimeter of triangle ABC.



26. One hose can fill a pool in 15 h. The second hose can fill the pool in 10 h. How long would it take to fill the pool using both hoses?

27. A car travels 315 mi in the same amount of time that a bus travels 245 mi. The rate of the car is 10 mph greater than that of the bus. Find the rate of the car.

28. The rate of a jet is 400 mph in calm air. Traveling with the wind, the jet can fly 2100 mi in the same amount of time it takes to fly 1900 mi against the wind. Find the rate of the wind.

29. A pitcher's earned run average (ERA) is the average number of runs allowed in 9 innings of pitching. If a pitcher allows 15 runs in 100 innings, find the pitcher's ERA.

30. The current (I) in an electric circuit varies inversely as the resistance (R). If the current in the circuit is 4 amps when the resistance is 50 ohms, find the current in the circuit when the resistance is 100 ohms.

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Chapter Test

1. Simplify: $\dfrac{16x^5y}{24x^2y^4}$

2. Simplify: $\dfrac{x^2 + 4x - 5}{1 - x^2}$

3. Multiply: $\dfrac{x^3y^4}{x^2 - 4x + 4} \cdot \dfrac{x^2 - x - 2}{x^6y^4}$

4. Multiply: $\dfrac{x^2 + 2x - 3}{x^2 + 6x + 9} \cdot \dfrac{2x^2 - 11x + 5}{2x^2 + 3x - 5}$

5. Divide: $\dfrac{x^2 + 3x + 2}{x^2 + 5x + 4} \div \dfrac{x^2 - x - 6}{x^2 + 2x - 15}$

6. Find the LCM of $6x - 3$ and $2x^2 + x - 1$.

7. Write each fraction in terms of the LCM of the denominators.

$$\frac{3}{x^2 - 2x}, \frac{x}{x^2 - 4}$$

8. Subtract: $\dfrac{2x}{x^2 + 3x - 10} - \dfrac{4}{x^2 + 3x - 10}$

9. Subtract: $\dfrac{2}{2x - 1} - \dfrac{3}{3x + 1}$

10. Subtract: $\dfrac{x}{x + 3} - \dfrac{2x - 5}{x^2 + x - 6}$

11. Simplify: $\dfrac{1 + \dfrac{1}{x} - \dfrac{12}{x^2}}{1 + \dfrac{2}{x} - \dfrac{8}{x^2}}$

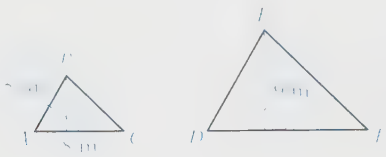
12. Solve: $\dfrac{6}{x} - 2 = 1$

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13. Solve: $\frac{2x}{x+1} - 3 = \frac{2}{x+1}$

14. Solve: $\frac{3}{x+4} = \frac{5}{x+6}$

15. Triangles *ABC* and *DEF* are similar. Find the area of triangle *DEF*.



16. Solve $d = s + rt$ for t.

17. An interior designer uses 2 rolls of wallpaper for every 45 ft^2 of wall space in an office. At this rate, how many rolls of wallpaper are needed for an office that has 315 ft^2 of wall space?

18. One landscaper can till the soil for a lawn in 30 min, whereas it takes a second landscaper 15 min to do the same job. How long would it take to till the soil for the lawn with both landscapers working together?

19. A cyclist travels 20 mi in the same amount of time as it takes a hiker to walk 6 mi. The rate of the cyclist is 7 mph faster than the rate of the hiker. Find the rate of the cyclist.

20. The electrical resistance (r) of a cable varies directly as its length (l) and inversely as the square of its diameter (d). If a cable 16,000 ft long and $\frac{1}{4}$ in. in diameter has a resistance of 3.2 ohms, what is the resistance of a cable that is 8000 ft long and $\frac{1}{2}$ in. in diameter?

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Cumulative Review

1. Simplify: $\left(\frac{2}{3}\right)^2 \div \left(\frac{3}{2} - \frac{2}{3}\right) + \frac{1}{2}$

2. Evaluate $-a^2 + (a - b)^2$ when $a = -2$ and $b = 3$.

3. Simplify: $-2x - (-3y) + 7x - 5y$

4. Simplify: $2[3x - 7(x - 3) - 8]$

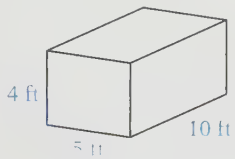
5. Solve: $4 - \frac{2}{3}x = 7$

6. Solve: $3[x - 2(x - 3)] = 2(3 - 2x)$

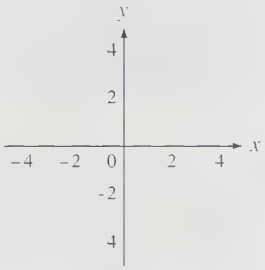
7. Find $16\frac{2}{3}\%$ of 60.

8. Solve: $x - 3(1 - 2x) \geq 1 - 4(3 - 2x)$

9. Find the volume of the rectangular solid shown in the figure.



10. Graph: $x - 2y = 2$



11. Given $P(x) = \frac{x - 1}{2x - 3}$, find $P(-2)$.

12. Find the equation of the line that contains the point $(-2, -1)$ and is parallel to the line $3x - 2y = 6$.

13. Evaluate the determinant:

$$\begin{vmatrix} 6 & 5 \\ 2 & -3 \end{vmatrix}$$

14. Multiply: $(a^2b^5)(ab^2)$

15. Simplify: $\frac{(2a^{-2}b^3)^{-2}}{(4a)^{-1}}$

16. Write 0.000000035 in scientific notation.

17. Multiply: $(2a^2 - 3a + 1)(-2a^2)$

18. Multiply: $(a - 3b)(a + 4b)$

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19. Divide: $(x^3 - 8) \div (x - 2)$

20. Factor: $y^2 - 7y + 6$

21. Factor: $12x^2 - x - 1$

22. Factor: $2a^3 + 7a^2 - 15a$

23. Factor: $4b^2 - 100$

24. Solve: $(x + 3)(2x - 5) = 0$

25. Simplify: $\dfrac{12x^4y^2}{18xy^7}$

26. Simplify: $\dfrac{x^2 - 7x + 10}{25 - x^2}$

27. Divide: $\dfrac{x^2 - x - 56}{x^2 + 8x + 7} \div \dfrac{x^2 - 13x + 40}{x^2 - 4x - 5}$

28. Subtract: $\dfrac{2}{2x - 1} - \dfrac{1}{x + 1}$

29. Simplify: $\dfrac{1 - \dfrac{2}{x} - \dfrac{15}{x^2}}{1 - \dfrac{25}{x^2}}$

30. Solve: $\dfrac{3x}{x - 3} - 2 = \dfrac{10}{x - 3}$

31. A silversmith mixes 60 g of an alloy that is 40% silver with 120 g of another silver alloy. The resulting alloy is 60% silver. Find the percent of silver in the 120 g of alloy.

32. A life insurance policy costs \$32 for every \$1000 of coverage. At this rate, how much money would a policy of \$5000 cost?

33. One water pipe can fill a tank in 9 min, whereas a second pipe requires 18 min to fill the tank. How long would it take both pipes, working together, to fill the tank?

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Chapter 9

Rational Exponents and Radicals



These floating icebergs stand out dramatically against the horizon. A submarine periscope would magnify and bring an object like this one clearly into focus. The periscope could also be used to determine the distance from the submarine to the iceberg. A simple formula involving a radical expression is used to calculate this distance, given the height of the periscope above the surface of the water. The **Project on page 534** shows how to use a radical equation to calculate the distance.

Objectives

Section 9.1

A To simplify expressions with rational exponents
B To write exponential expressions as radical expressions and to write radical expressions as exponential expressions
C To simplify radical expressions that are roots of perfect powers

Section 9.2

A To simplify radical expressions
B To add or subtract radical expressions
C To multiply radical expressions
D To divide radical expressions

Section 9.3

A To simplify a complex number
B To add or subtract complex numbers
C To multiply complex numbers
D To divide complex numbers

Section 9.4

A To solve a radical equation
B To solve application problems



Need help? For on-line student resources, such as section quizzes, visit this textbook's web site at **college.hmco.com/students.**

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Prep Test

1. Complete: $48 = ? \cdot 3$

For Exercises 2 to 7, simplify.

2. 2^5

3. $6\left(\frac{3}{2}\right)$

4. $\frac{1}{2} - \frac{2}{3} + \frac{1}{4}$

5. $(3 - 7x) - (4 - 2x)$

6. $\frac{3x^5y^6}{12x^4y}$

7. $(3x - 2)^2$

For Exercises 8 and 9, multiply.

8. $(2 + 4x)(5 - 3x)$

9. $(6x - 1)(6x + 1)$

10. Solve: $x^2 - 14x - 15 = 0$

Go Figure



You are planning a large dinner party. If you seat 5 people at each table, you end up with only 2 people at the last table. If you seat 3 people at each table you have 9 people left over with no place to sit. There are fewer than 10 tables. How many guests are coming to the dinner party?

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9.1 Rational Exponents and Radical Expressions

Objective A

To simplify expressions with rational exponents



Point of Interest

Nicolas Chuquet (c. 1475), a French physician, wrote an algebra text in which he used a notation for expressions with fractional exponents. He wrote $R^2 6$ to mean $6^{1/2}$ and $R^3 15$ to mean $15^{1/3}$. This was an improvement over earlier notations that used words for these expressions.

In this section, the definition of an exponent is extended beyond integers so that any rational number can be used as an exponent. The definition is expressed in such a way that the Rules of Exponents hold true for rational exponents.

Consider the expression $(a^{1/n})^n$ for $a > 0$ and n a positive integer. Now simplify, assuming that the Rule for Simplifying Powers of Exponential Expressions is true.

$$(a^{1/n})^n = a^{\frac{1}{n} \cdot n} = a^1 = a$$

Because $(a^{1/n})^n = a$, the number $a^{1/n}$ is the number whose nth power is a.

If $a > 0$ and n is a positive number, then $a^{1/n}$ is called the nth root of a.

$25^{1/2} = 5$ because $(5)^2 = 25$. $\quad 8^{1/3} = 2$ because $(2)^3 = 8$.

In the expression $a^{1/n}$, if a is a negative number and n is a positive *even* integer, then $a^{1/n}$ is not a real number.

$(-4)^{1/2}$ is not a real number, because there is no real number whose second power is -4.

When n is a positive *odd* integer, a can be a positive or a negative number.

$(-27)^{1/3} = -3$ because $(-3)^3 = -27$.

Using the definition of $a^{1/n}$ and the Rules of Exponents, it is possible to define any exponential expression that contains a rational exponent.

Rule for Rational Exponents

If m and n are positive integers and $a^{1/n}$ is a real number, then

$$a^{m/n} = (a^{1/n})^m$$

The expression $a^{m/n}$ can also be written $a^{m/n} = a^{m \cdot \frac{1}{n}} = (a^m)^{1/n}$. However, rewriting $a^{m/n}$ as $(a^m)^{1/n}$ is not so useful as rewriting it as $(a^{1/n})^m$. See the Take Note at the top of the next page.

As shown above, expressions that contain rational exponents do not always represent real numbers when the base of the exponential expression is a negative number. For this reason, **all variables in this chapter represent positive numbers unless otherwise stated.**

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TAKE NOTE

Although we can simplify an expression by rewriting $a^{m/n}$ in the form $(a^m)^{1/n}$, it is usually easier to simplify the form $(a^{1/n})^m$. For instance, simplifying $(27^{1/3})^2$ is easier than simplifying $(27^2)^{1/3}$.

➡ Simplify: $27^{2/3}$

$27^{2/3} = (3^3)^{2/3}$ • Rewrite 27 as 3^3.

$= 3^{3(2/3)}$ • Multiply the exponents.

$= 3^2$ • Simplify.

$= 9$

TAKE NOTE

Note that $32^{-2/5} = \frac{1}{4}$, a positive number. The negative exponent does not affect the sign of a number.

➡ Simplify: $32^{-2/5}$

$32^{-2/5} = (2^5)^{-2/5}$ • Rewrite 32 as 2^5.

$= 2^{-2}$ • Multiply the exponents.

$= \frac{1}{2^2}$ • Use the Rule of Negative Exponents.

$= \frac{1}{4}$ • Simplify.

➡ Simplify: $a^{1/2} \cdot a^{2/3} \cdot a^{-1/4}$

$a^{1/2} \cdot a^{2/3} \cdot a^{-1/4} = a^{1/2+2/3-1/4}$ • Use the Rule for Multiplying Exponential Expressions.

$= a^{6/12+8/12-3/12}$

$= a^{11/12}$ • Simplify.

➡ Simplify: $(x^6y^4)^{3/2}$

$(x^6y^4)^{3/2} = x^{6(3/2)}y^{4(3/2)}$ • Use the Rule for Simplifying Powers of Products.

$= x^9y^6$ • Simplify.

➡ Simplify: $\left(\frac{8a^3b^{-4}}{64a^{-9}b^2}\right)^{2/3}$

$\left(\frac{8a^3b^{-4}}{64a^{-9}b^2}\right)^{2/3} = \left(\frac{2^3a^3b^{-4}}{2^6a^{-9}b^2}\right)^{2/3}$ • Rewrite 8 as 2^3 and 64 as 2^6.

$= (2^{-3}a^{12}b^{-6})^{2/3}$ • Use the Rule for Dividing Exponential Expressions.

$= 2^{-2}a^8b^{-4}$ • Use the Rule for Simplifying Powers of Products.

$= \frac{a^8}{2^2b^4} = \frac{a^8}{4b^4}$ • Use the Rule of Negative Exponents and simplify.

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Example 1 Simplify: $64^{-2/3}$

Solution

$$64^{-2/3} = (2^6)^{-2/3} = 2^{-4}$$
$$= \frac{1}{2^4} = \frac{1}{16}$$

You Try It 1 Simplify: $16^{-3/4}$

Your solution

Example 2 Simplify: $(-49)^{3/2}$

Solution The base of the exponential expression is a negative number, while the denominator of the exponent is a positive even number.

Therefore, $(-49)^{3/2}$ is not a real number.

You Try It 2 Simplify: $(-81)^{3/4}$

Your solution

Example 3 Simplify: $(x^{1/2}y^{-3/2}z^{1/4})^{-3/2}$

Solution

$$(x^{1/2}y^{-3/2}z^{1/4})^{-3/2}$$
$$= x^{-3/4}y^{9/4}z^{-3/8}$$
$$= \frac{y^{9/4}}{x^{3/4}z^{3/8}}$$

You Try It 3 Simplify: $(x^{3/4}y^{1/2}z^{-2/3})^{-4/3}$

Your solution

Example 4

Simplify: $\dfrac{x^{1/2}y^{-5/4}}{x^{-4/3}y^{1/3}}$

Solution

$$\frac{x^{1/2}y^{-5/4}}{x^{-4/3}y^{1/3}}$$
$$= x^{3/6 - (-8/6)}y^{-15/12 - 4/12}$$
$$= x^{11/6}y^{-19/12} = \frac{x^{11/6}}{y^{19/12}}$$

You Try It 4

Simplify: $\left(\dfrac{16a^{-2}b^{4/3}}{9a^4b^{-2/3}}\right)^{-1/2}$

Your solution

Solutions on p. S27

To write exponential expressions as radical expressions and to write radical expressions as exponential expressions

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Point of Interest

The radical sign was introduced in 1525 in a book by Christoff Rudolff called *Coss*. He modified the symbol to indicate square roots, cube roots, and fourth roots. The idea of using an index, as we do in our modern notation, did not occur until some years later.

Recall that $a^{1/n}$ is the nth root of a. The expression $\sqrt[n]{a}$ is another symbol for the nth root of a.

If a is a real number, then $a^{1/n} = \sqrt[n]{a}$.

In the expression $\sqrt[n]{a}$, the symbol $\sqrt{\ }$ is called a **radical sign**, n is the **index** of the radical, and a is the **radicand**. When $n = 2$, the radical expression represents a square root and the index 2 is usually not written.

An exponential expression with a rational exponent can be written as a radical expression.

Rule for Writing Exponential Expressions as Radical Expressions

If $a^{1/n}$ is a real number, then $a^{m/n} = a^{m \cdot 1/n} = (a^m)^{1/n} = \sqrt[n]{a^m}$.

The expression $a^{m/n}$ can also be written $a^{m/n} = (a^{1/n})^m = (\sqrt[n]{a})^m$.

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The exponential expression at the right has been written as a radical expression.

$$y^{2/3} = (y^2)^{1/3} = \sqrt[3]{y^2}$$

The radical expressions at the right have been written as exponential expressions.

$$\sqrt[5]{x^6} = (x^6)^{1/5} = x^{6/5}$$
$$\sqrt{17} = (17)^{1/2} = 17^{1/2}$$

➡ Write $(5x)^{2/5}$ as a radical expression.

$(5x)^{2/5} = \sqrt[5]{(5x)^2}$ • The denominator of the rational exponent is the index of the radical. The numerator is the power of the radicand.

$= \sqrt[5]{25x^2}$ • Simplify.

➡ Write $\sqrt[3]{x^4}$ as an exponential expression with a rational exponent.

$\sqrt[3]{x^4} = (x^4)^{1/3}$ • The index of the radical is the denominator of the rational exponent. The power of the radicand is the numerator of the rational exponent.

$= x^{4/3}$ • Simplify.

➡ Write $\sqrt[3]{a^3 + b^3}$ as an exponential expression with a rational exponent.

$\sqrt[3]{a^3 + b^3} = (a^3 + b^3)^{1/3}$

Note that $(a^3 + b^3)^{1/3} \neq a + b$.

Example 5 Write $(3x)^{5/4}$ as a radical expression.

Solution $(3x)^{5/4} = \sqrt[4]{(3x)^5} = \sqrt[4]{243x^5}$

You Try It 5 Write $(2x^3)^{3/4}$ as a radical expression.

Your solution

Example 6 Write $-2x^{2/3}$ as a radical expression.

Solution $-2x^{2/3} = -2(x^2)^{1/3} = -2\sqrt[3]{x^2}$

You Try It 6 Write $-5a^{5/6}$ as a radical expression.

Your solution

Example 7 Write $\sqrt[4]{3a}$ as an exponential expression.

Solution $\sqrt[4]{3a} = (3a)^{1/4}$

You Try It 7 Write $\sqrt[3]{3ab}$ as an exponential expression.

Your solution

Example 8 Write $\sqrt{a^2 - b^2}$ as an exponential expression.

Solution $\sqrt{a^2 - b^2} = (a^2 - b^2)^{1/2}$

You Try It 8 Write $\sqrt[4]{x^4 + y^4}$ as an exponential expression.

Your solution

Solutions on pp. S27–S28

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Objective 1

To simplify radical expressions that are roots of perfect powers

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Every positive number has two square roots, one a positive number and one a negative number. For example, because $(5)^2 = 25$ and $(-5)^2 = 25$, there are two square roots of 25: 5 and -5.

The symbol $\sqrt{\ }$ is used to indicate the positive or **principal square root**. To indicate the negative square root of a number, a negative sign is placed in front of the radical.

$\sqrt{25} = 5$

$-\sqrt{25} = -5$

The square root of zero is zero.

$\sqrt{0} = 0$

The square root of a negative number is not a real number, because the square of a real number must be positive.

$\sqrt{-25}$ is not a real number.

Note that

$$\sqrt{(-5)^2} = \sqrt{25} = 5 \text{ and } \sqrt{5^2} = \sqrt{25} = 5$$

This is true for all real numbers and is stated as the following result.

For any real number a, $\sqrt{a^2} = |a|$ and $-\sqrt{a^2} = -|a|$. If a is a positive real number, then $\sqrt{a^2} = a$ and $(\sqrt{a})^2 = a$.

Besides square roots, we can also determine cube roots, fourth roots, and so on.

$\sqrt[3]{8} = 2$, because $2^3 = 8$. • The cube root of a positive number is positive.

$\sqrt[3]{-8} = -2$, because $(-2)^3 = -8$. • The cube root of a negative number is negative.

$\sqrt[4]{625} = 5$, because $5^4 = 625$.

$\sqrt[5]{243} = 3$, because $3^5 = 243$.

The following properties hold true for finding the nth root of a real number.

If n is an even integer, then $\sqrt[n]{a^n} = |a|$ and $-\sqrt[n]{a^n} = -|a|$. If n is an odd integer, then $\sqrt[n]{a^n} = a$.

TAKE NOTE

Note that when the index is an even natural number, the nth root requires absolute value symbols.

$\sqrt[6]{y^6} = |y|$ but $\sqrt[5]{y^5} = y$

Because we stated that variables within radicals represent *positive* numbers, we will omit the absolute value symbols when writing an answer.

For example,

$$\sqrt[6]{y^6} = |y| \qquad -\sqrt[12]{x^{12}} = -|x| \qquad \sqrt[5]{b^5} = b$$

For the remainder of this chapter, we will assume that variable expressions inside a radical represent positive numbers. Therefore, it is not necessary to use the absolute value signs.

➡ Simplify: $\sqrt[4]{x^4y^8}$

$\sqrt[4]{x^4y^8} = (x^4y^8)^{1/4}$ • The radicand is a perfect fourth power because the exponents on the variables are divisible by 4. Write the radical expression as an exponential expression.

$= x^{4/4}y^{8/4}$ • Use the Rule for Simplifying Powers of Products.

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Simplify: $\sqrt[3]{125c^9d^6}$

$\sqrt[3]{125c^9d^6} = (5^3c^9d^6)^{1/3}$ • The radicand is a perfect cube because 125 is a perfect cube ($125 = 5^3$) and all the exponents on the variables are divisible by 3.

$= 5c^3d^2$ • Use the Rule for Simplifying Powers of Products.

Note that **a variable expression is a perfect power if the exponents on the factors are evenly divisible by the index of the radical.**

The chart below shows roots of perfect powers. Knowledge of these roots is very helpful when simplifying radical expressions.

Square Roots		*Cube Roots*	*Fourth Roots*	*Fifth Roots*
$\sqrt{1} = 1$	$\sqrt{36} = 6$	$\sqrt[3]{1} = 1$	$\sqrt[4]{1} = 1$	$\sqrt[5]{1} = 1$
$\sqrt{4} = 2$	$\sqrt{49} = 7$	$\sqrt[3]{8} = 2$	$\sqrt[4]{16} = 2$	$\sqrt[5]{32} = 2$
$\sqrt{9} = 3$	$\sqrt{64} = 8$	$\sqrt[3]{27} = 3$	$\sqrt[4]{81} = 3$	$\sqrt[5]{243} = 3$
$\sqrt{16} = 4$	$\sqrt{81} = 9$	$\sqrt[3]{64} = 4$	$\sqrt[4]{256} = 4$	
$\sqrt{25} = 5$	$\sqrt{100} = 10$	$\sqrt[3]{125} = 5$	$\sqrt[4]{625} = 5$	

TAKE NOTE

From the chart, $\sqrt[5]{243} = 3$, which means that $3^5 = 243$. From this we know that $(-3)^5 = -243$, which means $\sqrt[5]{-243} = -3$.

Simplify: $\sqrt[5]{-243x^5y^{15}}$

$\sqrt[5]{-243x^5y^{15}} = -3xy^3$ • From the chart, 243 is a perfect fifth power, and each exponent is divisible by 5. Therefore, the radicand is a perfect fifth power.

Example 9 Simplify: $\sqrt[3]{-125a^6b^9}$

Solution The radicand is a perfect cube.

$\sqrt[3]{-125a^6b^9} = -5a^2b^3$ • Divide each exponent by 3.

You Try It 9 Simplify: $\sqrt[3]{-8x^{12}y^3}$

Your solution

Example 10 Simplify: $-\sqrt[4]{16a^4b^8}$

Solution The radicand is a perfect fourth power.

$-\sqrt[4]{16a^4b^8} = -2ab^2$ • Divide each exponent by 4.

You Try It 10 Simplify: $-\sqrt[4]{81x^{12}y^8}$

Your solution

Solutions on p. S28

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9.1 Exercises

Objective 1

Simplify.

1. $8^{1/3}$
2. $16^{1/2}$
3. $9^{3/2}$
4. $25^{3/2}$
5. $27^{-2/3}$
6. $64^{-1/3}$
7. $32^{2/5}$
8. $16^{3/4}$
9. $(-25)^{5/2}$
10. $(-36)^{1/4}$
11. $\left(\frac{25}{49}\right)^{-3/2}$
12. $\left(\frac{8}{27}\right)^{2/3}$
13. $x^{1/2}x^{1/2}$
14. $a^{1/3}a^{5/3}$
15. $y^{-1/4}y^{3/4}$
16. $x^{2/5} \cdot x^{-4/5}$
17. $x^{2/3} \cdot x^{3/4}$
18. $x \cdot x^{1/2}$
19. $a^{1/3} \cdot a^{3/4} \cdot a^{-1/2}$
20. $y^{-1/6} \cdot y^{2/3} \cdot y^{1/2}$
21. $\frac{a^{1/2}}{a^{3/2}}$
22. $\frac{b^{1/3}}{b^{4/3}}$
23. $\frac{y^{-3/4}}{y^{1/4}}$
24. $\frac{x^{-3/5}}{x^{1/5}}$
25. $\frac{y^{2/3}}{y^{-5/6}}$
26. $\frac{b^{3/4}}{b^{-3/2}}$
27. $(x^2)^{-1/2}$
28. $(a^8)^{-3/4}$
29. $(x^{-2/3})^6$
30. $(y^{-5/6})^{12}$
31. $(a^{-1/2})^{-2}$
32. $(b^{-2/3})^{-6}$
33. $(x^{-3/8})^{-4/5}$
34. $(y^{-3/2})^{-2/9}$
35. $(a^{1/2} \cdot a)^2$
36. $(b^{2/3} \cdot b^{1/6})^6$
37. $(x^{1/2} \cdot x^{3/4})^{-2}$
38. $(a^{1/2} \cdot a^{-2})^3$
39. $(y^{-1/2} \cdot y^{2/3})^{2/3}$
40. $(b^{-2/3} \cdot b^{1/4})^{-4/3}$
41. $(x^8y^2)^{1/2}$
42. $(a^3b^9)^{2/3}$

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43. $(x^4y^2z^6)^{3/2}$

44. $(a^8b^4c^4)^{3/4}$

45. $(x^{-3}y^6)^{-1/3}$

46. $(a^2b^{-6})^{-1/2}$

47. $(x^{-2}y^{1/3})^{-3/4}$

48. $(a^{-2/3}b^{2/3})^{3/2}$

49. $\left(\dfrac{x^{1/2}}{y^2}\right)^4$

50. $\left(\dfrac{b^{-3/4}}{a^{-1/2}}\right)^8$

51. $\dfrac{x^{1.4}\cdot x^{1.2}}{x^{2/3}}$

52. $\dfrac{b^{1/2}\cdot b^{-3/4}}{b^{1/4}}$

53. $\left(\dfrac{y^{2/3}\cdot y^{5/6}}{y^{1/9}}\right)^9$

54. $\left(\dfrac{a^{1/3}\cdot a^{-2/3}}{a^{1/2}}\right)^4$

55. $\left(\dfrac{b^2\cdot b^{-3/4}}{b^{-1/2}}\right)^{-1/2}$

56. $\dfrac{(x^{-5/6}\cdot x^3)^{-2/3}}{x^{4/3}}$

57. $(a^{2/3}b^2)^6(a^3b^3)^{1/3}$

58. $(x^3y^{1.2})^{.2}(x^{-3}y^2)^{1/6}$

59. $(16m^{.2}n^4)^{1/2}(mn^{1/2})$

60. $(27m^3n^{-6})^{1/3}(m^{1/3}n^{5/6})^6$

61. $\left(\dfrac{x^{1.2}y^{-3.4}}{y^{2/3}}\right)^6$

62. $\left(\dfrac{x^{1/2}y^{-5/4}}{y^{-3/4}}\right)^{-4}$

63. $\left(\dfrac{2^{.6}b^{-3}}{a^{-1/2}}\right)^{2/3}$

64. $\left(\dfrac{49c^{5/3}}{a^{-1/4}b^{5/6}}\right)^{-3/2}$

65. $y^{3/2}(y^{1/2}-y^{-1/2})$

66. $y^{3/5}(y^{2/5}+y^{-3/5})$

67. $a^{-1/4}(a^{5/4}-a^{9/4})$

68. $x^{4/3}(x^{2/3}+x^{-1/3})$

69. $x^n\cdot x^{3n}$

70. $a^{2n}\cdot a^{-5n}$

71. $x^n\cdot x^{n/2}$

72. $a^{n/2}\cdot a^{-n/3}$

73. $\dfrac{y^{n/2}}{y^{-n}}$

74. $\dfrac{b^{m/3}}{b^m}$

75. $(x^{2n})^n$

76. $(x^{5n})^{2n}$

77. $(x^{n/4}y^{n/8})^8$

78. $(x^{n/2}y^{n/3})^6$

79. $(x^{n.5}y^{n.10})^{20}$

80. $(x^{n/2}y^{n/5})^{10}$

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Objective B

Rewrite the exponential expression as a radical expression.

81. $3^{1/4}$ **82.** $5^{1/2}$ **83.** $a^{3/2}$ **84.** $b^{4/3}$

85. $(2t)^{5/2}$ **86.** $(3x)^{2/3}$ **87.** $-2x^{2/3}$ **88.** $-3a^{2/5}$

89. $(a^2b)^{2/3}$ **90.** $(x^2y^3)^{3/4}$ **91.** $(a^2b^4)^{3/5}$ **92.** $(a^3b^7)^{3/2}$

93. $(4x - 3)^{3/4}$ **94.** $(3x - 2)^{1/3}$ **95.** $x^{-2/3}$ **96.** $b^{-3/4}$

Rewrite the radical expression as an exponential expression.

97. $\sqrt{14}$ **98.** $\sqrt{7}$ **99.** $\sqrt[3]{x}$ **100.** $\sqrt[4]{x}$

101. $\sqrt[3]{x^4}$ **102.** $\sqrt[4]{a^3}$ **103.** $\sqrt[5]{b^3}$ **104.** $\sqrt[4]{b^5}$

105. $\sqrt[3]{2x^2}$ **106.** $\sqrt[5]{4y^7}$ **107.** $-\sqrt{3x^5}$ **108.** $-\sqrt[4]{4x^5}$

109. $3x\sqrt[3]{y^2}$ **110.** $2y\sqrt{x^3}$ **111.** $\sqrt{a^2 - 2}$ **112.** $\sqrt{3 - y^2}$

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Simplify.

113. $\sqrt{x^{16}}$ **114.** $\sqrt{y^{14}}$ **115.** $-\sqrt{x^8}$ **116.** $-\sqrt{a^6}$

117. $\sqrt[3]{x^3y^9}$ **118.** $\sqrt[3]{a^6b^{12}}$ **119.** $-\sqrt[3]{x^{15}y^3}$ **120.** $-\sqrt[3]{a^9b^9}$

121. $\sqrt{16a^4b^{12}}$ **122.** $\sqrt{25x^8y^2}$ **123.** $\sqrt{-16x^4y^2}$ **124.** $\sqrt{-9a^4b^8}$

125. $\sqrt[3]{27x^9}$ **126.** $\sqrt[3]{8a^{21}b^6}$ **127.** $\sqrt[3]{-64x^9y^{12}}$ **128.** $\sqrt[3]{-27a^3b^{15}}$

129. $-\sqrt[4]{x^8y^{12}}$ **130.** $-\sqrt[4]{a^{16}b^4}$ **131.** $\sqrt[5]{x^{20}y^{10}}$ **132.** $\sqrt[5]{a^5b^{25}}$

133. $\sqrt[4]{81x^4y^{20}}$ **134.** $\sqrt[4]{16a^8b^{20}}$ **135.** $\sqrt[5]{32a^5b^{10}}$ **136.** $\sqrt[5]{-32x^{15}y^{20}}$

APPLYING THE CONCEPTS

137. Determine whether the following statements are true or false. If the statement is false, correct the right-hand side of the equation.

a. $\sqrt{(-2)^2} = -2$ **b.** $\sqrt[3]{(-3)^3} = -3$ **c.** $\sqrt[n]{a} = a^{1/n}$

d. $\sqrt[n]{a^n + b^n} = a + b$ **e.** $(a^{1/2} + b^{1/2})^2 = a + b$ **f.** $\sqrt[m]{a^n} = a^{mn}$

138. Simplify.

a. $\sqrt[3]{\sqrt{x^6}}$ **b.** $\sqrt[4]{\sqrt{a^8}}$ **c.** $\sqrt{\sqrt{81y^8}}$

d. $\sqrt{\sqrt[n]{a^{4n}}}$ **e.** $\sqrt[n]{\sqrt{b^{6n}}}$ **f.** $\sqrt{\sqrt[3]{x^{12}y^{24}}}$

139. If x is any real number, is $\sqrt{x^2} = x$ always true? Show why or why not.

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9.2 Operations on Radical Expressions

Objective A

To simplify radical expressions

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Point of Interest

The Latin expression for irrational numbers was *numerus surdus*, which literally means "inaudible number." A prominent 16th-century mathematician wrote of irrational numbers, "Just as an infinite number is not a number, so an irrational number is not a true number, but lies hidden in some sort of cloud of infinity." In 1872, Richard Dedekind wrote a paper that established the first logical treatment of irrational numbers.

If a number is not a perfect power, its root can only be approximated; examples include $\sqrt{5}$ and $\sqrt[3]{3}$. These numbers are **irrational numbers**. Their decimal representations never terminate or repeat.

$$\sqrt{5} = 2.2360679\ldots \qquad \sqrt[3]{3} = 1.4422495\ldots$$

A radical expression is not in simplest form when the radicand contains a factor greater than 1 that is a perfect power. The Product Property of Radicals is used to simplify radical expressions whose radicands are not perfect powers.

The Product Property of Radicals

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are positive real numbers, then $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$ and $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$.

Simplify: $\sqrt{48}$

$\sqrt{48} = \sqrt{16 \cdot 3}$ • Write the radicand as the product of a perfect square and a factor that does not contain a perfect square.

$= \sqrt{16}\sqrt{3}$ • Use the Product Property of Radicals to write the expression as a product.

$= 4\sqrt{3}$ • Simplify $\sqrt{16}$.

Note that 48 must be written as the product of a perfect square and *a factor that does not contain a perfect square*. Therefore, it would not be correct to rewrite $\sqrt{48}$ as $\sqrt{4 \cdot 12}$ and simplify the expression as shown at the right. Although 4 is a perfect-square factor of 48, 12 contains a perfect square ($12 = 4 \cdot 3$) and $\sqrt{12}$ can be simplified. Remember to find the largest perfect power that is a factor of the radicand.

$$\begin{aligned}\sqrt{48} &= \sqrt{4 \cdot 12}\\ &= \sqrt{4}\sqrt{12}\\ &= 2\sqrt{12}\end{aligned}$$

Not in simplest form

Simplify: $\sqrt{18x^2y^3}$

$\sqrt{18x^2y^3} = \sqrt{9x^2y^2 \cdot 2y}$ • Write the radicand as the product of a perfect square and factors that do not contain a perfect square.

$= \sqrt{9x^2y^2}\sqrt{2y}$ • Use the Product Property of Radicals to write the expression as a product.

$= 3xy\sqrt{2y}$ • Simplify.

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Simplify: $\sqrt[3]{x^7}$

$\sqrt[3]{x^7} = \sqrt[3]{x^6 \cdot x}$ • Write the radicand as the product of a perfect cube and a factor that does not contain a perfect cube.

$= \sqrt[3]{x^6}\sqrt[3]{x}$ • Use the Product Property of Radicals to write the expression as a product.

$= x^2\sqrt[3]{x}$ • Simplify.

Simplify: $\sqrt[4]{32x^7}$

$\sqrt[4]{32x^7} = \sqrt[4]{2^5x^7}$ • Write the prime factorization of the coefficient of the radicand in exponential form.

$= \sqrt[4]{2^4x^4(2x^3)}$ • Write the radicand as the product of a perfect fourth power and factors that do not contain a perfect fourth power.

$= \sqrt[4]{2^4x^4}\sqrt[4]{2x^3}$ • Use the Product Property of Radicals to write the expression as a product.

$= 2x\sqrt[4]{2x^3}$ • Simplify.

Example 1 Simplify: $\sqrt[4]{x^9}$

Solution $\sqrt[4]{x^9} = \sqrt[4]{x^8 \cdot x} = \sqrt[4]{x^8}\sqrt[4]{x}$
$= x^2\sqrt[4]{x}$

You Try It 1 Simplify: $\sqrt[5]{x^7}$

Your solution

Example 2 Simplify: $\sqrt[3]{-27a^5b^{12}}$

Solution $\sqrt[3]{-27a^5b^{12}} = \sqrt[3]{(-3)^3a^5b^{12}}$
$= \sqrt[3]{(-3)^3a^3b^{12}(a^2)}$
$= \sqrt[3]{(-3)^3a^3b^{12}}\sqrt[3]{a^2}$
$= -3ab^4\sqrt[3]{a^2}$

You Try It 2 Simplify: $\sqrt[3]{-64x^8y^{18}}$

Your solution

Solutions on p. S28

To add or subtract radical expressions

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The Distributive Property is used to simplify the sum or difference of radical expressions that have the same radicand and the same index. For example,

$$3\sqrt{5} + 8\sqrt{5} = (3 + 8)\sqrt{5} = 11\sqrt{5}$$

$$2\sqrt[3]{3x} - 9\sqrt[3]{3x} = (2 - 9)\sqrt[3]{3x} = -7\sqrt[3]{3x}$$

TAKE NOTE
Adding and subtracting radicals is similar to combining like terms.

Radical expressions that are in simplest form and have unlike radicands or different indices cannot be simplified by the Distributive Property. The expressions below cannot be simplified by the Distributive Property.

$$3\sqrt[4]{2} - 6\sqrt[4]{3} \qquad 2\sqrt[4]{4x} + 3\sqrt[3]{4x}$$

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➡ Simplify: $3\sqrt{32x^2} - 2x\sqrt{2} + \sqrt{128x^2}$

$3\sqrt{32x^2} - 2x\sqrt{2} + \sqrt{128x^2}$

$= 3\sqrt{16x^2}\sqrt{2} - 2x\sqrt{2} + \sqrt{64x^2}\sqrt{2}$

$= 3 \cdot 4x\sqrt{2} - 2x\sqrt{2} + 8x\sqrt{2}$

$= 12x\sqrt{2} - 2x\sqrt{2} + 8x\sqrt{2}$

$= 18x\sqrt{2}$

• First simplify each term. Then combine like terms by using the Distributive Property.

Example 3

Simplify: $5b\sqrt[4]{32a^7b^5} - 2a\sqrt[4]{162a^3b^9}$

Solution

$5b\sqrt[4]{32a^7b^5} - 2a\sqrt[4]{162a^3b^9}$

$= 5b\sqrt[4]{16a^4b^4}\sqrt[4]{2a^3b} - 2a\sqrt[4]{81b^8}\sqrt[4]{2a^3b}$

$= 5b \cdot 2ab\sqrt[4]{2a^3b} - 2a \cdot 3b^2\sqrt[4]{2a^3b}$

$= 10ab^2\sqrt[4]{2a^3b} - 6ab^2\sqrt[4]{2a^3b}$

$= 4ab^2\sqrt[4]{2a^3b}$

You Try It 3

Simplify: $3xy\sqrt[3]{81x^5y} - \sqrt[3]{192x^8y^4}$

Your solution

Solution on p. S28

To multiply radical expressions

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The Product Property of Radicals is used to multiply radical expressions with the same index.

$\sqrt{3x} \cdot \sqrt{5y} = \sqrt{3x \cdot 5y} = \sqrt{15xy}$

➡ Simplify: $\sqrt[3]{2a^5b}\sqrt[3]{16a^2b^2}$

$\sqrt[3]{2a^5b}\sqrt[3]{16a^2b^2} = \sqrt[3]{32a^7b^3}$ • Use the Product Property of Radicals to multiply the radicands.

$= \sqrt[3]{8a^6b^3}\sqrt[3]{4a}$ • Simplify.

$= 2a^2b\sqrt[3]{4a}$

➡ Simplify: $\sqrt{2x}(\sqrt{8x} - \sqrt{3})$

$\sqrt{2x}(\sqrt{8x} - \sqrt{3}) = \sqrt{2x}(\sqrt{8x}) - \sqrt{2x}(\sqrt{3})$ • Use the Distributive Property.

$= \sqrt{16x^2} - \sqrt{6x}$ • Simplify.

$= 4x - \sqrt{6x}$

➡ Simplify: $(2\sqrt{5} - 3)(3\sqrt{5} + 4)$

$(2\sqrt{5} - 3)(3\sqrt{5} + 4) = 6(\sqrt{5})^2 + 8\sqrt{5} - 9\sqrt{5} - 12$ • Use the FOIL method to multiply the numbers.

$= 30 + 8\sqrt{5} - 9\sqrt{5} - 12$

$= 18 - \sqrt{5}$ • Combine like terms.

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➡ Simplify: $(4\sqrt{a} - \sqrt{b})(2\sqrt{a} + 5\sqrt{b})$

$$(4\sqrt{a} - \sqrt{b})(2\sqrt{a} + 5\sqrt{b})$$
$$= 8(\sqrt{a})^2 + 20\sqrt{ab} - 2\sqrt{ab} - 5(\sqrt{b})^2 \quad \text{• Use the FOIL method.}$$
$$= 8a + 18\sqrt{ab} - 5b$$

TAKE NOTE

The concept of conjugate is used in a number of different instances. Make sure you understand this idea.

The conjugate of $\sqrt{3} - 4$ is $\sqrt{3} + 4$.

The conjugate of $\sqrt{3} + 4$ is $\sqrt{3} - 4$.

The conjugate of $\sqrt{5a} + \sqrt{b}$ is $\sqrt{5a} - \sqrt{b}$.

The expressions $a + b$ and $a - b$ are **conjugates** of each other. Recall that $(a + b)(a - b) = a^2 - b^2$. This identity is used to simplify conjugate radical expressions.

➡ Simplify: $(\sqrt{11} - 3)(\sqrt{11} + 3)$

$$(\sqrt{11} - 3)(\sqrt{11} + 3) = (\sqrt{11})^2 - 3^2 = 11 - 9 \quad \text{• The radical expressions are conjugates.}$$
$$= 2$$

Example 4
Simplify: $\sqrt{3x}(\sqrt{27x^2} - \sqrt{3x})$

Solution
$$\sqrt{3x}(\sqrt{27x^2} - \sqrt{3x}) = \sqrt{81x^3} - \sqrt{9x^2}$$
$$= \sqrt{81x^2}\sqrt{x} - \sqrt{9x^2}$$
$$= 9x\sqrt{x} - 3x$$

You Try It 4
Simplify: $\sqrt{5b}(\sqrt{3b} - \sqrt{10})$

Your solution

Example 5
Simplify: $(2\sqrt[3]{x} - 3)(3\sqrt[3]{x} - 4)$

Solution
$$(2\sqrt[3]{x} - 3)(3\sqrt[3]{x} - 4)$$
$$= 6\sqrt[3]{x^2} - 8\sqrt[3]{x} - 9\sqrt[3]{x} + 12$$
$$= 6\sqrt[3]{x^2} - 17\sqrt[3]{x} + 12$$

You Try It 5
Simplify: $(2\sqrt[3]{2x} - 3)(\sqrt[3]{2x} - 5)$

Your solution

Example 6
Simplify: $(2\sqrt{x} - \sqrt{2y})(2\sqrt{x} + \sqrt{2y})$

Solution
$$(2\sqrt{x} - \sqrt{2y})(2\sqrt{x} + \sqrt{2y})$$
$$= (2\sqrt{x})^2 - (\sqrt{2y})^2$$
$$= 4x - 2y$$

You Try It 6
Simplify: $(\sqrt{a} - 3\sqrt{y})(\sqrt{a} + 3\sqrt{y})$

Your solution

Solutions on p. S28

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To divide radical expressions

VIDEO & DVD, CD TUTOR, WEB, SSM

The Quotient Property of Radicals is used to divide radical expressions with the same index.

The Quotient Property of Radicals

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, and $b \neq 0$, then

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \quad \text{and} \quad \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

➡ Simplify: $\sqrt[3]{\dfrac{81x^5}{y^6}}$

$$\sqrt[3]{\frac{81x^5}{y^6}} = \frac{\sqrt[3]{81x^5}}{\sqrt[3]{y^6}}$$

• Use the Quotient Property of Radicals.

$$= \frac{\sqrt[3]{27x^3}\sqrt[3]{3x^2}}{\sqrt[3]{y^6}} = \frac{3x\sqrt[3]{3x^2}}{y^2}$$

• Simplify each radical expression.

➡ Simplify: $\dfrac{\sqrt{5a^4b^7c^2}}{\sqrt{ab^3c}}$

$$\frac{\sqrt{5a^4b^7c^2}}{\sqrt{ab^3c}} = \sqrt{\frac{5a^4b^7c^2}{ab^3c}}$$

• Use the Quotient Property of Radicals.

$$= \sqrt{5a^3b^4c}$$

$$= \sqrt{a^2b^4}\sqrt{5ac} = ab^2\sqrt{5ac}$$

• Simplify the radicand.

A radical expression is in simplest form when no radical remains in the denominator of the radical expression. The procedure used to remove a radical from the denominator is called **rationalizing the denominator.**

➡ Simplify: $\dfrac{5}{\sqrt{2}}$

$$\frac{5}{\sqrt{2}} = \frac{5}{\sqrt{2}} \cdot 1 = \frac{5}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

• Multiply by $\frac{\sqrt{2}}{\sqrt{2}}$, which equals 1.

$$= \frac{5\sqrt{2}}{2}$$

• $\sqrt{2} \cdot \sqrt{2} = (\sqrt{2})^2 = 2$

➡ Simplify: $\dfrac{3}{\sqrt[3]{4x}}$

$$\frac{3}{\sqrt[3]{4x}} = \frac{3}{\sqrt[3]{4x}} \cdot \frac{\sqrt[3]{2x^2}}{\sqrt[3]{2x^2}}$$

• Because $\sqrt[3]{4x} \cdot \sqrt[3]{2x^2} = \sqrt[3]{8x^3}$, a perfect cube, multiply the expression by $\frac{\sqrt[3]{2x^2}}{\sqrt[3]{2x^2}}$, which equals 1.

$$= \frac{3\sqrt[3]{2x^2}}{\sqrt[3]{8x^3}} = \frac{3\sqrt[3]{2x^2}}{2x}$$

• Simplify.

TAKE NOTE

Multiplying by $\frac{\sqrt[3]{4x}}{\sqrt[3]{4x}}$ will not rationalize the denominator of $\frac{3x}{\sqrt[3]{4x}}$.

$$\frac{3}{\sqrt[3]{4x}} \cdot \frac{\sqrt[3]{4x}}{\sqrt[3]{4x}} = \frac{3\sqrt[3]{4x}}{\sqrt[3]{16x^2}}$$

Because $\sqrt[3]{16x^2}$ is not a perfect cube, the denominator still contains a radical expression.

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To simplify a fraction that has a square-root expression with two terms in the denominator, multiply the numerator and denominator by the conjugate of the denominator. Then simplify.

Simplify: $\dfrac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$

TAKE NOTE
Here is an example of using a conjugate to simplify a radical expression.

$$\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}=\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}\cdot\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}-\sqrt{y}}$$
$$=\frac{(\sqrt{x})^2-\sqrt{xy}-\sqrt{xy}+(\sqrt{y})^2}{(\sqrt{x})^2-(\sqrt{y})^2}=\frac{x-2\sqrt{xy}+y}{x-y}$$

Example 7

Simplify: $\dfrac{5}{\sqrt{5x}}$

Solution

$$\frac{5}{\sqrt{5x}}=\frac{5}{\sqrt{5x}}\cdot\frac{\sqrt{5x}}{\sqrt{5x}}=\frac{5\sqrt{5x}}{(\sqrt{5x})^2}$$
$$=\frac{5\sqrt{5x}}{5x}=\frac{\sqrt{5x}}{x}$$

You Try It 7

Simplify: $\dfrac{y}{\sqrt{3y}}$

Your solution

Example 8

Simplify: $\dfrac{3x}{\sqrt[4]{2x}}$

Solution

$$\frac{3x}{\sqrt[4]{2x}}=\frac{3x}{\sqrt[4]{2x}}\cdot\frac{\sqrt[4]{8x^3}}{\sqrt[4]{8x^3}}$$
$$=\frac{3x\sqrt[4]{8x^3}}{\sqrt[4]{16x^4}}=\frac{3x\sqrt[4]{8x^3}}{2x}$$
$$=\frac{3\sqrt[4]{8x^3}}{2}$$

You Try It 8

Simplify: $\dfrac{3x}{\sqrt[3]{3x^2}}$

Your solution

Example 9

Simplify: $\dfrac{3}{5-2\sqrt{3}}$

Solution

$$\frac{3}{5-2\sqrt{3}}=\frac{3}{5-2\sqrt{3}}\cdot\frac{5+2\sqrt{3}}{5+2\sqrt{3}}=\frac{15+6\sqrt{3}}{5^2-(2\sqrt{3})^2}$$
$$=\frac{15+6\sqrt{3}}{25-12}=\frac{15+6\sqrt{3}}{13}$$

You Try It 9

Simplify: $\dfrac{3+\sqrt{6}}{2-\sqrt{6}}$

Your solution

Solutions on p. S28

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9.2 Exercises

Objective A

Simplify.

1. $\sqrt{x^4y^3z^5}$
2. $\sqrt{x^3y^6z^9}$
3. $\sqrt{8a^3b^8}$
4. $\sqrt{24a^9b^6}$
5. $\sqrt{45x^2y^3z^5}$
6. $\sqrt{60xy^7z^{12}}$
7. $\sqrt{-9x^3}$
8. $\sqrt{-x^2y^5}$
9. $\sqrt[3]{a^{16}b^8}$
10. $\sqrt[3]{a^5b^8}$
11. $\sqrt[3]{-125x^2y^4}$
12. $\sqrt[3]{-216x^5y^9}$
13. $\sqrt[3]{a^4b^5c^6}$
14. $\sqrt[3]{a^8b^{11}c^{15}}$
15. $\sqrt[4]{16x^9y^5}$
16. $\sqrt[4]{64x^8y^{10}}$

Objective B

Add or subtract.

17. $2\sqrt{x} - 8\sqrt{x}$
18. $3\sqrt{y} + 12\sqrt{y}$
19. $\sqrt{8} - \sqrt{32}$
20. $\sqrt{27a} - \sqrt{8a}$
21. $\sqrt{18b} + \sqrt{75b}$
22. $2\sqrt{2x^3} + 4x\sqrt{8x}$
23. $3\sqrt{8x^2y^3} - 2x\sqrt{32y^3}$
24. $2\sqrt{32x^2y^3} - xy\sqrt{98y}$
25. $2a\sqrt{27ab^5} + 3b\sqrt{3a^3b}$
26. $\sqrt[3]{128} + \sqrt[3]{250}$
27. $\sqrt[3]{16} - \sqrt[3]{54}$
28. $2\sqrt[3]{3a^4} - 3a\sqrt[3]{81a}$
29. $2b\sqrt[3]{16b^2} + \sqrt[3]{128b^5}$
30. $3\sqrt[3]{x^5y^7} - 8xy\sqrt[3]{x^2y^4}$

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31. $3\sqrt[4]{32a^5} - a\sqrt[4]{162a}$

32. $2a\sqrt[4]{16ab^5} + 3b\sqrt[4]{256a^5b}$

33. $2\sqrt{50} - 3\sqrt{125} + \sqrt{98}$

34. $3\sqrt{108} - 2\sqrt{18} - 3\sqrt{48}$

35. $\sqrt{9b^3} - \sqrt{25b^3} + \sqrt{49b^3}$

36. $\sqrt{4x^7y^5} + 9x^2\sqrt{x^3y^5} - 5xy\sqrt{x^5y^3}$

37. $2x\sqrt{8xy^2} - 3y\sqrt{32x^3} + \sqrt{4x^3y^3}$

38. $5a\sqrt{3a^3b} + 2a^2\sqrt{27ab} - 4\sqrt{75a^5b}$

39. $\sqrt[3]{54xy^3} - 5\sqrt[3]{2xy^3} + y\sqrt[3]{128x}$

40. $2\sqrt[3]{24x^3y^4} + 4x\sqrt[3]{81y^4} - 3y\sqrt[3]{24x^3y}$

41. $2a\sqrt[4]{32b^5} - 3b\sqrt[4]{162a^4b} + \sqrt[4]{2a^4b^5}$

42. $6y\sqrt[4]{48x^5} - 2x\sqrt[4]{243xy^4} - 4\sqrt[4]{3x^5y^4}$

Objective C

Multiply.

43. $\sqrt{8}\sqrt{32}$

44. $\sqrt{14}\sqrt{35}$

45. $\sqrt[3]{4}\sqrt[3]{8}$

46. $\sqrt[3]{6}\sqrt[3]{36}$

47. $\sqrt{x^2y^5}\sqrt{xy}$

48. $\sqrt{a^3b}\sqrt{ab^4}$

49. $\sqrt{2x^2y}\sqrt{32xy}$

50. $\sqrt{5x^3y}\sqrt{10x^3y^4}$

51. $\sqrt[3]{x^2y}\sqrt[3]{16x^4y^2}$

52. $\sqrt[3]{4a^2b^3}\sqrt[3]{8ab^5}$

53. $\sqrt[4]{12ab^3}\sqrt[4]{4a^5b^2}$

54. $\sqrt[4]{36a^2b^4}\sqrt[4]{12a^5b^3}$

55. $\sqrt{3}(\sqrt{27} - \sqrt{3})$

56. $\sqrt{10}(\sqrt{10} - \sqrt{5})$

57. $\sqrt{x}(\sqrt{x} - \sqrt{2})$

58. $\sqrt{y}(\sqrt{y} - \sqrt{5})$

59. $\sqrt{2x}(\sqrt{8x} - \sqrt{32})$

60. $\sqrt{3a}(\sqrt{27a^2} - \sqrt{a})$

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61. $(\sqrt{x} - 3)^2$ **62.** $(\sqrt{2x} + 4)^2$ **63.** $(4\sqrt{5} + 2)^2$

64. $2\sqrt{3x^2} \cdot 3\sqrt{12xy^3} \cdot \sqrt{6x^3y}$ **65.** $2\sqrt{14xy} \cdot 4\sqrt{7x^2y} \cdot 3\sqrt{8xy^2}$

66. $\sqrt[3]{8ab}\sqrt[3]{4a^2b^3}\sqrt[3]{9ab^4}$ **67.** $\sqrt[3]{2a^2b}\sqrt[3]{4a^3b^2}\sqrt[3]{8a^5b^6}$

68. $(\sqrt{2} - 3)(\sqrt{2} + 4)$ **69.** $(\sqrt{5} - 5)(2\sqrt{5} + 2)$

70. $(\sqrt{y} - 2)(\sqrt{y} + 2)$ **71.** $(\sqrt{x} - y)(\sqrt{x} + y)$

72. $(\sqrt{2x} - 3\sqrt{y})(\sqrt{2x} + 3\sqrt{y})$ **73.** $(2\sqrt{3x} - \sqrt{y})(2\sqrt{3x} + \sqrt{y})$

Objective D

74. When is a radical expression in simplest form?

75. Explain what it means to rationalize the denominator of a radical expression and how to do so.

Simplify.

76. $\dfrac{\sqrt{32x^2}}{\sqrt{2x}}$ **77.** $\dfrac{\sqrt{60y^4}}{\sqrt{12y}}$ **78.** $\dfrac{\sqrt{42a^3b^5}}{\sqrt{14a^2b}}$ **79.** $\dfrac{\sqrt{65ab^4}}{\sqrt{5ab}}$

80. $\dfrac{1}{\sqrt{5}}$ **81.** $\dfrac{1}{\sqrt{2}}$ **82.** $\dfrac{1}{\sqrt{2x}}$ **83.** $\dfrac{2}{\sqrt{3y}}$

84. $\dfrac{5}{\sqrt{5x}}$ **85.** $\dfrac{9}{\sqrt{3a}}$ **86.** $\sqrt{\dfrac{x}{5}}$ **87.** $\sqrt{\dfrac{y}{2}}$

88. $\dfrac{3}{\sqrt[3]{2}}$ **89.** $\dfrac{5}{\sqrt[3]{9}}$ **90.** $\dfrac{3}{\sqrt[3]{4x^2}}$ **91.** $\dfrac{5}{\sqrt[3]{3y}}$

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92. $\dfrac{\sqrt{40x^3y^2}}{\sqrt{80x^2y^3}}$ **93.** $\dfrac{\sqrt{15a^2b^5}}{\sqrt{30a^5b^3}}$ **94.** $\dfrac{\sqrt{24a^2b}}{\sqrt{18ab^4}}$ **95.** $\dfrac{\sqrt{12x^3y}}{\sqrt{20x^4y}}$

96. $\dfrac{5}{\sqrt{3}-2}$ **97.** $\dfrac{-2}{1-\sqrt{2}}$ **98.** $\dfrac{-3}{2-\sqrt{3}}$ **99.** $\dfrac{-4}{3-\sqrt{2}}$

100. $\dfrac{2}{\sqrt{5}+2}$ **101.** $\dfrac{5}{2-\sqrt{7}}$ **102.** $\dfrac{3}{\sqrt{y}-2}$ **103.** $\dfrac{-7}{\sqrt{x}-3}$

104. $\dfrac{\sqrt{2}-\sqrt{3}}{\sqrt{2}+\sqrt{3}}$ **105.** $\dfrac{\sqrt{3}+\sqrt{4}}{\sqrt{2}+\sqrt{3}}$ **106.** $\dfrac{2+3\sqrt{7}}{5-2\sqrt{7}}$

107. $\dfrac{2+3\sqrt{5}}{1-\sqrt{5}}$ **108.** $\dfrac{2\sqrt{3}-1}{3\sqrt{3}+2}$ **109.** $\dfrac{2\sqrt{a}-\sqrt{b}}{4\sqrt{a}+3\sqrt{b}}$

110. $\dfrac{2\sqrt{x}-4}{\sqrt{x}+2}$ **111.** $\dfrac{3\sqrt{y}-y}{\sqrt{y}+2y}$ **112.** $\dfrac{3\sqrt{x}-4\sqrt{y}}{3\sqrt{x}-2\sqrt{y}}$

APPLYING THE CONCEPTS

113. Determine whether the following statements are true or false. If the statement is false, correct the right side of the equation.

a. $\sqrt[2]{3}\cdot\sqrt[3]{4}=\sqrt[5]{12}$ **b.** $\sqrt{3}\cdot\sqrt{3}=3$ **c.** $\sqrt[3]{x}\cdot\sqrt[3]{x}=x$

d. $\sqrt{x}+\sqrt{y}=\sqrt{x+y}$ **e.** $\sqrt[2]{2}\cdot\sqrt[3]{3}=\sqrt[5]{2+3}$ **f.** $8\sqrt[5]{a}-2\sqrt[5]{a}=6\sqrt[5]{a}$

114. Multiply: $(\sqrt[3]{a}+\sqrt[3]{b})(\sqrt[3]{a^2}-\sqrt[3]{ab}+\sqrt[3]{b^2})$

115. Rewrite $\dfrac{\sqrt[4]{(a+b)^3}}{\sqrt{a+b}}$ as an expression with a single radical.

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9.3 Complex Numbers

Objective A **To simplify a complex number**

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The radical expression $\sqrt{-4}$ is not a real number, because there is no real number whose square is -4. However, the solution of an algebraic equation is sometimes the square root of a negative number.

For example, the equation $x^2 + 1 = 0$ does not have a real number solution, because there is no real number whose square is a negative number.

$$x^2 + 1 = 0$$
$$x^2 = -1$$

Around the 17th century, a new number, called an **imaginary number**, was defined so that a negative number would have a square root. The letter i was chosen to represent the number whose square is -1.

$$\boldsymbol{i^2 = -1}$$

An imaginary number is defined in terms of i.

Point of Interest

The first written occurrence of an imaginary number was in a book published in 1545 by Girolamo Cardan, where he wrote (in our modern notation) $5 + \sqrt{-15}$. He went on to say that the number "is as refined as it is useless." It was not until the 20th century that applications of complex numbers were found.

Definition of $\sqrt{-a}$

If a is a positive real number, then the principal square root of negative a is the imaginary number $i\sqrt{a}$.

$$\sqrt{-a} = i\sqrt{a}$$

Here are some examples.

$$\sqrt{-16} = i\sqrt{16} = 4i$$
$$\sqrt{-12} = i\sqrt{12} = 2i\sqrt{3}$$
$$\sqrt{-21} = i\sqrt{21}$$
$$\sqrt{-1} = i\sqrt{1} = i$$

It is customary to write i in front of a radical to avoid confusing $\sqrt{a}\,i$ with $\sqrt{ai}$.

The real numbers and imaginary numbers make up the complex numbers.

Complex Number

A **complex number** is a number of the form $\boldsymbol{a + bi}$, where a and b are real numbers and $i = \sqrt{-1}$. The number a is the **real part** of $a + bi$, and b is the **imaginary part**.

TAKE NOTE

The *imaginary part* of a complex number is a real number. As another example, the imaginary part of $6 - 8i$ is -8.

Examples of complex numbers are shown at the right.

Real Part	*Imaginary Part*
$a + bi$	
$3 + 2i$	
$8 - 10i$	

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Complex numbers $a + bi$
- Real Numbers $a + 0i$
- Imaginary Numbers $0 + bi$

A *real number* is a complex number in which $b = 0$.

An *imaginary number* is a complex number in which $a = 0$.

➡ Simplify: $\sqrt{20} - \sqrt{-50}$

$$\sqrt{20} - \sqrt{-50} = \sqrt{20} - i\sqrt{50}$$
$$= \sqrt{4 \cdot 5} - i\sqrt{25 \cdot 2}$$
$$= 2\sqrt{5} - 5i\sqrt{2}$$

- Write the complex number in the form $a + bi$.
- Use the Product Property of Radicals to simplify each radical.

Example 1
Simplify: $\sqrt{-80}$

Solution
$\sqrt{-80} = i\sqrt{80} = i\sqrt{16 \cdot 5} = 4i\sqrt{5}$

You Try It 1
Simplify: $\sqrt{-45}$

Your solution

Example 2
Simplify: $\sqrt{25} + \sqrt{-40}$

Solution
$$\sqrt{25} + \sqrt{-40} = \sqrt{25} + i\sqrt{40}$$
$$= \sqrt{25} + i\sqrt{4 \cdot 10}$$
$$= 5 + 2i\sqrt{10}$$

You Try It 2
Simplify: $\sqrt{98} - \sqrt{-60}$

Your solution

Solutions on p. S28

Objective V

To add or subtract complex numbers

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Addition and Subtraction of Complex Numbers

To add two complex numbers, add the real parts and add the imaginary parts. To subtract two complex numbers, subtract the real parts and subtract the imaginary parts.

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$
$$(a + bi) - (c + di) = (a - c) + (b - d)i$$

➡ Subtract: $(3 - 7i) - (4 - 2i)$

$$(3 - 7i) - (4 - 2i) = (3 - 4) + [-7 - (-2)]i$$
$$= -1 - 5i$$

- Subtract the real parts and subtract the imaginary parts of the complex numbers.

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Add: $(3 + \sqrt{-12}) + (7 - \sqrt{-27})$

$$(3 + \sqrt{-12}) + (7 - \sqrt{-27})$$
$$= (3 + i\sqrt{12}) + (7 - i\sqrt{27})$$ • Write each complex number in the form $a + bi$.
$$= (3 + i\sqrt{4 \cdot 3}) + (7 - i\sqrt{9 \cdot 3})$$ • Use the Product Property of Radicals to simplify each radical.
$$= (3 + 2i\sqrt{3}) + (7 - 3i\sqrt{3})$$
$$= 10 - i\sqrt{3}$$ • Add the complex numbers.

Example 3
Add: $(3 + 2i) + (6 - 5i)$

Solution
$(3 + 2i) + (6 - 5i) = 9 - 3i$

You Try It 3
Subtract: $(-4 + 2i) - (6 - 8i)$

Your solution

Example 4
Subtract: $(9 - \sqrt{-8}) - (5 + \sqrt{-32})$

Solution
$$(9 - \sqrt{-8}) - (5 + \sqrt{-32})$$
$$= (9 - i\sqrt{8}) - (5 + i\sqrt{32})$$
$$= (9 - i\sqrt{4 \cdot 2}) - (5 + i\sqrt{16 \cdot 2})$$
$$= (9 - 2i\sqrt{2}) - (5 + 4i\sqrt{2})$$
$$= 4 - 6i\sqrt{2}$$

You Try It 4
Subtract: $(16 - \sqrt{-45}) - (3 + \sqrt{-20})$

Your solution

Example 5
Add: $(6 + 4i) + (-6 - 4i)$

Solution
$(6 + 4i) + (-6 - 4i) = 0 + 0i = 0$

This illustrates that the additive inverse of $a + bi$ is $-a - bi$.

You Try It 5
Add: $(3 - 2i) + (-3 + 2i)$

Your solution

Solutions on p. S28

Objective C To multiply complex numbers



When multiplying complex numbers, we often find that i^2 is a part of the product. Recall that $i^2 = -1$.

Multiply: $2i \cdot 3i$

$$2i \cdot 3i = 6i^2$$ • Multiply the imaginary numbers.
$$= 6(-1)$$ • Replace i^2 by -1.
$$= -6$$ • Simplify.

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TAKE NOTE
This example illustrates an important point. When working with an expression that has a square root of a negative number, always rewrite the number as the product of a real number and i before continuing.

Multiply: $\sqrt{-6} \cdot \sqrt{-24}$

$$\sqrt{-6} \cdot \sqrt{-24} = i\sqrt{6} \cdot i\sqrt{24}$$
- Write each radical as the product of a real number and i.

$$= i^2\sqrt{144}$$
- Multiply the imaginary numbers.

$$= -\sqrt{144}$$
- Replace i^2 by -1.

$$= -12$$
- Simplify the radical expression.

Note from the last example that it would have been incorrect to multiply the radicands of the two radical expressions. To illustrate,

$$\sqrt{-6} \cdot \sqrt{-24} = \sqrt{(-6)(-24)} = \sqrt{144} = 12, \textit{ not } -12$$

Multiply: $4i(3 - 2i)$

$$4i(3 - 2i) = 12i - 8i^2$$
- Use the Distributive Property to remove parentheses.

$$= 12i - 8(-1)$$
- Replace i^2 by -1.

$$= 8 + 12i$$
- Write the answer in the form $a + bi$.

The product of two complex numbers is defined as follows.

The Product of Two Complex Numbers

$$(a + bi)(c + di) = (ac - bd) + (ad + bc)i$$

One way to remember this rule is to use the FOIL method.

Multiply: $(2 + 4i)(3 - 5i)$

$$(2 + 4i)(3 - 5i) = 6 - 10i + 12i - 20i^2$$
- Use the FOIL method to find the product.

$$= 6 + 2i - 20i^2$$

$$= 6 + 2i - 20(-1)$$
- Replace i^2 by -1.

$$= 26 + 2i$$
- Write the answer in the form $a + bi$.

The conjugate of $a + bi$ is $a - bi$.
The product of conjugates, $(a + bi)(a - bi)$, is the real number $a^2 + b^2$.

$$(a + bi)(a - bi) = a^2 - b^2i^2$$
$$= a^2 - b^2(-1)$$
$$= a^2 + b^2$$

Multiply: $(2 + 3i)(2 - 3i)$

$$(2 + 3i)(2 - 3i) = 2^2 + 3^2$$
- The product of conjugates is $a^2 + b^2$.

$$= 4 + 9$$

$$= 13$$

Note that the product of a complex number and its conjugate is a real number.

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Example 6
Multiply: $(2i)(-5i)$

Solution
$(2i)(-5i) = -10i^2 = (-10)(-1) = 10$

You Try It 6
Multiply: $(-3i)(-10i)$

Your solution

Example 7
Multiply: $\sqrt{-10} \cdot \sqrt{-5}$

Solution
$$\sqrt{-10} \cdot \sqrt{-5} = i\sqrt{10} \cdot i\sqrt{5}$$
$$= i^2\sqrt{50} = -\sqrt{25 \cdot 2} = -5\sqrt{2}$$

You Try It 7
Multiply: $-\sqrt{-8} \cdot \sqrt{-5}$

Your solution

Example 8
Multiply: $3i(2 - 4i)$

Solution:
$$3i(2 - 4i) = 6i - 12i^2 = 6i - 12(-1)$$
$$= 12 + 6i$$

You Try It 8
Multiply: $-6i(3 + 4i)$

Your solution

Example 9
Multiply: $\sqrt{-8}(\sqrt{6} - \sqrt{-2})$

Solution
$$\sqrt{-8}(\sqrt{6} - \sqrt{-2}) = i\sqrt{8}(\sqrt{6} - i\sqrt{2})$$
$$= i\sqrt{48} - i^2\sqrt{16}$$
$$= i\sqrt{16 \cdot 3} - (-1)\sqrt{16}$$
$$= 4i\sqrt{3} + 4 = 4 + 4i\sqrt{3}$$

You Try It 9
Multiply: $\sqrt{-3}(\sqrt{27} - \sqrt{-6})$

Your solution

Example 10
Multiply: $(3 - 4i)(2 + 5i)$

Solution
$$(3 - 4i)(2 + 5i) = 6 + 15i - 8i - 20i^2$$
$$= 6 + 7i - 20i^2$$
$$= 6 + 7i - 20(-1) = 26 + 7i$$

You Try It 10
Multiply: $(4 - 3i)(2 - i)$

Your solution

Example 11
Multiply: $(4 + 5i)(4 - 5i)$

Solution
$(4 + 5i)(4 - 5i) = 4^2 + 5^2 = 16 + 25 = 41$

You Try It 11
Multiply: $(3 + 6i)(3 - 6i)$

Your solution

Example 12
Multiply: $\left(\frac{9}{10} + \frac{3}{10}i\right)\left(1 - \frac{1}{3}i\right)$

Solution
$$\left(\frac{9}{10} + \frac{3}{10}i\right)\left(1 - \frac{1}{3}i\right) = \frac{9}{10} - \frac{3}{10}i + \frac{3}{10}i - \frac{1}{10}i^2$$
$$= \frac{9}{10} - \frac{1}{10}i^2 = \frac{9}{10} - \frac{1}{10}(-1)$$
$$= \frac{9}{10} + \frac{1}{10} = 1$$

You Try It 12
Multiply: $(3 - i)\left(\frac{3}{10} + \frac{1}{10}i\right)$

Your solution

Solutions on pp. S28–S29

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To divide complex numbers

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A rational expression containing one or more complex numbers is in simplest form when no imaginary number remains in the denominator.

Simplify: $\frac{2 - 3i}{2i}$

$$\frac{2 - 3i}{2i} = \frac{2 - 3i}{2i} \cdot \frac{i}{i}$$ • Multiply the expression by $\frac{i}{i}$.

$$= \frac{2i - 3i^2}{2i^2}$$

$$= \frac{2i - 3(-1)}{2(-1)}$$ • Replace i^2 by -1.

$$= \frac{3 + 2i}{-2}$$ • Simplify.

$$= -\frac{3}{2} - i$$ • Write the answer in the form $a + bi$.

Simplify: $\frac{3 + 2i}{1 + i}$

$$\frac{3 + 2i}{1 + i} = \frac{3 + 2i}{1 + i} \cdot \frac{1 - i}{1 - i}$$ • Multiply the numerator and denominator by the conjugate of $1 + i$.

$$= \frac{3 - 3i + 2i - 2i^2}{1^2 + 1^2}$$ • In $1 + i$, $a = 1$ and $b = 1$. $a^2 + b^2 = 1^2 + 1^2$.

$$= \frac{3 - i - 2(-1)}{2}$$ • Replace i^2 by -1 and simplify.

$$= \frac{5 - i}{2} = \frac{5}{2} - \frac{1}{2}i$$ • Write the answer in the form $a + bi$.

Example 13

Simplify: $\frac{5 + 4i}{3i}$

Solution

$$\frac{5 + 4i}{3i} = \frac{5 + 4i}{3i} \cdot \frac{i}{i} = \frac{5i + 4i^2}{3i^2}$$

$$= \frac{5i + 4(-1)}{3(-1)} = \frac{-4 + 5i}{-3} = \frac{4}{3} - \frac{5}{3}i$$

You Try It 13

Simplify: $\frac{2 - 3i}{4i}$

Your solution

Example 14

Simplify: $\frac{5 - 3i}{4 + 2i}$

Solution

$$\frac{5 - 3i}{4 + 2i} = \frac{5 - 3i}{4 + 2i} \cdot \frac{4 - 2i}{4 - 2i}$$

$$= \frac{20 - 10i - 12i + 6i^2}{4^2 + 2^2}$$

$$= \frac{20 - 22i + 6(-1)}{20}$$

$$= \frac{14 - 22i}{20} = \frac{14}{20} - \frac{22}{20}i = \frac{7}{10} - \frac{11}{10}i$$

You Try It 14

Simplify: $\frac{2 + 5i}{3 - 2i}$

Your solution

Solutions on p. S29

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9.3 Exercises

Objective A

1. What is an imaginary number? What is a complex number?

2. Are all real numbers also complex numbers? Are all complex numbers also real numbers?

Simplify.

3. $\sqrt{-4}$ **4.** $\sqrt{-64}$ **5.** $\sqrt{-98}$ **6.** $\sqrt{-72}$

7. $\sqrt{-27}$ **8.** $\sqrt{-75}$ **9.** $\sqrt{16} + \sqrt{-4}$ **10.** $\sqrt{25} + \sqrt{-9}$

11. $\sqrt{12} - \sqrt{-18}$ **12.** $\sqrt{60} - \sqrt{-48}$ **13.** $\sqrt{160} - \sqrt{-147}$ **14.** $\sqrt{96} - \sqrt{-125}$

Objective B

Add or subtract.

15. $(2 + 4i) + (6 - 5i)$ **16.** $(6 - 9i) + (4 + 2i)$

17. $(-2 - 4i) - (6 - 8i)$ **18.** $(3 - 5i) + (8 - 2i)$

19. $(8 - \sqrt{-4}) - (2 + \sqrt{-16})$ **20.** $(5 - \sqrt{-25}) - (11 - \sqrt{-36})$

21. $(12 - \sqrt{-50}) + (7 - \sqrt{-8})$ **22.** $(5 - \sqrt{-12}) - (9 + \sqrt{-108})$

23. $(\sqrt{8} + \sqrt{-18}) + (\sqrt{32} - \sqrt{-72})$ **24.** $(\sqrt{40} - \sqrt{-98}) - (\sqrt{90} + \sqrt{-32})$

Objective C

Multiply.

25. $(7i)(-9i)$ **26.** $(-6i)(-4i)$ **27.** $\sqrt{-2}\sqrt{-8}$

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28. $\sqrt{-5}\sqrt{-45}$ **29.** $\sqrt{-3}\sqrt{-6}$ **30.** $\sqrt{-5}\sqrt{-10}$

31. $2i(6 + 2i)$ **32.** $-3i(4 - 5i)$ **33.** $\sqrt{-2}(\sqrt{8} + \sqrt{-2})$

34. $\sqrt{-3}(\sqrt{12} - \sqrt{-6})$ **35.** $(5 - 2i)(3 + i)$ **36.** $(2 - 4i)(2 - i)$

37. $(6 + 5i)(3 + 2i)$ **38.** $(4 - 7i)(2 + 3i)$ **39.** $(1 - i)\left(\frac{1}{2} + \frac{1}{2}i\right)$

40. $\left(\frac{4}{5} - \frac{2}{5}i\right)\left(1 + \frac{1}{2}i\right)$ **41.** $\left(\frac{6}{5} + \frac{3}{5}i\right)\left(\frac{2}{3} - \frac{1}{3}i\right)$ **42.** $(2 - i)\left(\frac{2}{5} + \frac{1}{5}i\right)$

Simplify.

43. $\frac{3}{i}$ **44.** $\frac{4}{5i}$ **45.** $\frac{2 - 3i}{-4i}$ **46.** $\frac{16 + 5i}{-3i}$

47. $\frac{4}{5 + i}$ **48.** $\frac{6}{5 + 2i}$ **49.** $\frac{2}{2 - i}$ **50.** $\frac{5}{4 - i}$

51. $\frac{1 - 3i}{3 + i}$ **52.** $\frac{2 + 12i}{5 + i}$ **53.** $\frac{\sqrt{-10}}{\sqrt{8} - \sqrt{-2}}$ **54.** $\frac{\sqrt{-2}}{\sqrt{12} - \sqrt{-8}}$

55. $\frac{2 - 3i}{3 + i}$ **56.** $\frac{3 + 5i}{1 - i}$ **57.** $\frac{5 + 3i}{3 - i}$ **58.** $\frac{3 - 2i}{2i + 3}$

APPLYING THE CONCEPTS

59. **a.** Is $3i$ a solution of $2x^2 + 18 = 0$?
b. Is $3 + i$ a solution of $x^2 - 6x + 10 = 0$?

60. Evaluate i^n for $n = 0, 1, 2, 3, 4, 5, 6$, and 7. Make a conjecture about the value of i^n for any natural number. Using your conjecture, evaluate i^{76}.

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9.4 Equations Containing Radical Expressions

Objective A

To solve a radical equation









An equation that contains a variable expression in a radicand is a **radical equation**.

$\sqrt[3]{2x-5}+x=7$
$\sqrt{x+1}-\sqrt{x}=4$
} Radical Equations

The following property is used to solve a radical equation.

The Property of Raising Each Side of an Equation to a Power

If two numbers are equal, then the same powers of the numbers are equal.

If $a = b$, then $a^n = b^n$.

➡ Solve: $\sqrt{x-2}-6=0$

$\sqrt{x-2}-6=0$

$\sqrt{x-2}=6$ • Isolate the radical by adding 6 to each side of the equation.

$(\sqrt{x-2})^2=6^2$ • Square each side of the equation.

$x-2=36$ • Simplify and solve for x.

$x=38$

Check:

$\sqrt{x-2}-6=0$

$\sqrt{38-2}-6$	0
$\sqrt{36}-6$	0
$6-6$	0
$0=0$	

38 checks as a solution. The solution is 38.

➡ Solve: $\sqrt[3]{x+2}=-3$

$\sqrt[3]{x+2}=-3$

$(\sqrt[3]{x+2})^3=(-3)^3$ • Cube each side of the equation.

$x+2=-27$ • Solve the resulting equation.

$x=-29$

Check:

$\sqrt[3]{x+2}=-3$

$\sqrt[3]{-29+2}$	-3
$\sqrt[3]{-27}$	-3
$-3=-3$	

-29 checks as a solution. The solution is -29.

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Raising each side of an equation to an even power may result in an equation that has a solution that is not a solution of the original equation. This is called an **extraneous solution**. Here is an example:

➡ Solve: $\sqrt{2x-1}+\sqrt{x}=2$

$$\sqrt{2x-1}+\sqrt{x}=2$$

$$\sqrt{2x-1}=2-\sqrt{x}$$ • Solve for one of the radical expressions.

$$(\sqrt{2x-1})^2=(2-\sqrt{x})^2$$ • Square each side. Recall that $(a-b)^2=a^2-2ab+b^2$.

$$2x-1=4-4\sqrt{x}+x$$

$$x-5=-4\sqrt{x}$$

$$(x-5)^2=(-4\sqrt{x})^2$$ • Square each side.

$$x^2-10x+25=16x$$

$$x^2-26x+25=0$$ • Solve the quadratic equation by factoring.

$$(x-25)(x-1)=0$$

$$x=25 \quad \text{or} \quad x=1$$

Check:

$$\begin{array}{r|l} \sqrt{2x-1}+\sqrt{x}=2 & \\ \hline \sqrt{2(25)-1}+\sqrt{25} & 2 \\ 7+5 & 2 \\ 12\neq 2 & \end{array} \qquad \begin{array}{r|l} \sqrt{2x-1}+\sqrt{x}=2 & \\ \hline \sqrt{2(1)-1}+\sqrt{1} & 2 \\ 1+1 & 2 \\ 2=2 & \end{array}$$

25 does not check as a solution. 1 checks as a solution. The solution is 1.

TAKE NOTE

Note that

$(2-\sqrt{x})^2=(2-\sqrt{x})(2-\sqrt{x})=4-4\sqrt{x}+x$

TAKE NOTE

You must always check the proposed solutions to radical equations. The proposed solutions of the equation on the right were 1 and 25. However, 25 did not check as a solution. Here 25 is an extraneous solution.

Example 1

Solve: $\sqrt[3]{3x-1}=-4$

Solution

$$\sqrt[3]{3x-1}=-4$$

$$(\sqrt[3]{3x-1})^3=(-4)^3$$

$$3x-1=-64$$

$$3x=-63$$

$$x=-21$$

Check:

$$\begin{array}{r|l} \sqrt[3]{3x-1}=-4 & \\ \hline \sqrt[3]{3(-21)-1} & -4 \\ \sqrt[3]{-64} & -4 \\ -4=-4 & \end{array}$$

The solution is -21.

You Try It 1

Solve: $\sqrt[4]{x-8}=3$

Your solution

Example 2

Solve: $\sqrt{x-1}+\sqrt{x+4}=5$

Solution

$$\sqrt{x-1}+\sqrt{x+4}=5$$

$$\sqrt{x+4}=5-\sqrt{x-1}$$

$$(\sqrt{x+4})^2=(5-\sqrt{x-1})^2$$

$$x+4=25-10\sqrt{x-1}+x-1$$

$$2=\sqrt{x-1}$$

$$2^2=(\sqrt{x-1})^2$$

$$4=x-1$$

$$5=x$$

5 checks as a solution. The solution is 5.

You Try It 2

Solve: $\sqrt{x}-\sqrt{x+5}=1$

Your solution

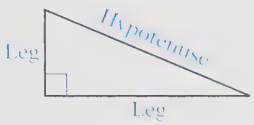
Solutions on p. S29

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Objective B To solve application problems

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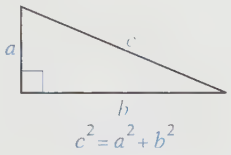
A right triangle contains one 90° angle. The side opposite the 90° angle is called the **hypotenuse**. The other two sides are called **legs**.



Point of Interest

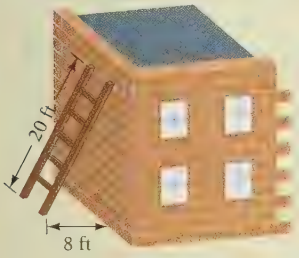
The first known proof of this theorem occurs in a Chinese text, *Arithmetic Classic*, which was first written around 600 B.C.

Pythagoras, a Greek mathematician, discovered that the square of the hypotenuse of a right triangle is equal to the sum of the squares of the two legs. This is called the **Pythagorean Theorem**.



Example 3

A ladder 20 ft long is leaning against a building. How high on the building will the ladder reach when the bottom of the ladder is 8 ft from the building? Round to the nearest tenth.



Strategy

To find the distance, use the Pythagorean Theorem. The hypotenuse is the length of the ladder. One leg is the distance from the bottom of the ladder to the base of the building. The distance along the building from the ground to the top of the ladder is the unknown leg.

Solution

$$\begin{aligned} c^2 &= a^2 + b^2 \\ 20^2 &= 8^2 + b^2 \\ 400 &= 64 + b^2 \\ 336 &= b^2 \\ (336)^{1/2} &= (b^2)^{1/2} \\ \sqrt{336} &= b \\ 18.3 &\approx b \end{aligned}$$

The distance is 18.3 ft.

You Try It 3

Find the diagonal of a rectangle that is 6 cm long and 3 cm wide. Round to the nearest tenth.

Your strategy

Your solution

Solution on p. S29

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Example 4

An object is dropped from a high building. Find the distance the object has fallen when its speed reaches 96 ft/s. Use the equation $v = \sqrt{64d}$, where v is the speed of the object in feet per second and d is the distance in feet.

Strategy

To find the distance the object has fallen, replace v in the equation with the given value and solve for d.

Solution

$$v = \sqrt{64d}$$
$$96 = \sqrt{64d}$$
$$(96)^2 = (\sqrt{64d})^2$$
$$9216 = 64d$$
$$144 = d$$

The object has fallen 144 ft.

You Try It 4

How far would a submarine periscope have to be above the water for the lookout to locate a ship 5.5 mi away? The equation for the distance in miles that the lookout can see is $d = \sqrt{1.5h}$, where h is the height in feet above the surface of the water. Round to the nearest hundredth.

Your strategy

Your solution

Example 5

Find the length of a pendulum that makes one swing in 1.5 s. The equation for the time of one swing is given by $T = 2\pi\sqrt{\frac{L}{32}}$, where T is the time in seconds and L is the length in feet. Round to the nearest hundredth.

Strategy

To find the length of the pendulum, replace T in the equation with the given value and solve for L.

Solution

$$T = 2\pi\sqrt{\frac{L}{32}}$$
$$1.5 = 2\pi\sqrt{\frac{L}{32}}$$
$$\frac{1.5}{2\pi} = \sqrt{\frac{L}{32}}$$ • Divide each side by 2π.
$$\left(\frac{1.5}{2\pi}\right)^2 = \frac{L}{32}$$ • Square each side.
$$32\left(\frac{1.5}{2\pi}\right)^2 = L$$ • Multiply each side by 32.
$$1.82 \approx L$$ • Use the π key on your calculator.

The length of the pendulum is 1.82 ft.

You Try It 5

Find the distance required for a car to reach a velocity of 88 ft/s when the acceleration is 22 ft/s^2. Use the equation $v = \sqrt{2as}$, where v is the velocity in feet per second, a is the acceleration, and s is the distance in feet.

Your strategy

Your solution

Solutions on p. S29

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9.4 Exercises

Objective A

Solve.

1. $\sqrt{x} = 5$
2. $\sqrt{y} = 2$
3. $\sqrt[3]{a} = 3$
4. $\sqrt[3]{y} = 5$
5. $\sqrt{3x} = 12$
6. $\sqrt{5x} = 10$
7. $\sqrt[3]{4x} = -2$
8. $\sqrt[3]{6x} = -3$
9. $\sqrt{2x} = -4$
10. $\sqrt{5x} = -5$
11. $\sqrt{3x - 2} = 5$
12. $\sqrt{5x - 4} = 9$
13. $\sqrt{3 - 2x} = 7$
14. $\sqrt{9 - 4x} = 4$
15. $7 = \sqrt{1 - 3x}$
16. $6 = \sqrt{8 - 7x}$
17. $\sqrt[3]{4x - 1} = 2$
18. $\sqrt[3]{5x + 2} = 3$
19. $\sqrt[3]{1 - 2x} = -3$
20. $\sqrt[3]{3 - 2x} = -2$
21. $\sqrt[3]{9x + 1} = 4$
22. $\sqrt{3x + 9} - 12 = 0$
23. $\sqrt{4x - 3} - 5 = 0$
24. $\sqrt{x - 2} = 4$
25. $\sqrt[3]{x - 3} + 5 = 0$
26. $\sqrt[3]{x - 2} = 3$
27. $\sqrt[3]{2x - 6} = 4$
28. $\sqrt{x^2 - 8x} = 3$
29. $\sqrt{x^2 + 7x + 11} = 1$
30. $\sqrt[4]{4x + 1} = 2$
31. $\sqrt[4]{2x - 9} = 3$
32. $\sqrt{2x - 3} - 2 = 1$
33. $\sqrt{3x - 5} - 5 = 3$

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34. $\sqrt[3]{2x-3}+5=2$ **35.** $\sqrt[3]{x-4}+7=5$ **36.** $\sqrt{5x-16}+1=4$

37. $\sqrt{3x-5}-2=3$ **38.** $\sqrt{2x-1}-8=-5$ **39.** $\sqrt{7x+2}-10=-7$

40. $\sqrt[3]{4x-3}-2=3$ **41.** $\sqrt[3]{1-3x}+5=3$ **42.** $1-\sqrt{4x+3}=-5$

43. $7-\sqrt{3x+1}=-1$ **44.** $\sqrt{x+1}=2-\sqrt{x}$ **45.** $\sqrt{2x+4}=3-\sqrt{2x}$

46. $\sqrt{x^2+3x-2}-x=1$ **47.** $\sqrt{x^2-4x-1}+3=x$ **48.** $\sqrt{x^2-3x-1}=3$

49. $\sqrt{x^2-2x+1}=3$ **50.** $\sqrt{2x+5}-\sqrt{3x-2}=1$ **51.** $\sqrt{4x+1}-\sqrt{2x+4}=1$

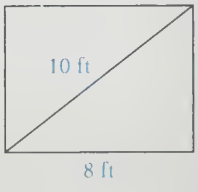
52. $\sqrt{5x-1}-\sqrt{3x-2}=1$ **53.** $\sqrt{5x+4}-\sqrt{3x+1}=1$ **54.** $\sqrt[3]{x^2+2}-3=0$

55. $\sqrt[3]{x^2+4}-2=0$ **56.** $\sqrt[4]{x^2+2x+8}-2=0$ **57.** $\sqrt[4]{x^2+x-1}-1=0$

58. $4\sqrt{x+1}-x=1$ **59.** $3\sqrt{x-2}+2=x$ **60.** $x+3\sqrt{x-2}=12$

Application Problems

61. Find the width of a rectangle that has a diagonal of 10 ft and a length of 8 ft.

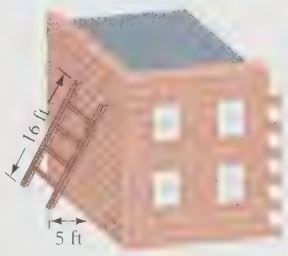


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62. Find the length of a rectangle that has a diagonal of 15 m and a width of 9 m.

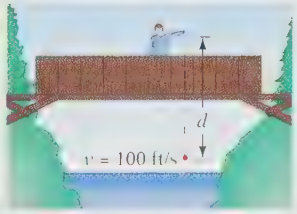
63. A 26-foot ladder is leaning against a building. How far is the bottom of the ladder from the building when the ladder reaches a height of 24 ft on the building?

64. A 16-foot ladder is leaning against a building. How high on the building will the ladder reach when the bottom of the ladder is 5 ft from the building? Round to the nearest tenth.



65. An object is dropped from an airplane. Find the distance the object has fallen when its speed reaches 400 ft/s. Use the equation $v = \sqrt{64d}$, where v is the speed of the object and d is the distance.

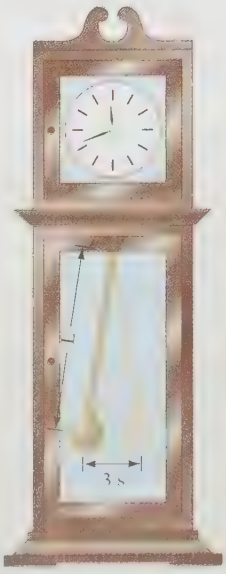
66. An object is dropped from a bridge. Find the distance the object has fallen when its speed reaches 100 ft/s. Use the equation $v = \sqrt{64d}$, where v is the speed of the object and d is the distance.



67. How far would a submarine periscope have to be above the water for the lookout to locate a ship 3.6 mi away? The equation for the distance in miles that the lookout can see is $d = \sqrt{1.5h}$, where h is the height in feet above the surface of the water.

68. How far would a submarine periscope have to be above the water for the lookout to locate a ship 4.2 mi away? The equation for the distance in miles that the lookout can see is $d = \sqrt{1.5h}$, where h is the height in feet above the surface of the water.

69. Find the length of a pendulum on a clock that makes one swing in 2.4 s. The equation for the time of one swing of a pendulum is given by $T = 2\pi\sqrt{\frac{L}{32}}$, where T is the time in seconds and L is the length in feet. Round to the nearest hundredth.



70. Find the length of a pendulum that makes one swing in 3 s. The equation for the time of one swing of a pendulum is given by $T = 2\pi\sqrt{\frac{L}{32}}$, where T is the time in seconds and L is the length in feet. Round to the nearest hundredth.

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71. Find the distance required for a car to reach a velocity of 60 m/s when the acceleration is 10 m/s^2. Use the equation $v = \sqrt{2as}$, where v is the velocity, a is the acceleration, and s is the distance.

72. The time it takes for an object to fall a certain distance is given by the equation $t = \sqrt{\frac{2d}{g}}$, where t is the time in seconds, d is the distance in feet, and g is the acceleration due to gravity. The acceleration due to gravity on Earth is 32 feet per second. If an object is dropped from the top of a tall building, how far will it fall in 6 s?

73. High definition television (HDTV) gives consumers a wider viewing area, more like a film in a theater. A regular television with a 27-inch diagonal measurement has a screen 16.2 in. tall. An HDTV screen with the same 16.2-inch height would have a diagonal measuring 33 in. How many inches wider is the HDTV screen? Round to the nearest hundredth.



74. At what height above Earth's surface would a satellite be in orbit if it is traveling at a speed of 7500 m/s? Use the equation $v = \sqrt{\frac{4 \times 10^{14}}{h + 6.4 \times 10^6}}$, where v is the speed of the satellite in meters per second and h is the height above Earth's surface in meters. Round to the nearest thousand.

APPLYING THE CONCEPTS

75. Solve the following equations. Describe the solution by using the following terms: integer, rational number, irrational number, real number, and imaginary number. Note that more than one term may be used to describe the answer.

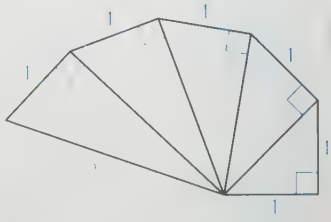
a. $x^2 + 3 = 7$ **b.** $x^2 + 1 = 0$ **c.** $x^{3/4} = 8$

76. Solve: $\sqrt{3x - 2} = \sqrt{2x - 3} + \sqrt{x - 1}$

77. Solve $a^2 + b^2 = c^2$ for a.

78. Solve $V = \frac{4}{3}\pi r^3$ for r.

79. Find the length of the side labeled x in the diagram at the right.



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Focus on Problem Solving

Another Look at Polya's Four-Step Process

Polya's four general steps to follow when attempting to solve a problem are to understand the problem, devise a plan, carry out the plan, and review the solution. (See the Focus on Problem Solving in the chapter entitled "Factoring.") In the process of devising a plan (Step 2), it may be appropriate to write a mathematical expression or an equation. We will illustrate this with the following problem.

Number the letters of the alphabet in sequence from 1 to 26. (See the list at the left.) Find a word for which the product of the numerical values of the letters of the word equals 1,000,000. We will agree that a "word" is any sequence of letters that contains at least one vowel; it need not be in the dictionary.

A = 1
B = 2
C = 3
D = 4
E = 5
F = 6
G = 7
H = 8
I = 9
J = 10
K = 11
L = 12
M = 13
N = 14
O = 15
P = 16
Q = 17
R = 18
S = 19
T = 20
U = 21
V = 22
W = 23
X = 24
Y = 25
Z = 26

1. Understand the Problem

Consider REZB. The product of the values of the letters is $18 \cdot 5 \cdot 26 \cdot 2 = 4680$. This "word" is a sequence of letters with at least one vowel. However, the product of the numerical values of the letters is not 1,000,000. Thus this word does not solve our problem.

2. Devise a Plan

Actually, we should have known that the product of the values of the letters in REZB could not equal 1,000,000. The letter R has a factor of 9, and the letter Z has a factor of 13. Neither of these two numbers is a factor of 1,000,000. Consequently, R and Z cannot be letters in the word we are trying to find. This observation leads to an important observation: Each of the letters that make up our word must be a factor of 1,000,000. To find these letters, consider the prime factorization of 1,000,000.

$$1{,}000{,}000 = 2^6 \cdot 5^6$$

Looking at the prime factorization, we note that only letters that contain 2 or 5 as factors are possible candidates. These letters are B, D, E, H, J, P, T, and Y. One additional point: Because 1 times any number is the number, the letter A can be part of any word we construct.

Our task is now to construct a word from these letters such that the product is 1,000,000. From the prime factorization above, we must have 2 as a factor six times and 5 as a factor six times.

3. Carry Out the Plan

We must construct a word with the characteristics described in our plan. Here is a possibility:

THEBEYE

4. Review Your Solution

You should multiply the values of all the letters and verify that the product is 1,000,000. To ensure that you have an understanding of the problem, try to find other "words" that satisfy the conditions of the problem.

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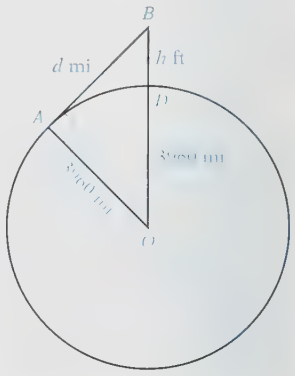
Projects and Group Activities

Distance to the Horizon

In Section 9.4, we used the formula $d = \sqrt{1.5h}$ to calculate the approximate distance d (in miles) that a person using a periscope h feet above the water could see. That formula is derived by using the Pythagorean Theorem.

Consider the diagram (not to scale) at the right, which shows Earth as a sphere and the periscope extending h feet above the surface. From geometry, because AB is tangent to the circle and OA is a radius, triangle AOB is a right triangle. Therefore,

$$(OA)^2 + (AB)^2 = (OB)^2$$



Substituting into this formula, we have

$$3960^2 + d^2 = \left(3960 + \frac{h}{5280}\right)^2$$

• Because h is in feet, $\frac{h}{5280}$ is in miles.

$$3960^2 + d^2 = 3960^2 + \frac{2 \cdot 3960}{5280}h + \left(\frac{h}{5280}\right)^2$$

$$d^2 = \frac{3}{2}h + \left(\frac{h}{5280}\right)^2$$

$$d = \sqrt{\frac{3}{2}h + \left(\frac{h}{5280}\right)^2}$$

At this point, an assumption is made that $\sqrt{\frac{3}{2}h + \left(\frac{h}{5280}\right)^2} \approx \sqrt{1.5h}$, where we have written $\frac{3}{2}$ as 1.5. Thus $d \approx \sqrt{1.5h}$ is used to approximate the distance that can be seen using a periscope h feet above the water.

1. Write a paragraph that justifies the assumption that

$$\sqrt{\frac{3}{2}h + \left(\frac{h}{5280}\right)^2} \approx \sqrt{1.5h}$$

(*Suggestion:* Evaluate each expression for various values of h. Because h is the height of a periscope above the water, it is unlikely that $h > 25$ ft.)

2. The distance d is the distance from the top of the periscope to A. The distance along the surface of the water is given by arc AD. This distance, L, can be approximated by the equation

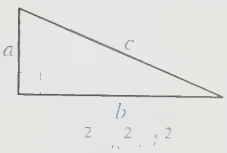
$$L \approx \sqrt{1.5h} + 0.306186\left(\sqrt{\frac{h}{5280}}\right)^3$$

Using this formula, calculate L when $h = 10$.

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Pythagorean Triples

The Pythagorean Theorem states that if a and b are the legs of a right triangle and c is the length of the hypotenuse, then $c^2 = a^2 + b^2$.



For instance, the triangle with legs 3 and 4 and hypotenuse 5 is a right triangle because $3^2 + 4^2 = 5^2$. The numbers 3, 4, and 5 are called a **Pythagorean triple** because they are natural numbers that satisfy the equation of the Pythagorean Theorem.

1. Determine whether the numbers are a Pythagorean triple.
- **a.** 5, 7, and 9
- **b.** 8, 15, and 17
- **c.** 11, 60, and 61
- **d.** 28, 45, and 53

Mathematicians have investigated Pythagorean triples and have found formulas that will generate these triples. One such formula is

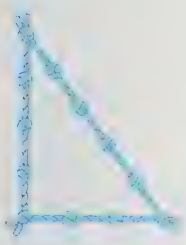
$$a = m^2 - n^2 \qquad b = 2mn \qquad c = m^2 + n^2, \text{ where } m > n$$

For instance, let $m = 2$ and $n = 1$. Then $a = 2^2 - 1^2 = 3$, $b = 2(2)(1) = 4$, and $c = 2^2 + 1^2 = 5$. This is the Pythagorean triple given above.

2. Find the Pythagorean triple produced by each of the following.
- **a.** $m = 3$ and $n = 1$
- **b.** $m = 5$ and $n = 2$
- **c.** $m = 4$ and $n = 2$
- **d.** $m = 6$ and $n = 1$

3. Find values of m and n that yield the Pythagorean triple 11, 60, 61.

4. Verify that $a^2 + b^2 = c^2$ when $a = m^2 - n^2$, $b = 2mn$, and $c = m^2 + n^2$.



5. The early Greek builders used a rope with 12 equally spaced knots to make right-angle corners for buildings. Explain how they used the rope.

6. Find three odd integers, a, b, c, such that $a^2 + b^2 = c^2$.

Chapter Summary

Key Words

The *nth root of a* is $a^{1/n}$. The expression $\sqrt[n]{a}$ is another symbol for the nth root of a. [pp. 497, 499]

In the expression $\sqrt[n]{a}$, the symbol $\sqrt{\ }$ is called a *radical sign*, n is the *index*, and a is the *radicand*. [p. 499]

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The symbol $\sqrt{\ }$ is used to indicate the positive or *principal square root* of a number. [p. 501]

The expressions $a + b$ and $a - b$ are called *conjugates* of each other. The product of conjugates of the form $(a + b)(a - b)$ is $a^2 - b^2$. [p. 510]

The procedure used to remove a radical from the denominator of a radical expression is called *rationalizing the denominator.* [p. 511]

A *complex number* is a number of the form $a + bi$, where a and b are real numbers and $i = \sqrt{-1}$. For the complex number $a + bi$, a is the *real part* of the complex number, and b is the *imaginary part* of the complex number. [p. 517]

A *radical equation* is an equation that contains a variable expression in a radicand. [p. 525]

Essential Rules

Rule for Rational Exponents [p. 497]

If m and n are positive integers and $a^{1/n}$ is a real number, then $a^{m/n} = (a^{1/n})^m$.

Rule for Writing Exponential Expressions as Radical Expressions [p. 499]

If $a^{1/n}$ is a real number, then $a^{m/n} = \sqrt[n]{a^m}$.

The Product Property of Radicals [p. 507]

If a and b are positive real numbers, then $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$.

The Quotient Property of Radicals [p. 511]

If a and b are positive real numbers, then $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$.

Definition of $\sqrt{-a}$ [p. 517]

If a is a positive real number, then the principal square root of negative a is the imaginary number $i\sqrt{a}$: $\sqrt{-a} = i\sqrt{a}$.

Addition of Complex Numbers [p. 518]

If $a + bi$ and $c + di$ are complex numbers, then
$(a + bi) + (c + di) = (a + c) + (b + d)i$.

Subtraction of Complex Numbers [p. 518]

If $a + bi$ and $c + di$ are complex numbers, then
$(a + bi) - (c + di) = (a - c) + (b - d)i$.

The Property of Raising Each Side of an Equation to a Power [p. 525]

If a and b are real numbers and $a = b$, then $a^n = b^n$.

The Pythagorean Theorem [p. 527]

The square of the hypotenuse of a right triangle is equal to the sum of the squares of the two legs.
$c^2 = a^2 + b^2$

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Chapter Review

1. Simplify: $(16x^{-4}y^{12})^{1/4}(100x^6y^{-2})^{1/2}$

2. Solve: $\sqrt[4]{3x-5}=2$

3. Multiply: $(6-5i)(4+3i)$

4. Rewrite $7y\sqrt[3]{x^2}$ as an exponential expression.

5. Multiply: $(\sqrt{3}+8)(\sqrt{3}-2)$

6. Solve: $\sqrt{4x+9}+10=11$

7. Simplify: $\dfrac{x^{-3/2}}{x^{7/2}}$

8. Simplify: $\dfrac{8}{\sqrt{3y}}$

9. Simplify: $\sqrt[3]{-8a^6b^{12}}$

10. Subtract: $\sqrt{50a^4b^3}-ab\sqrt{18a^2b}$

11. Simplify: $\dfrac{x+2}{\sqrt{x}+\sqrt{2}}$

12. Simplify: $\dfrac{5+2i}{3i}$

13. Simplify: $\sqrt{18a^3b^6}$

14. Subtract: $(\sqrt{50}+\sqrt{-72})-(\sqrt{162}-\sqrt{-8})$

15. Subtract: $3x\sqrt[3]{54x^8y^{10}}-2x^2y\sqrt[3]{16x^5y^7}$

16. Multiply: $\sqrt[3]{16x^4y}\,\sqrt[3]{4xy^5}$

17. Multiply: $i(3-7i)$

18. Rewrite $3x^{3/4}$ as a radical expression.

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19. Simplify: $\sqrt[5]{-64a^8b^{12}}$

20. Simplify: $\frac{5 + 9i}{1 - i}$

21. Multiply: $\sqrt{-12}\sqrt{-6}$

22. Solve: $\sqrt{x - 5} + \sqrt{x + 6} = 11$

23. Simplify: $\sqrt[4]{81a^8b^{12}}$

24. Simplify: $\sqrt{-50}$

25. Subtract: $(-8 + 3i) - (4 - 7i)$

26. Simplify: $(5 - \sqrt{6})^2$

27. Simplify: $4x\sqrt{12x^2y} + \sqrt{3x^4y} - x^2\sqrt{27y}$

28. The velocity of the wind determines the amount of power generated by a windmill. A typical equation for this relationship is $v = 4.05\sqrt[3]{P}$, where v is the velocity in miles per hour and P is the power in watts. Find the amount of power generated by a 20-mph wind. Round to the nearest whole number.

29. Find the distance required for a car to reach a velocity of 88 ft/s when the acceleration is 16 ft/s^2. Use the equation $v = \sqrt{2as}$, where v is the velocity in feet per second, a is the acceleration, and s is the distance in feet.

30. A 12-foot ladder is leaning against a building. How far from the building is the bottom of the ladder when the top of the ladder touches the building 10 ft above the ground? Round to the nearest hundredth.

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Chapter Test

1. Write $\frac{1}{2}\sqrt[4]{x^3}$ as an exponential expression.

2. Subtract: $\sqrt[3]{54x^7y^3} - x\sqrt[3]{128x^4y^3} - x^2\sqrt[3]{2xy^3}$

3. Write $3y^{2/5}$ as a radical expression.

4. Multiply: $(2 + 5i)(4 - 2i)$

5. Simplify: $(2\sqrt{x} + \sqrt{y})^2$

6. Simplify: $\frac{r^{2/3}r^{-1}}{r^{-1/2}}$

7. Solve: $\sqrt{x + 12} - \sqrt{x} = 2$

8. Simplify: $\sqrt[3]{8x^3y^6}$

9. Multiply: $\sqrt{3x}(\sqrt{x} - \sqrt{25x})$

10. Subtract: $(5 - 2i) - (8 - 4i)$

11. Simplify: $\sqrt{32x^4y^7}$

12. Multiply: $(2\sqrt{3} + 4)(3\sqrt{3} - 1)$

13. Add: $(2 + i) + (2 - i)$

14. Simplify: $\frac{4 - 2\sqrt{5}}{2 - \sqrt{5}}$

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15. Add: $\sqrt{18a^3} + a\sqrt{50a}$

16. Multiply: $(\sqrt{a} - 3\sqrt{b})(2\sqrt{a} + 5\sqrt{b})$

17. Simplify: $\dfrac{(2x^{1/3}y^{-2/3})^6}{(x^{-4}y^8)^{1/4}}$

18. Simplify: $\dfrac{\sqrt{x}}{\sqrt{x} - \sqrt{y}}$

19. Simplify: $\dfrac{2 + 3i}{1 - 2i}$

20. Solve: $\sqrt[3]{2x - 2} + 4 = 2$

21. Simplify: $\left(\dfrac{4a^4}{b^2}\right)^{-3/2}$

22. Simplify: $\sqrt[3]{27a^4b^3c^7}$

23. Simplify: $\dfrac{\sqrt{32x^5y}}{\sqrt{2xy^3}}$

24. Multiply: $(\sqrt{-8})(\sqrt{-2})$

25. An object is dropped from a high building. Find the distance the object has fallen when its speed reaches 192 ft/s. Use the equation $v = \sqrt{64d}$, where v is the speed of the object in feet per second and d is the distance in feet.

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Cumulative Review

1. Simplify: $2^3 \cdot 3 - 4(3 - 4 \cdot 5)$

2. Evaluate $4a^2b - a^3$ when $a = -2$ and $b = 3$.

3. Simplify: $-3(4x - 1) - 2(1 - x)$

4. Solve: $5 - \frac{2}{3}x = 4$

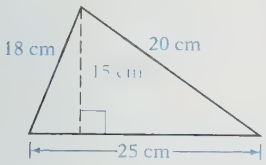
5. Solve: $2[4 - 2(3 - 2x)] = 4(1 - x)$

6. Solve: $6x - 3(2x + 2) > 3 - 3(x + 2)$

7. Solve: $2 + |4 - 3x| = 5$

8. Solve: $|2x + 3| \leq 9$

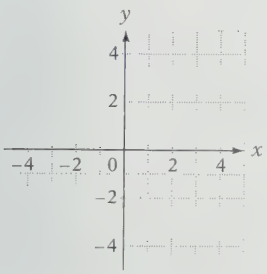
9. Find the area of the triangle shown in the figure below.



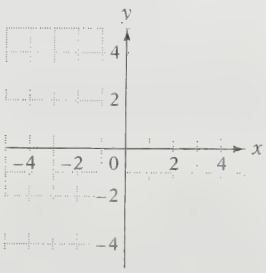
10. Find the value of the determinant:

$$\begin{vmatrix} 1 & 2 & -3 \\ 0 & -1 & 2 \\ 3 & 1 & -2 \end{vmatrix}$$

11. Graph $3x - 2y = -6$. State the slope and y-intercept.



12. Graph the solution set of $3x + 2y \leq 4$.



13. Find the equation of the line that passes through the points (2, 3) and (−1, 2).

14. Solve by using Cramer's Rule:

$$2x - y = 4$$
$$-2x + 3y = 5$$

15. Simplify: $(2^{-1}x^2y^{-6})(2^{-1}y^{-4})^{-2}$

16. Factor: $81x^2 - y^2$

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17. Factor: $x^5 + 2x^3 - 3x$

18. Solve $P = \dfrac{R - C}{n}$ for C.

19. Simplify: $\left(\dfrac{x^{-2/3}y^{1/2}}{y^{-1/3}}\right)^6$

20. Subtract: $\sqrt{40x^3} - x\sqrt{90x}$

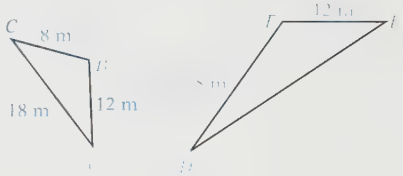
21. Multiply: $(\sqrt{3} - 2)(\sqrt{3} - 5)$

22. Simplify: $\dfrac{4}{\sqrt{6} - \sqrt{2}}$

23. Simplify: $\dfrac{2i}{3 - i}$

24. Solve: $\sqrt[3]{3x - 4} + 5 = 1$

25. The two triangles are similar triangles. Find the length of side DE.

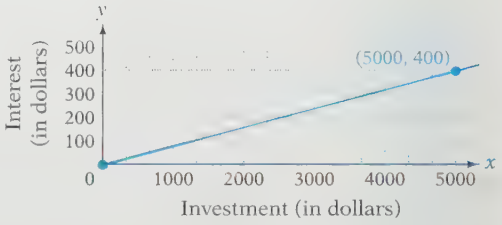


26. An investment of \$2500 is made at an annual simple interest rate of 7.2%. How much additional money must be invested at an annual simple interest rate of 8.4% so that the total interest earned is \$516?

27. A sales executive traveled 25 mi by car and then an additional 625 mi by plane. The rate of the plane was five times the rate of the car. The total time of the trip was 3 h. Find the rate of the plane.

28. How long does it take light to travel to Earth from the moon when the moon is 232,500 mi from Earth? Light travels 1.86×10^5 mi/s.

29. The graph shows the amount invested and the annual interest earned on the investment. Find the slope of the line between the two points shown on the graph. Then write a sentence that states the meaning of the slope.



30. How far would a submarine periscope have to be above the water for the lookout to locate a ship 7 mi away? The equation for the distance in miles that the lookout can see is $d = \sqrt{1.5h}$, where h is the height in feet above the surface of the water. Round to the nearest tenth of a foot.

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Chapter 10 Quadratic Equations



A rainstorm like the one shown here directly affects highway driving conditions. How quickly a car is able to stop once the brakes are applied is determined by many factors, including how wet the road is, the type of pavement, and the speed of the car. Some factors, such as the tread on a car's tires, can be controlled, and good tread can minimize the adverse effects of braking on dangerously wet roads. The braking distance of a car can be modeled by quadratic equations such as the ones in **Exercises 5 and 9 on page 573.**

Objectives

Section 10.1

A To solve a quadratic equation by factoring
B To write a quadratic equation given its solutions
C To solve a quadratic equation by taking square roots

Section 10.2

A To solve a quadratic equation by completing the square

Section 10.3

A To solve a quadratic equation by using the quadratic formula

Section 10.4

A To solve an equation that is quadratic in form
B To solve a radical equation that is reducible to a quadratic equation
C To solve a fractional equation that is reducible to a quadratic equation

Section 10.5

A To solve application problems

Section 10.6

A To solve a nonlinear inequality

WEB Need help? For on-line student resources, such as section quizzes, visit this textbook's web site at **college.hmco.com/students.**

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Prep Test

1. Simplify: $\sqrt{18}$

2. Simplify: $\sqrt{-9}$

3. Simplify: $\dfrac{3x-2}{x-1}-1$

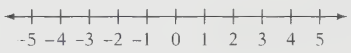
4. Evaluate b^2-4ac when $a=2$, $b=-4$, and $c=1$.

5. Is $4x^2+28x+49$ a perfect-square trinomial?

6. Factor: $4x^2-4x+1$

7. Factor: $9x^2-4$

8. Graph: $\{x|x<-1\}\cap\{x|x<4\}$



9. Solve: $x(x-1)=x+15$

10. Solve: $\dfrac{4}{x-3}=\dfrac{16}{x}$

Go Figure

The numeral $0.AAA\ldots$ is a repeating decimal. Given that $A\neq B$ and $\sqrt{0.AAA\ldots}=\sqrt{B}$, find the value of A.

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10.1 Solving Quadratic Equations by Factoring or by Taking Square Roots

Objective A

To solve a quadratic equation by factoring

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Recall that a **quadratic equation** is an equation of the form $ax^2 + bx + c = 0$, where a and b are coefficients, c is a constant, and $a \neq 0$.

Quadratic equations
$3x^2 - x + 2 = 0$, $a = 3$, $b = -1$, $c = 2$
$-x^2 + 4 = 0$, $a = -1$, $b = 0$, $c = 4$
$6x^2 - 5x = 0$, $a = 6$, $b = -5$, $c = 0$

A quadratic equation is in **standard form** when the polynomial is in descending order and equal to zero. Because the degree of the polynomial $ax^2 + bx + c$ is 2, a quadratic equation is also called a **second-degree equation**.

As we discussed earlier, quadratic equations sometimes can be solved by using the Principle of Zero Products. This method is reviewed here.

The Principle of Zero Products

If a and b are real numbers and $ab = 0$, then $a = 0$ or $b = 0$.

The Principle of Zero Products states that if the product of two factors is zero, then at least one of the factors must be zero.

TAKE NOTE

Recall that the steps involved in solving a quadratic equation by factoring are

1. Write the equation in standard form.
2. Factor.
3. Use the Principle of Zero Products to set each factor equal to 0.
4. Solve each equation.
5. Check the solutions.

➡ Solve by factoring: $3x^2 = 2 - 5x$

$$3x^2 = 2 - 5x$$
$$3x^2 + 5x - 2 = 0$$ • Write the equation in standard form.
$$(3x - 1)(x + 2) = 0$$ • Factor.
$$3x - 1 = 0 \qquad x + 2 = 0$$ • Use the Principle of Zero Products to write two equations.
$$3x = 1 \qquad x = -2$$ • Solve each equation.
$$x = \frac{1}{3}$$

$\frac{1}{3}$ and -2 check as solutions. The solutions are $\frac{1}{3}$ and -2.

TAKE NOTE

When a quadratic equation has two solutions that are the same number, the solution is called a **double root** of the equation. 3 is a double root of $x^2 - 6x = -9$.

➡ Solve by factoring: $x^2 - 6x = -9$

$$x^2 - 6x = -9$$
$$x^2 - 6x + 9 = 0$$ • Write the equation in standard form.
$$(x - 3)(x - 3) = 0$$ • Factor.
$$x - 3 = 0 \qquad x - 3 = 0$$ • Use the Principle of Zero Products.
$$x = 3 \qquad x = 3$$ • Solve each equation.

3 checks as a solution. The solution is 3.

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Example 1

Solve by factoring: $2x(x - 3) = x + 4$

Solution

$$2x(x - 3) = x + 4$$
$$2x^2 - 6x = x + 4$$
$$2x^2 - 7x - 4 = 0$$
$$(2x + 1)(x - 4) = 0$$
$$2x + 1 = 0 \qquad x - 4 = 0$$
$$2x = -1 \qquad x = 4$$
$$x = -\frac{1}{2}$$

The solutions are $-\frac{1}{2}$ and 4.

You Try It 1

Solve by factoring: $2x^2 = 7x - 3$

Your solution

Example 2

Solve for x by factoring: $x^2 - 4ax - 5a^2 = 0$

Solution

This is a literal equation. Solve for x in terms of a.

$$x^2 - 4ax - 5a^2 = 0$$
$$(x + a)(x - 5a) = 0$$
$$x + a = 0 \qquad x - 5a = 0$$
$$x = -a \qquad x = 5a$$

The solutions are $-a$ and $5a$.

You Try It 2

Solve for x by factoring: $x^2 - 3ax - 4a^2 = 0$

Your solution

Solutions on p. S30

To write a quadratic equation given its solutions

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As shown below, the solutions of the equation $(x - r_1)(x - r_2) = 0$ are r_1 and r_2.

$$(x - r_1)(x - r_2) = 0$$
$$x - r_1 = 0 \qquad x - r_2 = 0$$
$$x = r_1 \qquad x = r_2$$

Check:

$$\frac{(x - r_1)(x - r_2) = 0}{(r_1 - r_1)(r_1 - r_2) \mid 0}$$
$$0 \cdot (r_1 - r_2) \mid 0$$
$$0 = 0$$

$$\frac{(x - r_1)(x - r_2) = 0}{(r_2 - r_1)(r_2 - r_2) \mid 0}$$
$$(r_2 - r_1) \cdot 0 \mid 0$$
$$0 = 0$$

Using the equation $(x - r_1)(x - r_2) = 0$ and the fact that r_1 and r_2 are solutions of this equation, it is possible to write a quadratic equation given its solutions.

➪ Write a quadratic equation that has solutions 4 and -5.

$$(x - r_1)(x - r_2) = 0$$
$$(x - 4)[x - (-5)] = 0 \quad \text{• Replace } r_1 \text{ by 4 and } r_2 \text{ by } -5.$$
$$(x - 4)(x + 5) = 0 \quad \text{• Simplify.}$$
$$x^2 + x - 20 = 0 \quad \text{• Multiply.}$$

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➡ Write a quadratic equation with integer coefficients and solutions $\frac{2}{3}$ and $\frac{1}{2}$.

$$(x - r_1)(x - r_2) = 0$$
$$\left(x - \frac{2}{3}\right)\left(x - \frac{1}{2}\right) = 0$$ • Replace r_1 by $\frac{2}{3}$ and r_2 by $\frac{1}{2}$.
$$x^2 - \frac{7}{6}x + \frac{1}{3} = 0$$ • Multiply.
$$\therefore \left(x^2 - \frac{7}{6}x + \frac{1}{3}\right) = \quad \cdot 0$$ • Multiply each side of the equation by the LCM of the denominators.

Example 3

Write a quadratic equation with integer coefficients and solutions $\frac{1}{2}$ and -4.

Solution

$$(x - r_1)(x - r_2) = 0$$
$$\left(x - \frac{1}{2}\right)[x - (-4)] = 0$$
$$\left(x - \frac{1}{2}\right)(x + 4) = 0$$
$$x^2 + \frac{7}{2}x - 2 = 0$$
$$2\left(x^2 + \frac{7}{2}x - 2\right) = 2 \cdot 0$$
$$2x^2 + 7x - 4 = 0$$

You Try It 3

Write a quadratic equation with integer coefficients and solutions 3 and $-\frac{1}{2}$.

Your solution

Solution on p. S30

To solve a quadratic equation by taking square roots

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The solution of the quadratic equation $x^2 = 16$ is shown at the right.

$$x^2 = 16$$
$$x^2 - 16 = 0$$
$$(x - 4)(x + 4) = 0$$
$$x - 4 = 0 \qquad x + 4 = 0$$
$$x = 4 \qquad x = -4$$

The solutions can also be found by taking the square root of each side of the equation and writing the positive and negative square roots of the number. The notation $x = \pm 4$ means $x = 4$ or $x = -4$.

$$x^2 = 16$$
$$\sqrt{x^2} = \sqrt{16}$$
$$x = \pm 4$$

The solutions are 4 and -4.

➡ Solve by taking square roots: $3x^2 = 54$

$$3x^2 = 54$$
$$x^2 = 18$$ • Solve for x^2.
$$\sqrt{x^2} = \sqrt{18}$$ • Take the square root of each side of the equation.
$$x = \pm\sqrt{18} = \pm 3\sqrt{2}$$ • Simplify.

The solutions are $3\sqrt{2}$ and $-3\sqrt{2}$. • $3\sqrt{2}$ and $-3\sqrt{2}$ check as solutions.

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Solving a quadratic equation by taking the square root of each side of the equation can lead to solutions that are complex numbers.

➡ Solve by taking square roots: $2x^2 + 18 = 0$

$$2x^2 + 18 = 0$$
$$2x^2 = -18 \quad \text{• Solve for } x^2.$$
$$x^2 = -9$$
$$\sqrt{x^2} = \sqrt{-9} \quad \text{• Take the square root of each side of the equation.}$$
$$x = \pm\sqrt{-9} \quad \text{• Simplify.}$$
$$x = \pm 3i$$

Check:

$2x^2 + 18 = 0$	
$2(3i)^2 + 18$	0
$2(-9) + 18$	0
$-18 + 18$	0
$0 = 0$	

$2x^2 + 18 = 0$	
$2(-3i)^2 + 18$	0
$2(-9) + 18$	0
$-18 + 18$	0
$0 = 0$	

The solutions are $3i$ and $-3i$.

An equation containing the square of a binomial can be solved by taking square roots.

➡ Solve by taking square roots: $(x + 2)^2 - 24 = 0$

$$(x + 2)^2 - 24 = 0$$
$$(x + 2)^2 = 24 \quad \text{• Solve for } (x + 2)^2.$$
$$\sqrt{(x + 2)^2} = \sqrt{24} \quad \text{• Take the square root of each side of the equation. Then simplify.}$$
$$x + 2 = \pm\sqrt{24}$$
$$x + 2 = \pm 2\sqrt{6}$$
$$x + 2 = 2\sqrt{6} \qquad x + 2 = -2\sqrt{6} \quad \text{• Solve for } x.$$
$$x = -2 + 2\sqrt{6} \qquad x = -2 - 2\sqrt{6}$$

The solutions are $-2 + 2\sqrt{6}$ and $-2 - 2\sqrt{6}$.

Example 4

Solve by taking square roots:
$3(x - 2)^2 + 12 = 0$

Solution

$$3(x - 2)^2 + 12 = 0$$
$$3(x - 2)^2 = -12$$
$$(x - 2)^2 = -4$$
$$\sqrt{(x - 2)^2} = \sqrt{-4}$$
$$x - 2 = \pm\sqrt{-4}$$
$$x - 2 = \pm 2i$$
$$x - 2 = 2i \qquad x - 2 = -2i$$
$$x = 2 + 2i \qquad x = 2 - 2i$$

The solutions are $2 + 2i$ and $2 - 2i$.

You Try It 4

Solve by taking square roots:
$2(x + 1)^2 - 24 = 0$

Your solution

Solution on p. S30

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10.1 Exercises

Objective 1

1. Explain why the restriction $a \neq 0$ is necessary in the definition of a quadratic equation.

2. What does the Principle of Zero Products state? How is it used to solve a quadratic equation?

Write the quadratic equation in standard form with the coefficient of x^2 positive. Give the values of a, b, and c.

3. $2x^2 - 4x = 5$

4. $x^2 = 3x + 1$

5. $5x = 4x^2 + 6$

6. $3x^2 = 7$

Solve by factoring.

7. $x^2 - 4x = 0$

8. $y^2 + 6y = 0$

9. $t^2 - 25 = 0$

10. $p^2 - 81 = 0$

11. $s^2 - s - 6 = 0$

12. $v^2 + 4v - 5 = 0$

13. $y^2 - 6y + 9 = 0$

14. $x^2 + 10x + 25 = 0$

15. $9z^2 - 18z = 0$

16. $4y^2 + 20y = 0$

17. $r^2 - 3r = 10$

18. $p^2 + 5p = 6$

19. $v^2 + 10 = 7v$

20. $t^2 - 16 = 15t$

21. $2x^2 - 9x - 18 = 0$

22. $3y^2 - 4y - 4 = 0$

23. $4z^2 - 9z + 2 = 0$

24. $2s^2 - 9s + 9 = 0$

25. $3w^2 + 11w = 4$

26. $2r^2 + r = 6$

27. $6x^2 = 23x + 18$

28. $6x^2 = 7x - 2$

29. $4 - 15u - 4u^2 = 0$

30. $3 - 2y - 8y^2 = 0$

31. $x + 18 = x(x - 6)$

32. $t + 24 = t(t + 6)$

33. $4s(s + 3) = s - 6$

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34. $3v(v - 2) = 11v + 6$

35. $u^2 - 2u + 4 = (2u - 3)(u + 2)$

36. $(3v - 2)(2v + 1) = 3v^2 - 11v - 10$

37. $(3x - 4)(x + 4) = x^2 - 3x - 28$

Solve for x by factoring.

38. $x^2 + 14ax + 48a^2 = 0$

39. $x^2 - 9bx + 14b^2 = 0$

40. $x^2 + 9xy - 36y^2 = 0$

41. $x^2 - 6cx - 7c^2 = 0$

42. $x^2 - ax - 20a^2 = 0$

43. $2x^2 + 3bx + b^2 = 0$

44. $3x^2 - 4cx + c^2 = 0$

45. $3x^2 - 14ax + 8a^2 = 0$

46. $3x^2 - 11xy + 6y^2 = 0$

47. $3x^2 - 8ax - 3a^2 = 0$

48. $3x^2 - 4bx - 4b^2 = 0$

49. $4x^2 + 8xy + 3y^2 = 0$

50. $6x^2 - 11cx + 3c^2 = 0$

51. $6x^2 + 11ax + 4a^2 = 0$

52. $12x^2 - 5xy - 2y^2 = 0$

Write a quadratic equation that has integer coefficients and has as solutions the given pair of numbers.

53. 2 and 5

54. 3 and 1

55. -2 and -4

56. -1 and -3

57. 6 and -1

58. -2 and 5

59. 3 and -3

60. 5 and -5

61. 4 and 4

62. 2 and 2

63. 0 and 5

64. 0 and -2

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65. 0 and 3

66. 0 and -1

67. 3 and $\frac{1}{2}$

68. 2 and $\frac{2}{3}$

69. $-\frac{3}{4}$ and 2

70. $-\frac{1}{2}$ and 5

71. $-\frac{5}{3}$ and -2

72. $-\frac{3}{2}$ and -1

73. $-\frac{2}{3}$ and $\frac{2}{3}$

74. $-\frac{1}{2}$ and $\frac{1}{2}$

75. $\frac{1}{2}$ and $\frac{1}{3}$

76. $\frac{3}{4}$ and $\frac{2}{3}$

77. $\frac{6}{5}$ and $-\frac{1}{2}$

78. $\frac{3}{4}$ and $-\frac{3}{2}$

79. $-\frac{1}{4}$ and $-\frac{1}{2}$

80. $-\frac{5}{6}$ and $-\frac{2}{3}$

81. $\frac{3}{5}$ and $-\frac{1}{10}$

82. $\frac{7}{2}$ and $-\frac{1}{4}$

Objective C

Solve by taking square roots.

83. $y^2 = 49$

84. $x^2 = 64$

85. $z^2 = -4$

86. $v^2 = -16$

87. $s^2 - 4 = 0$

88. $r^2 - 36 = 0$

89. $4x^2 - 81 = 0$

90. $9x^2 - 16 = 0$

91. $y^2 + 49 = 0$

92. $z^2 + 16 = 0$

93. $v^2 - 48 = 0$

94. $s^2 - 32 = 0$

95. $r^2 - 75 = 0$

96. $u^2 - 54 = 0$

97. $z^2 + 18 = 0$

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98. $t^2 + 27 = 0$ **99.** $(x - 1)^2 = 36$ **100.** $(x + 2)^2 = 25$

101. $3(y + 3)^2 = 27$ **102.** $4(s - 2)^2 = 36$ **103.** $5(z + 2)^2 = 125$

104. $2(y - 3)^2 = 18$ **105.** $\left(v - \frac{1}{2}\right)^2 = \frac{1}{4}$ **106.** $\left(r + \frac{2}{3}\right)^2 = \frac{1}{9}$

107. $(x + 5)^2 - 6 = 0$ **108.** $(t - 1)^2 - 15 = 0$

109. $(v - 3)^2 + 45 = 0$ **110.** $(x + 5)^2 + 32 = 0$

111. $\left(u - \frac{2}{3}\right)^2 - 18 = 0$ **112.** $\left(z - \frac{1}{2}\right)^2 - 20 = 0$

APPLYING THE CONCEPTS

Write a quadratic equation that has as solutions the given pair of numbers.

113. $\sqrt{2}$ and $-\sqrt{2}$ **114.** $2i$ and $-2i$ **115.** $3\sqrt{2}$ and $-3\sqrt{2}$ **116.** $2i\sqrt{3}$ and $-2i\sqrt{3}$

Solve for x.

117. $4a^2x^2 = 36b^2, a > 0, b > 0$ **118.** $(x + a)^2 - 4 = 0$ **119.** $(2x - 1)^2 = (2x + 3)^2$

120. Show that the solutions of the equation $ax^2 + bx = 0, a > 0, b > 0$, are 0 and $-\frac{b}{a}$.

121. Show that the solutions of the equation $ax^2 + c = 0, a > 0, c > 0$, are $\frac{\sqrt{ca}}{a}i$ and $-\frac{\sqrt{ca}}{a}i$.

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10.2 Solving Quadratic Equations by Completing the Square

Objective A

To solve a quadratic equation by completing the square

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Recall that a perfect-square trinomial is the square of a binomial.

Perfect-Square Trinomial		*Square of a Binomial*
$x^2 + 8x + 16$	=	$(x + 4)^2$
$x^2 - 10x + 25$	=	$(x - 5)^2$
$x^2 + 2ax + a^2$	=	$(x + a)^2$

For each perfect-square trinomial, the square of $\frac{1}{2}$ of the coefficient of x equals the constant term.

$$\left(\frac{1}{2}\text{ coefficient of } x\right)^2 = \text{constant term}$$

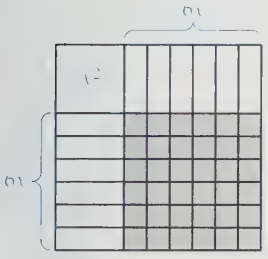
$x^2 + 8x + 16, \quad \left(\frac{1}{2} \cdot 8\right)^2 = 16$

$x^2 - 10x + 25, \quad \left[\frac{1}{2}(-10)\right]^2 = 25$

$x^2 + 2ax + a^2, \quad \left(\frac{1}{2} \cdot 2a\right)^2 = a^2$

Point of Interest

Early attempts to solve quadratic equations were primarily geometric. The Persian mathematician al-Khowarizmi (c. A.D. 800) essentially completed a square of $x^2 + 12x$ as follows.



This relationship can be used to write the constant term for a perfect-square trinomial. Adding to a binomial the constant term that makes it a perfect-square trinomial is called **completing the square**.

➡ Complete the square on $x^2 + 12x$. Write the resulting perfect-square trinomial as the square of a binomial.

$\left[\frac{1}{2}(12)\right]^2 = (6)^2 = 36$ • Find the constant term.

$x^2 + 12x + 36$ • Complete the square on $x^2 + 12x$ by adding the constant term.

$x^2 + 12x + 36 = (x + 6)^2$ • Write the resulting perfect-square trinomial as the square of a binomial.

➡ Complete the square on $z^2 - 3z$. Write the resulting perfect-square trinomial as the square of a binomial.

$\left[\frac{1}{2} \cdot (-3)\right]^2 = \left(-\frac{3}{2}\right)^2 = \frac{9}{4}$ • Find the constant term.

$z^2 - 3z + \frac{9}{4}$ • Complete the square on $z^2 - 3z$ by adding the constant term.

$z^2 - 3z + \frac{9}{4} = \left(z - \frac{3}{2}\right)^2$ • Write the resulting perfect-square trinomial as the square of a binomial.

Any quadratic equation can be solved by completing the square. Add to each side of the equation the term that completes the square. Rewrite the equation in the form $(x + a)^2 = b$. Then take the square root of each side of the equation.

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Solve by completing the square: $x^2 - 6x - 15 = 0$

$$x^2 - 6x - 15 = 0$$

$$x^2 - 6x = 15$$ • Add 15 to each side of the equation.

$$x^2 - 6x + 9 = 15 + 9$$ • Complete the square. Add $\left[\frac{1}{2}(-6)\right]^2 = (-3)^2 = 9$ to each side of the equation.

$$(x - 3)^2 = 24$$ • Factor the perfect-square trinomial.

$$\sqrt{(x - 3)^2} = \sqrt{24}$$ • Take the square root of each side of the equation.

$$x - 3 = \pm\sqrt{24}$$ • Solve for x.

$$x - 3 = \pm 2\sqrt{6}$$

$$x - 3 = 2\sqrt{6} \qquad x - 3 = -2\sqrt{6}$$

$$x = 3 + 2\sqrt{6} \qquad x = 3 - 2\sqrt{6}$$

Check:

$$\begin{array}{r|l} x^2 - 6x - 15 = 0 & \\ \hline (3 + 2\sqrt{6})^2 - 6(3 + 2\sqrt{6}) - 15 & 0 \\ 9 + 12\sqrt{6} + 24 - 18 - 12\sqrt{6} - 15 & 0 \\ 0 = 0 & \end{array}$$

$$\begin{array}{r|l} x^2 - 6x - 15 = 0 & \\ \hline (3 - 2\sqrt{6})^2 - 6(3 - 2\sqrt{6}) - 15 & 0 \\ 9 - 12\sqrt{6} + 24 - 18 + 12\sqrt{6} - 15 & 0 \\ 0 = 0 & \end{array}$$

The solutions are $3 + 2\sqrt{6}$ and $3 - 2\sqrt{6}$.

TAKE NOTE
The exact solutions of the equation $x^2 - 6x - 15 = 0$ are $3 + 2\sqrt{6}$ and $3 - 2\sqrt{6}$. 7.899 and −1.899 are approximate solutions of the equation.

In the example above, the solutions of the equation $x^2 - 6x - 15 = 0$ are $3 + 2\sqrt{6}$ and $3 - 2\sqrt{6}$. These are the exact solutions. However, in some situations it may be preferable to have decimal approximations of the solutions to a quadratic equation. Approximate solutions can be found by using a calculator and then rounding to the desired degree of accuracy.

$$3 + 2\sqrt{6} \approx 7.899 \quad \text{and} \quad 3 - 2\sqrt{6} \approx -1.899$$

To the nearest thousandth, the solutions of the equation $x^2 - 6x - 15 = 0$ are 7.899 and −1.899.

Solve $2x^2 - x - 2 = 0$ by completing the square. Find the exact solutions and approximate the solutions to the nearest thousandth.

In order for us to complete the square on an expression, the coefficient of the squared term must be 1. After adding the opposite of the constant term to each side of the equation, multiply each side of the equation by $\frac{1}{2}$.

$$2x^2 - x - 2 = 0$$

$$2x^2 - x = 2$$ • Add 2 to each side of the equation.

$$\frac{1}{2}(2x^2 - x) = \frac{1}{2} \cdot 2$$ • Multiply each side of the equation by $\frac{1}{2}$.

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TAKE NOTE

This example illustrates all the steps required to solve a quadratic equation by completing the square.

1. Write the equation in the form $ax^2 + bx = -c$.
2. Multiply both sides of the equation by $\frac{1}{a}$.
3. Complete the square on $x^2 + \frac{b}{a}x$. Add the number that completes the square to both sides of the equation.
4. Factor the perfect-square trinomial.
5. Take the square root of each side of the equation.
6. Solve the resulting equation for x.
7. Check the solutions.

$$x^2 - \frac{1}{2}x = 1$$

- The coefficient of x^2 is now 1.

$$x^2 - \frac{1}{2}x + \frac{1}{16} = 1 + \frac{1}{16}$$

- Complete the square. Add $\left[\frac{1}{2}\left(-\frac{1}{2}\right)\right]^2 = \left(-\frac{1}{4}\right)^2 = \frac{1}{16}$ to each side of the equation.

$$\left(x - \frac{1}{4}\right)^2 = \frac{17}{16}$$

- Factor the perfect-square trinomial.

$$\sqrt{\left(x - \frac{1}{4}\right)^2} = \sqrt{\frac{17}{16}}$$

- Take the square root of each side of the equation.

$$x - \frac{1}{4} = \pm\frac{\sqrt{17}}{4}$$

- Solve for x.

$$x - \frac{1}{4} = \frac{\sqrt{17}}{4} \qquad x - \frac{1}{4} = -\frac{\sqrt{17}}{4}$$

$$x = \frac{1}{4} + \frac{\sqrt{17}}{4} \qquad x = \frac{1}{4} - \frac{\sqrt{17}}{4}$$

The exact solutions are $\frac{1 + \sqrt{17}}{4}$ and $\frac{1 - \sqrt{17}}{4}$.

$$\frac{1 + \sqrt{17}}{4} \approx 1.281 \qquad \frac{1 - \sqrt{17}}{4} \approx -0.781$$

To the nearest thousandth, the solutions are 1.281 and −0.781.

Point of Interest

Mathematicians have studied quadratic equations for centuries. Many of the initial equations were a result of trying to solve a geometry problem. One of the most famous, which dates from around 500 B.C., is "squaring the circle." The question was "Is it possible to construct a square whose area is that of a given circle?" For these early mathematicians, to *construct* meant to draw with only a straightedge and a compass. It was approximately 2300 years later that mathematicians were able to prove that such a construction is impossible.

Solve by completing the square: $x^2 - 3x + 5 = 0$

$$x^2 - 3x + 5 = 0$$

$$x^2 - 3x = -5$$

- Add the opposite of the constant term to each side of the equation.

$$x^2 - 3x + \frac{9}{4} = -5 + \frac{9}{4}$$

- Add to each side of the equation the term that completes the square on $x^2 - 3x$.

$$\left(x - \frac{3}{2}\right)^2 = -\frac{11}{4}$$

- Factor the perfect-square trinomial.

$$\sqrt{\left(x - \frac{3}{2}\right)^2} = \sqrt{-\frac{11}{4}}$$

- Take the square root of each side of the equation.

$$x - \frac{3}{2} = \pm\frac{i\sqrt{11}}{2}$$

- Simplify.

$$x - \frac{3}{2} = \frac{i\sqrt{11}}{2} \qquad x - \frac{3}{2} = -\frac{i\sqrt{11}}{2}$$

- Solve for x.

$$x = \frac{3}{2} + \frac{i\sqrt{11}}{2} \qquad x = \frac{3}{2} - \frac{i\sqrt{11}}{2}$$

$\frac{3}{2} + \frac{\sqrt{11}}{2}i$ and $\frac{3}{2} - \frac{\sqrt{11}}{2}i$ check as solutions.

The solutions are $\frac{3}{2} + \frac{\sqrt{11}}{2}i$ and $\frac{3}{2} - \frac{\sqrt{11}}{2}i$

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Example 1

Solve by completing the square:

$4x^2 - 8x + 1 = 0$

Solution

$4x^2 - 8x + 1 = 0$

$4x^2 - 8x = -1$

$\frac{1}{4}(4x^2 - 8x) = \frac{1}{4}(-1)$ • The coefficient of x^2 must be 1.

$x^2 - 2x = -\frac{1}{4}$

$x^2 - 2x + 1 = -\frac{1}{4} + 1$ • Complete the square.

$(x - 1)^2 = \frac{3}{4}$

$\sqrt{(x - 1)^2} = \sqrt{\frac{3}{4}}$

$x - 1 = \pm\frac{\sqrt{3}}{2}$

$x - 1 = \frac{\sqrt{3}}{2}$ $\qquad$ $x - 1 = -\frac{\sqrt{3}}{2}$

$x = 1 + \frac{\sqrt{3}}{2}$ $\qquad$ $x = 1 - \frac{\sqrt{3}}{2}$

$= \frac{2 + \sqrt{3}}{2}$ $\qquad$ $= \frac{2 - \sqrt{3}}{2}$

The solutions are $\frac{2 + \sqrt{3}}{2}$ and $\frac{2 - \sqrt{3}}{2}$.

You Try It 1

Solve by completing the square:

$4x^2 - 4x - 1 = 0$

Your solution

Example 2

Solve by completing the square:

$x^2 + 4x + 5 = 0$

Solution

$x^2 + 4x + 5 = 0$

$x^2 + 4x = -5$

$x^2 + 4x + 4 = -5 + 4$ • Complete the square.

$(x + 2)^2 = -1$

$\sqrt{(x + 2)^2} = \sqrt{-1}$

$x + 2 = \pm i$

$x + 2 = i$ $\qquad$ $x + 2 = -i$

$x = -2 + i$ $\qquad$ $x = -2 - i$

The solutions are $-2 + i$ and $-2 - i$.

You Try It 2

Solve by completing the square:

$2x^2 + x - 5 = 0$

Your solution

Solutions on p. S30

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10.2 Exercises

Objective A

1. What is the meaning of the phrase *complete the square*?

2. Write out the steps for solving a quadratic equation by completing the square.

Solve by completing the square.

3. $x^2 - 4x - 5 = 0$

4. $y^2 + 6y + 5 = 0$

5. $v^2 + 8v - 9 = 0$

6. $w^2 - 2w - 24 = 0$

7. $z^2 - 6z + 9 = 0$

8. $u^2 + 10u + 25 = 0$

9. $r^2 + 4r - 7 = 0$

10. $s^2 + 6s - 1 = 0$

11. $x^2 - 6x + 7 = 0$

12. $y^2 + 8y + 13 = 0$

13. $z^2 - 2z + 2 = 0$

14. $t^2 - 4t + 8 = 0$

15. $s^2 - 5s - 24 = 0$

16. $v^2 + 7v - 44 = 0$

17. $x^2 + 5x - 36 = 0$

18. $y^2 - 9y + 20 = 0$

19. $p^2 - 3p + 1 = 0$

20. $r^2 - 5r - 2 = 0$

21. $t^2 - t - 1 = 0$

22. $u^2 - u - 7 = 0$

23. $y^2 - 6y = 4$

24. $w^2 + 4w = 2$

25. $x^2 = 8x - 15$

26. $z^2 = 4z - 3$

27. $v^2 = 4v - 13$

28. $x^2 = 2x - 17$

29. $p^2 + 6p = -13$

30. $x^2 + 4x = -20$

31. $y^2 - 2y = 17$

32. $x^2 + 10x = 7$

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33. $z^2 = z + 4$

34. $r^2 = 3r - 1$

35. $x^2 + 13 = 2x$

36. $6v^2 - 7v = 3$

37. $4x^2 - 4x + 5 = 0$

38. $4t^2 - 4t + 17 = 0$

39. $9x^2 - 6x + 2 = 0$

40. $9y^2 - 12y + 13 = 0$

41. $2s^2 = 4s + 5$

42. $3u^2 = 6u + 1$

43. $2r^2 = 3 - r$

44. $2x^2 = 12 - 5x$

45. $y - 2 = (y - 3)(y + 2)$

46. $8s - 11 = (s - 4)(s - 2)$

47. $6t - 2 = (2t - 3)(t - 1)$

48. $2z + 9 = (2z + 3)(z + 2)$

49. $(x - 4)(x + 1) = x - 3$

50. $(y - 3)^2 = 2y + 10$

Solve by completing the square. Approximate the solutions to the nearest thousandth.

51. $z^2 + 2z = 4$

52. $t^2 - 4t = 7$

53. $2x^2 = 4x - 1$

54. $3y^2 = 5y - 1$

55. $4z^2 + 2z = 1$

56. $4w^2 - 8w = 3$

APPLYING THE CONCEPTS

Solve for x by completing the square.

57. $x^2 - ax - 2a^2 = 0$

58. $x^2 + 3ax - 4a^2 = 0$

59. $x^2 + 3ax - 10a^2 = 0$

60. When a baseball is hit, the height h, in feet, of the ball above the ground t seconds after it is hit can be approximated by the equation $h = -16t^2 + 70t + 4$. The distance s, in feet, that the ball is from home plate t seconds after it is hit can be approximated by the equation $s = 44.5t$.

a. Determine when the ball will hit the ground. Round to the nearest hundredth. (*Hint*: The ball hits the ground when $h = 0$.)

b. Determine whether the ball will clear a 6-foot fence 325 ft from home plate.

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10.3 Solving Quadratic Equations by Using the Quadratic Formula

Objective A

To solve a quadratic equation by using the quadratic formula



A general formula known as the **quadratic formula** can be derived by applying the method of completing the square to the standard form of a quadratic equation. This formula can be used to solve any quadratic equation. The equation $ax^2 + bx + c = 0$ is solved by completing the square as follows.

Add the opposite of the constant term to each side of the equation.

$$ax^2 + bx + c = 0$$
$$ax^2 + bx + c + (-c) = 0 + (-c)$$

Multiply each side of the equation by the reciprocal of a, the coefficient of x^2.

$$ax^2 + bx = -c$$
$$\frac{1}{a}(ax^2 + bx) = \frac{1}{a}(-c)$$
$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

Complete the square by adding $\left(\frac{1}{2} \cdot \frac{b}{a}\right)^2 = \frac{b^2}{4a^2}$ to each side of the equation.

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$$

Simplify the right side of the equation.

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \left(\frac{c}{a} \cdot \frac{4a}{4a}\right)$$
$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{4ac}{4a^2}$$
$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2 - 4ac}{4a^2}$$

Factor the perfect-square trinomial on the left side of the equation.

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Take the square root of each side of the equation.

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2 - 4ac}{4a^2}}$$
$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

Solve for x.

$$x + \frac{b}{2a} = \frac{\sqrt{b^2 - 4ac}}{2a} \qquad x + \frac{b}{2a} = -\frac{\sqrt{b^2 - 4ac}}{2a}$$
$$x = -\frac{b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a} \qquad x = -\frac{b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-b + \sqrt{b^2 - 4ac}}{2a} \qquad = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Point of Interest

Although mathematicians have studied quadratic equations since around 500 B.C., it was not until the 18th century that the formula was written as it is today. Of further note, the word *quadratic* has the same Latin root as does the word *square*.

The Quadratic Formula

The solutions of $ax^2 + bx + c = 0$, $a \neq 0$, are

$$\frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

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The quadratic formula is frequently written as $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

➡ Solve by using the quadratic formula: $2x^2 + 5x + 3 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(5) \pm \sqrt{(5)^2 - 4(2)(3)}}{2(2)}$$
$$= \frac{-5 \pm \sqrt{25 - 24}}{4}$$
$$= \frac{-5 \pm \sqrt{1}}{4} = \frac{-5 \pm 1}{4}$$
$$x = \frac{-5 + 1}{4} = \frac{-4}{4} = -1 \qquad x = \frac{-5 - 1}{4} = \frac{-6}{4} = -\frac{3}{2}$$

The solutions are -1 and $-\frac{3}{2}$.

- The equation $2x^2 + 5x + 3 = 0$ is in standard form. $a = 2$, $b = 5$, $c = 3$ Replace a, b, and c in the quadratic formula with these values.

TAKE NOTE
The solutions of this quadratic equation are rational numbers. When this happens, the equation could have been solved by factoring and using the Principle of Zero Products. This may be easier than applying the quadratic formula.

➡ Solve $3x^2 = 4x + 6$ by using the quadratic formula. Find the exact solutions and approximate the solutions to the nearest thousandth.

$$3x^2 = 4x + 6$$
$$3x^2 - 4x - 6 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-6)}}{2(3)}$$
$$= \frac{4 \pm \sqrt{16 - (-72)}}{6}$$
$$= \frac{4 \pm \sqrt{88}}{6} = \frac{4 \pm 2\sqrt{22}}{6}$$
$$= \frac{2(2 \pm \sqrt{22})}{2 \cdot 3} = \frac{2 \pm \sqrt{22}}{3}$$

- Write the equation in standard form. Subtract $4x$ and 6 from each side of the equation. $a = 3$, $b = -4$, $c = -6$ Replace a, b, and c in the quadratic formula with these values.

Check:

$3x^2 = 4x + 6$

$3\left(\frac{2 + \sqrt{22}}{3}\right)^2 \mid 4\left(\frac{2 + \sqrt{22}}{3}\right) + 6$

$3\left(\frac{4 + 4\sqrt{22} + 22}{9}\right) \mid \frac{8}{3} + \frac{4\sqrt{22}}{3} + \frac{18}{3}$

$3\left(\frac{26 + 4\sqrt{22}}{9}\right) \mid \frac{26}{3} + \frac{4\sqrt{22}}{3}$

$\frac{26 + 4\sqrt{22}}{3} = \frac{26 + 4\sqrt{22}}{3}$

$3x^2 = 4x + 6$

$3\left(\frac{2 - \sqrt{22}}{3}\right)^2 \mid 4\left(\frac{2 - \sqrt{22}}{3}\right) + 6$

$3\left(\frac{4 - 4\sqrt{22} + 22}{9}\right) \mid \frac{8}{3} - \frac{4\sqrt{22}}{3} + \frac{18}{3}$

$3\left(\frac{26 - 4\sqrt{22}}{9}\right) \mid \frac{26}{3} - \frac{4\sqrt{22}}{3}$

$\frac{26 - 4\sqrt{22}}{3} = \frac{26 - 4\sqrt{22}}{3}$

The solutions are $\frac{2 + \sqrt{22}}{3}$ and $\frac{2 - \sqrt{22}}{3}$.

$$\frac{2 + \sqrt{22}}{3} \approx 2.230 \qquad \frac{2 - \sqrt{22}}{3} \approx -0.897$$

To the nearest thousandth, the solutions are 2.230 and -0.897.

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➡ Solve by using the quadratic formula: $4x^2 = 8x - 13$

$$4x^2 = 8x - 13$$
$$4x^2 - 8x + 13 = 0$$ • Write the equation in standard form.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ • Use the quadratic formula.

$$= \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 4 \cdot 13}}{2 \cdot 4}$$ • $a = 4, b = -8, c = 13$

$$= \frac{8 \pm \sqrt{64 - 208}}{8} = \frac{8 \pm \sqrt{-144}}{8}$$
$$= \frac{8 \pm 12i}{8} = \frac{2 + 3i}{2}$$

The solutions are $1 + \frac{3}{2}i$ and $1 - \frac{3}{2}i$.

Of the three preceding examples, the first two had real number solutions; the last one had complex number solutions.

In the quadratic formula, the quantity $b^2 - 4ac$ is called the **discriminant**. When a, b, and c are real numbers, the discriminant determines whether a quadratic equation will have a double root, two real number solutions that are not equal, or two complex number solutions.

THE EFFECT OF THE DISCRIMINANT ON THE SOLUTIONS OF A QUADRATIC EQUATION

1. If $b^2 - 4ac = 0$, the equation has one real number solution, a double root.
2. If $b^2 - 4ac > 0$, the equation has two unequal real number solutions.
3. If $b^2 - 4ac < 0$, the equation has two complex number solutions.

➡ Use the discriminant to determine whether $x^2 - 4x - 5 = 0$ has one real number solution, two real number solutions, or two complex number solutions.

$b^2 - 4ac$
$(-4)^2 - 4(1)(-5) = 16 + 20 = 36$ • Evaluate the discriminant. $a = 1, b = -4, c = -5$
$36 > 0$

Because $b^2 - 4ac > 0$, the equation has two real number solutions.

Example 1 Solve by using the quadratic formula: $2x^2 - x + 5 = 0$

Solution

$2x^2 - x + 5 = 0$
$a = 2, b = -1, c = 5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(5)}}{2 \cdot 2}$$
$$= \frac{1 \pm \sqrt{1 - 40}}{4} = \frac{1 \pm \sqrt{-39}}{4}$$
$$= \frac{1 \pm i\sqrt{39}}{4}$$

The solutions are $\frac{1}{4} + \frac{\sqrt{39}}{4}i$ and $\frac{1}{4} - \frac{\sqrt{39}}{4}i$.

You Try It 1 Solve by using the quadratic formula: $x^2 - 2x + 10 = 0$

Your solution

Solution on p. S31

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Example 2

Solve by using the quadratic formula:
$2x^2 = (x - 2)(x - 3)$

Solution

$2x^2 = (x - 2)(x - 3)$
$2x^2 = x^2 - 5x + 6$

$x^2 + 5x - 6 = 0$
$a = 1, b = 5, c = -6$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-5 \pm \sqrt{5^2 - 4(1)(-6)}}{2 \cdot 1}$$
$$= \frac{-5 \pm \sqrt{25 + 24}}{2} = \frac{-5 \pm \sqrt{49}}{2}$$
$$= \frac{-5 \pm 7}{2}$$

$$x = \frac{-5 + 7}{2} \qquad x = \frac{-5 - 7}{2}$$
$$= \frac{2}{2} = 1 \qquad = \frac{-12}{2} = -6$$

The solutions are 1 and -6.

You Try It 2

Solve by using the quadratic formula:
$4x^2 = 4x - 1$

Your solution

Example 3

Use the discriminant to determine whether $4x^2 - 2x + 5 = 0$ has one real number solution, two real number solutions, or two complex number solutions.

Solution

$a = 4, b = -2, c = 5$

$b^2 - 4ac = (-2)^2 - 4(4)(5)$
$= 4 - 80$
$= -76$

$-76 < 0$

Because the discriminant is less than zero, the equation has two complex number solutions.

You Try It 3

Use the discriminant to determine whether $3x^2 - x - 1 = 0$ has one real number solution, two real number solutions, or two complex number solutions.

Your solution

Solutions on p. S31

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10.3 Exercises

1. Write the quadratic formula. What does each variable in the formula represent?

2. Write the expression that appears under the radical symbol in the quadratic formula. What is this quantity called? What can it be used to determine?

Solve by using the quadratic formula.

3. $x^2 - 3x - 10 = 0$

4. $z^2 - 4z - 8 = 0$

5. $y^2 + 5y - 36 = 0$

6. $z^2 - 3z - 40 = 0$

7. $w^2 = 8w + 72$

8. $t^2 = 2t + 35$

9. $v^2 = 24 - 5v$

10. $x^2 = 18 - 7x$

11. $2y^2 + 5y - 1 = 0$

12. $4p^2 - 7p + 1 = 0$

13. $8s^2 = 10s + 3$

14. $12t^2 = 5t + 2$

15. $x^2 = 14x - 4$

16. $v^2 = 12v - 24$

17. $2z^2 - 2z - 1 = 0$

18. $6w^2 = 9w - 1$

19. $z^2 + 2z + 2 = 0$

20. $p^2 - 4p + 5 = 0$

21. $y^2 - 2y + 5 = 0$

22. $x^2 + 6x + 13 = 0$

23. $s^2 - 4s + 13 = 0$

24. $t^2 - 6t + 10 = 0$

25. $2w^2 - 2w - 5 = 0$

26. $v^2 + 8v + 3 = 0$

27. $2x^2 + 6x + 5 = 0$

28. $2y^2 + 2y + 13 = 0$

29. $4t^2 - 6t + 9 = 0$

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Solve by using the quadratic formula. Approximate the solutions to the nearest thousandth.

30. $x^2 - 6x - 6 = 0$

31. $p^2 - 8p + 3 = 0$

32. $r^2 - 2r = 4$

33. $w^2 + 4w = 1$

34. $3t^2 = 7t + 1$

35. $2y^2 = y + 5$

Use the discriminant to determine whether the quadratic equation has one real number solution, two real number solutions, or two complex number solutions.

36. $2z^2 - z + 5 = 0$

37. $3y^2 + y + 1 = 0$

38. $9x^2 - 12x + 4 = 0$

39. $4x^2 + 20x + 25 = 0$

40. $2v^2 - 3v - 1 = 0$

41. $3w^2 + 3w - 2 = 0$

APPLYING THE CONCEPTS

42. Name the three methods of solving a quadratic equation that have been discussed. What are the advantages and disadvantages of each?

For what values of p does the quadratic equation have two real number solutions that are not equal? Write the answer in set-builder notation.

43. $x^2 - 6x + p = 0$

44. $x^2 + 10x + p = 0$

For what values of p does the quadratic equation have two complex number solutions? Write the answer in set-builder notation.

45. $x^2 - 2x + p = 0$

46. $x^2 + 4x + p = 0$

47. Find all values of x that satisfy the equation $x^2 + ix + 2 = 0$.

48. Show that the equation $x^2 + bx - 1 = 0$ always has real number solutions regardless of the value of b.

49. Can the quadratic formula be used to solve *any* quadratic equation? If so, explain why. If not, give an example of a quadratic equation that cannot be solved by using the quadratic formula.

50. One of the steps in the derivation of the quadratic formula is $x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$ or $\pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$. Carefully explain the occurrence of the $\pm$ sign. (*Hint:* Recall that $\sqrt{x^2} = |x|$.)

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10.4 Solving Equations That Are Reducible to Quadratic Equations

Objective A

To solve an equation that is quadratic in form

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Certain equations that are not quadratic can be expressed in quadratic form by making suitable substitutions. An equation is **quadratic in form** if it can be written as $au^2 + bu + c = 0$.

The equation $x^4 - 4x^2 - 5 = 0$ is quadratic in form.

$$\begin{aligned} x^4 - 4x^2 - 5 &= 0 \\ (x^2)^2 - 4(x^2) - 5 &= 0 \\ u^2 - 4u - 5 &= 0 \end{aligned}$$

• Let $x^2 = u$.

The equation $y - y^{1/2} - 6 = 0$ is quadratic in form.

$$\begin{aligned} y - y^{1/2} - 6 &= 0 \\ (y^{1/2})^2 - (y^{1/2}) - 6 &= 0 \\ u^2 - u - 6 &= 0 \end{aligned}$$

• Let $y^{1/2} = u$.

Here is the key to recognizing equations that are quadratic in form: When the equation is written in standard form, the exponent on one variable term is $\frac{1}{2}$ the exponent on the other variable term.

Solve: $z + 7z^{1/2} - 18 = 0$

$$z + 7z^{1/2} - 18 = 0$$

• The equation is quadratic in form.

$$(z^{1/2})^2 + 7(z^{1/2}) - 18 = 0$$

$$u^2 + 7u - 18 = 0$$

• Let $z^{1/2} = u$.

$$(u - 2)(u + 9) = 0$$

• Solve by factoring.

$$\begin{array}{ll} u - 2 = 0 & u + 9 = 0 \\ u = 2 & u = -9 \end{array}$$

$$\begin{array}{ll} z^{1/2} = 2 & z^{1/2} = -9 \\ \sqrt{z} = 2 & \sqrt{z} = -9 \end{array}$$

• Replace u by $z^{1/2}$.

$$\begin{array}{ll} (\sqrt{z})^2 = 2^2 & (\sqrt{z})^2 = (-9)^2 \\ z = 4 & z = 81 \end{array}$$

• Solve for z.

TAKE NOTE
When each side of an equation is squared, the resulting equation may have a solution that is not a solution of the original equation.

Check:

$$\begin{array}{r|l} z + 7z^{1/2} - 18 = 0 & \\ \hline 4 + 7(4)^{1/2} - 18 & 0 \\ 4 + 7 \cdot 2 - 18 & \\ 4 + 14 - 18 & \\ 0 = 0 & \end{array}$$

$$\begin{array}{r|l} z + 7z^{1/2} - 18 = 0 & \\ \hline 81 + 7(81)^{1/2} - 18 & 0 \\ 81 + 7 \cdot 9 - 18 & \\ 81 + 63 - 18 & \\ 126 \neq 0 & \end{array}$$

4 checks as a solution, but 81 does not check as a solution.
The solution is 4.

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Example 1 Solve: $x^4 + x^2 - 12 = 0$

Solution

$$x^4 + x^2 - 12 = 0$$
$$(x^2)^2 + (x^2) - 12 = 0$$
$$u^2 + u - 12 = 0$$
$$(u - 3)(u + 4) = 0$$

$u - 3 = 0$ $\quad$ $u + 4 = 0$
$u = 3$ $\quad$ $u = -4$

Replace u by x^2.

$x^2 = 3$ $\quad$ $x^2 = -4$
$\sqrt{x^2} = \sqrt{3}$ $\quad$ $\sqrt{x^2} = \sqrt{-4}$
$x = \pm\sqrt{3}$ $\quad$ $x = \pm 2i$

The solutions are $\sqrt{3}$, $-\sqrt{3}$, $2i$, and $-2i$.

You Try It 1 Solve: $x - 5x^{1/2} + 6 = 0$

Your solution

Solution on p. S31

To solve a radical equation that is reducible to a quadratic equation

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Certain equations that contain radicals can be expressed as quadratic equations.

Solve: $\sqrt{x + 2} + 4 = x$

$$\sqrt{x + 2} + 4 = x$$
$$\sqrt{x + 2} = x - 4$$ • Solve for the radical expression.
$$(\sqrt{x + 2})^2 = (x - 4)^2$$ • Square each side of the equation.
$$x + 2 = x^2 - 8x + 16$$ • Simplify.
$$0 = x^2 - 9x + 14$$ • Write the equation in standard form.
$$0 = (x - 7)(x - 2)$$ • Solve for x.

$x - 7 = 0$ $\quad$ $x - 2 = 0$
$x = 7$ $\quad$ $x = 2$

TAKE NOTE
You should always check your solutions by substituting the proposed solutions back into the *original* equation.

Check: $\sqrt{x + 2} + 4 = x$

$\sqrt{7 + 2} + 4$	7
$\sqrt{9} + 4$	
$3 + 4$	
$7 = 7$	

$\sqrt{x + 2} + 4 = x$

$\sqrt{2 + 2} + 4$	2
$\sqrt{4} + 4$	
$2 + 4$	
$6 \neq 2$	

7 checks as a solution, but 2 does not check as a solution.
The solution is 7.

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Example 2

Solve: $\sqrt{7y - 3} + 3 = 2y$

Solution

$$\sqrt{7y - 3} + 3 = 2y$$
$$\sqrt{7y - 3} = 2y - 3$$
$$(\sqrt{7y - 3})^2 = (2y - 3)^2$$
$$7y - 3 = 4y^2 - 12y + 9$$
$$0 = 4y^2 - 19y + 12$$
$$0 = (4y - 3)(y - 4)$$

$4y - 3 = 0$ $\qquad$ $y - 4 = 0$

$4y = 3$ $\qquad$ $y = 4$

$y = \frac{3}{4}$

4 checks as a solution.

$\frac{3}{4}$ does not check as a solution.

The solution is 4.

You Try It 2

Solve: $\sqrt{2x + 1} + x = 7$

Your solution

Example 3

Solve: $\sqrt{2y + 1} - \sqrt{y} = 1$

Solution

$$\sqrt{2y + 1} - \sqrt{y} = 1$$

Solve for one of the radical expressions.

$$\sqrt{2y + 1} = \sqrt{y} + 1$$
$$(\sqrt{2y + 1})^2 = (\sqrt{y} + 1)^2$$
$$2y + 1 = y + 2\sqrt{y} + 1$$
$$y = 2\sqrt{y}$$

Square each side of the equation.

$$y^2 = (2\sqrt{y})^2$$
$$y^2 = 4y$$
$$y^2 - 4y = 0$$
$$y(y - 4) = 0$$

$y = 0$ $\qquad$ $y - 4 = 0$

$y = 4$

0 and 4 check as solutions.

The solutions are 0 and 4.

You Try It 3

Solve: $\sqrt{2x - 1} + \sqrt{x} = 2$

Your solution

Solutions on p. S31

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To solve a fractional equation that is reducible to a quadratic equation



After each side of a fractional equation has been multiplied by the LCM of the denominators, the resulting equation may be a quadratic equation.

Solve: $\frac{1}{r} + \frac{1}{r+1} = \frac{3}{2}$

$$\frac{1}{r} + \frac{1}{r+1} = \frac{3}{2}$$

$$2r(r+1)\left(\frac{1}{r} + \frac{1}{r+1}\right) = 2r(r+1) \cdot \frac{3}{2}$$

- Multiply each side of the equation by the LCM of the denominators.

$$2(r+1) + 2r = r(r+1) \cdot 3$$
$$2r + 2 + 2r = 3r(r+1)$$
$$4r + 2 = 3r^2 + 3r$$
$$0 = 3r^2 - r - 2$$

- Write the equation in standard form.
- Solve for r by factoring.

$$0 = (3r+2)(r-1)$$

$3r + 2 = 0$ $\qquad$ $r - 1 = 0$

$3r = -2$ $\qquad$ $r = 1$

$r = -\frac{2}{3}$

$-\frac{2}{3}$ and 1 check as solutions. The solutions are $-\frac{2}{3}$ and 1.

Example 4

Solve: $\frac{9}{x-3} = 2x + 1$

Solution

$$\frac{9}{x-3} = 2x + 1$$

$$(x-3)\frac{9}{x-3} = (x-3)(2x+1)$$
$$9 = 2x^2 - 5x - 3$$
$$0 = 2x^2 - 5x - 12$$
$$0 = (2x+3)(x-4)$$

$2x + 3 = 0$ $\qquad$ $x - 4 = 0$

$2x = -3$ $\qquad$ $x = 4$

$x = -\frac{3}{2}$

$-\frac{3}{2}$ and 4 check as solutions.

The solutions are $-\frac{3}{2}$ and 4.

You Try It 4

Solve: $3y + \frac{25}{3y-2} = -8$

Your solution

Solution on p. S31

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10.4 Exercises

Objective A

Solve.

1. $x^4 - 13x^2 + 36 = 0$
2. $y^4 - 5y^2 + 4 = 0$
3. $z^4 - 6z^2 + 8 = 0$
4. $t^4 - 12t^2 + 27 = 0$
5. $p - 3p^{1/2} + 2 = 0$
6. $v - 7v^{1/2} + 12 = 0$
7. $x - x^{1/2} - 12 = 0$
8. $w - 2w^{1/2} - 15 = 0$
9. $z^4 + 3z^2 - 4 = 0$
10. $y^4 + 5y^2 - 36 = 0$
11. $x^4 + 12x^2 - 64 = 0$
12. $x^4 - 81 = 0$
13. $p + 2p^{1/2} - 24 = 0$
14. $v + 3v^{1/2} - 4 = 0$
15. $y^{2/3} - 9y^{1/3} + 8 = 0$
16. $z^{2/3} - z^{1/3} - 6 = 0$
17. $9w^4 - 13w^2 + 4 = 0$
18. $4y^4 - 7y^2 - 36 = 0$

Objective B

Solve.

19. $\sqrt{x + 1} + x = 5$
20. $\sqrt{x - 4} + x = 6$
21. $x = \sqrt{x} + 6$
22. $\sqrt{2y - 1} = y - 2$
23. $\sqrt{3w + 3} = w + 1$
24. $\sqrt{2s + 1} = s - 1$
25. $\sqrt{4y + 1} - y = 1$
26. $\sqrt{3s + 4} + 2s = 12$
27. $\sqrt{10x + 5} - 2x = 1$

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28. $\sqrt{t+8} = 2t+1$

29. $\sqrt{p+11} = 1-p$

30. $x-7 = \sqrt{x-5}$

31. $\sqrt{x-1} - \sqrt{x} = -1$

32. $\sqrt{y}+1 = \sqrt{y+5}$

33. $\sqrt{2x-1} = 1-\sqrt{x-1}$

34. $\sqrt{x+6} + \sqrt{x+2} = 2$

35. $\sqrt{t+3} + \sqrt{2t+7} = 1$

36. $\sqrt{5-2x} = \sqrt{2-x}+1$

Solve.

37. $x = \dfrac{10}{x-9}$

38. $z = \dfrac{5}{z-4}$

39. $\dfrac{t}{t+1} = \dfrac{-2}{t-1}$

40. $\dfrac{2v}{v-1} = \dfrac{5}{v+2}$

41. $\dfrac{y-1}{y+2} + y = 1$

42. $\dfrac{2p-1}{p-2} + p = 8$

43. $\dfrac{3r+2}{r+2} - 2r = 1$

44. $\dfrac{2v+3}{v+4} + 3v = 4$

45. $\dfrac{2}{2x+1} + \dfrac{1}{x} = 3$

46. $\dfrac{3}{s} - \dfrac{2}{2s-1} = 1$

47. $\dfrac{16}{z-2} + \dfrac{16}{z+2} = 6$

48. $\dfrac{2}{y+1} + \dfrac{1}{y-1} = 1$

49. $\dfrac{t}{t-2} + \dfrac{2}{t-1} = 4$

50. $\dfrac{4t+1}{t+4} + \dfrac{3t-1}{t+1} = 2$

51. $\dfrac{5}{2p-1} + \dfrac{4}{p+1} = 2$

APPLYING THE CONCEPTS

52. Solve: $(\sqrt{x}-2)^2 - 5\sqrt{x} + 14 = 0$. (*Hint:* Let $u = \sqrt{x} - 2$.)

53. Solve: $(\sqrt{x}+3)^2 - 4\sqrt{x} - 17 = 0$. (*Hint:* Let $u = \sqrt{x} + 3$.)

54. The fourth power of a number is twenty-five less than ten times the square of the number. Find the number.

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10.5 Applications of Quadratic Equations

To solve application problems

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The application problems in this section are similar to problems solved earlier in the text. Each of the strategies for the problems in this section will result in a quadratic equation.

A small pipe takes 16 min longer to empty a tank than does a larger pipe. Working together, the pipes can empty the tank in 6 min. How long would it take the smaller pipe working alone to empty the tank?

Strategy for Solving an Application Problem

1. Determine the type of problem. Is it a uniform motion problem, a geometry problem, an integer problem, or a work problem?

The problem is a work problem.

2. Choose a variable to represent the unknown quantity. Write numerical or variable expressions for all the remaining quantities. These results can be recorded in a table.

The unknown time of the larger pipe: t
The unknown time of the smaller pipe: $t + 16$

	Rate of Work	·	*Time Worked*	=	*Part of Task Completed*
Larger pipe	$\frac{1}{t}$	·	6	=	$\frac{6}{t}$
Smaller pipe	$\frac{1}{t+16}$	·	6	=	$\frac{6}{t+16}$

3. Determine how the quantities are related.

$$\frac{6}{t} + \frac{6}{t+16} = 1$$

• The sum of the parts of the task completed must equal 1.

$$t(t+16)\left(\frac{6}{t} + \frac{6}{t+16}\right) = t(t+16) \cdot 1$$
$$(t+16)6 + 6t = t^2 + 16t$$
$$6t + 96 + 6t = t^2 + 16t$$
$$0 = t^2 + 4t - 96$$
$$0 = (t+12)(t-8)$$
$$t + 12 = 0 \qquad t - 8 = 0$$
$$t = -12 \qquad t = 8$$

Because time cannot be negative, the solution $t = -12$ is not possible.

The time for the smaller pipe is $t + 16$.

$t + 16 = 8 + 16 = 24$ • Replace t by 8 and evaluate.

The smaller pipe requires 24 min to empty the tank.

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Example 1

In 8 h, two campers rowed 15 mi down a river and then rowed back to their campsite. The rate of the river's current was 1 mph. Find the rate at which the campers rowed.

Strategy

- This is a uniform motion problem.
- Unknown rowing rate of the campers: r

	Distance	*Rate*	*Time*
Down river	15	$r + 1$	$\frac{15}{r+1}$
Up river	15	$r - 1$	$\frac{15}{r-1}$

- The total time of the trip was 8 h.

Solution

$$\frac{15}{r+1} + \frac{15}{r-1} = 8$$

$$(r+1)(r-1)\left(\frac{15}{r+1} + \frac{15}{r-1}\right) = (r+1)(r-1)8$$

$$(r-1)15 + (r+1)15 = (r^2 - 1)8$$

$$15r - 15 + 15r + 15 = 8r^2 - 8$$

$$30r = 8r^2 - 8$$

$$0 = 8r^2 - 30r - 8$$

$$0 = 2(4r^2 - 15r - 4)$$

$$0 = 2(4r + 1)(r - 4)$$

$4r + 1 = 0$ $\qquad$ $r - 4 = 0$

$4r = -1$ $\qquad$ $r = 4$

$r = -\frac{1}{4}$

The solution $r = -\frac{1}{4}$ is not possible, because the rate cannot be a negative number.

The rowing rate was 4 mph.

You Try It 1

The length of a rectangle is 3 m more than the width. The area is 54 m^2. Find the length of the rectangle.

Your strategy

Your solution

Solution on p. S32

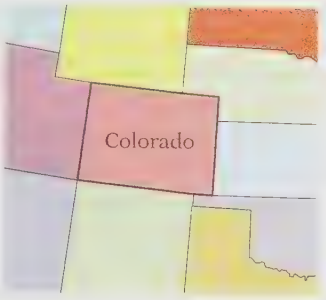
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10.5 Exercises

Objective 1 *Application Problems*

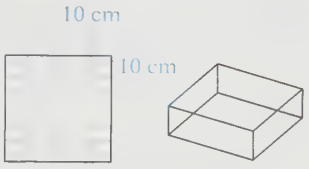
1. The base of a triangle is 1 cm less than five times the height of the triangle. The area of the triangle is 21 cm^2. Find the height and the length of the base of the triangle.

2. The length of a rectangle is 2 ft less than three times the width of the rectangle. The area of the rectangle is 65 ft^2. Find the length and width of the rectangle.



3. The state of Colorado is almost perfectly rectangular, with its north border 111 mi longer than its west border. If the state encompasses 104,000 mi^2, estimate the dimensions of Colorado. Round to the nearest mile.

4. A square piece of cardboard is formed into a box by cutting 10-centimeter squares from each of the four corners and then folding up the sides, as shown in the figure. If the volume, V, of the box is to be 49,000 cm^3, what size square piece of cardboard is needed? Recall that $V = LWH$.



5. A car with good tire tread can stop in less distance than a car with poor tread. The formula for the stopping distance d, in feet, of a car with good tread on dry cement is approximated by $d = 0.04v^2 + 0.5v$, where v is the speed of the car. If the driver must be able to stop within 60 ft, what is the maximum safe speed, to the nearest mile per hour, of the car?



6. A model rocket is launched with an initial velocity of 200 ft/s. The height h, in feet, of the rocket t seconds after the launch is given by $h(t) = -16t^2 + 200t$. How many seconds after the launch will the rocket be 300 ft above the ground? Round to the nearest hundredth of a second.



7. The height of a projectile fired upward is given by the formula $s = v_0t - 16t^2$, where s is the height in feet, v_0 is the initial velocity, and t is the time in seconds. Find the time for a projectile to return to Earth if it has an initial velocity of 200 ft/s.

8. The height of a projectile fired upward is given by the formula $s = v_0t - 16t^2$, where s is the height in feet, v_0 is the initial velocity, and t is the time in seconds. Find the time for a projectile to reach a height of 64 ft if it has an initial velocity of 128 ft/s. Round to the nearest hundredth of a second.

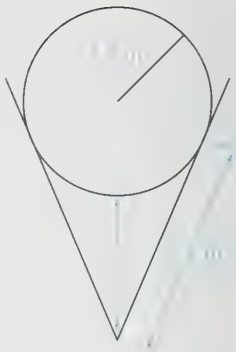
9. In Germany, there is no speed limit on some portions of the autobahn (highway). Other portions have a speed limit of 180 km/h (approximately 112 mph). The distance d, in meters, required to stop a car traveling at v kilometers per hour is $d(v) = 0.019v^2 + 0.69v$. Approximate, to the nearest tenth, the maximum speed a driver can be traveling and still be able to stop within 150 m.



German Autobahn System

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10. A perfectly spherical scoop of mint chocolate chip ice cream is placed in a cone, as shown in the figure. How far is the bottom of the scoop of ice cream from the bottom of the cone? Round to the nearest tenth. (*Hint:* A line segment from the center of the ice cream to the point at which the ice cream touches the cone is perpendicular to the edge of the cone.)



11. A small pipe can fill a tank in 6 min more time than it takes a larger pipe to fill the same tank. Working together, both pipes can fill the tank in 4 min. How long would it take each pipe working alone to fill the tank?

12. A cruise ship made a trip of 100 mi in 8 h. The ship traveled the first 40 mi at a constant rate before increasing its speed by 5 mph. Then it traveled another 60 mi at the increased speed. Find the rate of the cruise ship for the first 40 mi.

13. The Concorde's speed in calm air is 1320 mph. Flying with the wind, the Concorde can fly from New York to London, a distance of approximately 4000 mi, in 0.5 h less than the time required to make the return trip. Find the rate of the wind to the nearest mile per hour.



14. A car travels 120 mi. A second car, traveling 10 mph faster than the first car, makes the same trip in 1 h less time. Find the speed of each car.

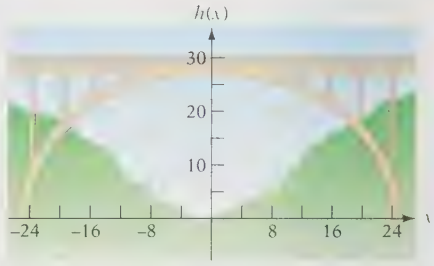
15. For a portion of the Green River in Utah, the rate of the river's current is 4 mph. A tour guide can row 5 mi down this river and back in 3 h. Find the rowing rate of the guide in calm water.

16. The height h, in feet, of an arch is given by the equation $h(x) = -\frac{3}{64}x^2 + 27$, where $|x|$ is the distance in feet from the center of the arch.

a. What is the maximum height of the arch?

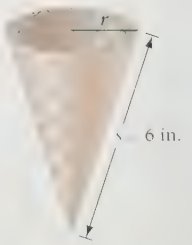
b. What is the height of the arch 8 ft to the right of the center?

c. How far from the center is the arch 8 ft tall?

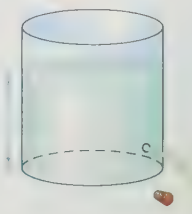


APPLYING THE CONCEPTS

17. The surface area of the ice cream cone shown at the right is given by $A = \pi r^2 + \pi rs$, where r is the radius of the circular top of the cone and s is the slant height of the cone. If the surface area of the cone is 11.25π in^2 and its slant height is 6 in., find the radius of the cone.



18. Using Torricelli's Principle, it can be shown that the depth, d, of a liquid in a bottle with a hole of area 0.5 cm^2 in its side can be approximated by $d = 0.0034t^2 - 0.52518t + 20$, where t is the time since a stopper was removed from the hole. When will the depth be 10 cm? Round to the nearest tenth of a second.



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10.6 Quadratic Inequalities and Rational Inequalities

Objective A To solve a nonlinear inequality

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A **quadratic inequality** is one that can be written in the form $ax^2 + bx + c < 0$ or $ax^2 + bx + c > 0$, where $a \neq 0$. The symbols $\leq$ and $\geq$ can also be used. The solution set of a quadratic inequality can be found by solving a compound inequality.

To solve $x^2 - 3x - 10 > 0$, first factor the trinomial.

$$x^2 - 3x - 10 > 0$$
$$(x + 2)(x - 5) > 0$$

There are two cases for which the product of the factors will be positive: (1) both factors are positive, or (2) both factors are negative.

(1) $x + 2$ () and $x - 5$ ()
(2) $x + 2$ () and $x - 5$ ()

Solve each pair of compound inequalities.

(1) $x + 2 > 0$ and $x - 5 > 0$
$x > -2$ $x > 5$
$\{x|x > -2\} \cap \{x|x > 5\} = \{x|x > 5\}$

(2) $x + 2 < 0$ and $x - 5 < 0$
$x < -2$ $x < 5$
$\{x|x < -2\} \cap \{x|x < 5\} = \{x|x < -2\}$

Because the two cases for which the product will be positive are connected by *or*, the solution set is the union of the solution sets of the individual inequalities.

$$\{x|x > 5\} \cup \{x|x < -2\} = \{x|x > 5 \text{ or } x < -2\}$$

Although the solution set of any quadratic inequality can be found by using the method outlined above, a graphical method is often easier to use.

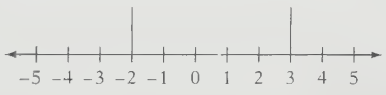
➡ Solve and graph the solution set of $x^2 - x - 6 < 0$.

$$x^2 - x - 6 < 0$$

Factor the trinomial.

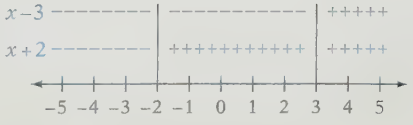
$$(x - 3)(x + 2) < 0$$

On a number line, draw lines indicating the numbers that make each factor equal to zero.



$x - 3 = 0$ $\quad x + 2 = 0$
$x = 3$ $\quad x = -2$

For each factor, place plus signs above the number line for those regions where the factor is positive and minus signs where the factor is negative.



Because $x^2 - x - 6 < 0$, the solution set will be the regions where one factor is positive and the other factor is negative.

Write the solution set.

$$\{x|-2 < x < 3\}$$

The graph of the solution set of $x^2 - x - 6 < 0$ is shown at the right.



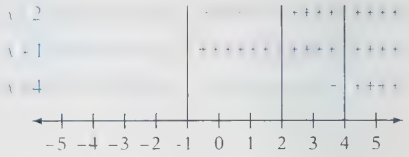
TAKE NOTE

For each factor, choose a number in each region. For example: when $x = -4$, $x - 3$ is negative; when $x = 1$, $x - 3$ is negative; and when $x = 4$, $x - 3$ is positive. When $x = -4$, $x + 2$ is negative; when $x = 1$, $x + 2$ is positive; and when $x = 4$, $x + 2$ is positive.

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➨ Solve and graph the solution set of $(x - 2)(x + 1)(x - 4) > 0$.

On a number line, identify for each factor the regions where the factor is positive and those where the factor is negative.



There are two regions where the product of the three factors is positive.

Write the solution set. $\{x \mid -1 < x < 2 \text{ or } x > 4\}$

The graph of the solution set of $(x - 2)(x + 1)(x - 4) > 0$ is shown at the right.



➨ Solve: $\frac{2x - 5}{x - 4} \leq 1$

$$\frac{2x - 5}{x - 4} \leq 1$$

Rewrite the inequality so that 0 appears on the right side of the inequality.

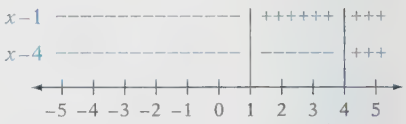
$$\frac{2x - 5}{x - 4} - 1 \leq 0$$

Simplify.

$$\frac{2x - 5}{x - 4} - \frac{x - 4}{x - 4} \leq 0$$

$$\frac{x - 1}{x - 4} \leq 0$$

On a number line, identify for each factor of the numerator and each factor of the denominator the regions where the factor is positive and those where the factor is negative.



The region where the quotient of the two factors is negative is between 1 and 4.

Write the solution set. $\{x \mid 1 \leq x < 4\}$

Note that 1 is part of the solution set but 4 is not because the denominator of the rational expression is zero when $x = 4$.

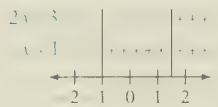
Example 1

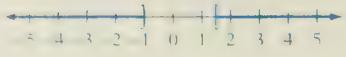
Solve and graph the solution set of $2x^2 - x - 3 \geq 0$.

Solution

$2x^2 - x - 3 \geq 0$

$(2x - 3)(x + 1) \geq 0$

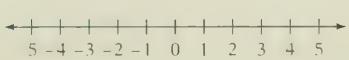




You Try It 1

Solve and graph the solution set of $2x^2 - x - 10 \leq 0$.

Your solution



Solution on p. S32

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10.6 Exercises

Objective A

1. If $(x - 3)(x - 5) > 0$, what must be true of the values of $x - 3$ and $x - 5$?

2. For the inequality $\frac{x - 2}{x - 3} \leq 1$, which of the values 1, 2, and 3 is not a possible element of the solution set? Why?

Solve and graph the solution set.

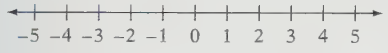
3. $(x - 4)(x + 2) > 0$



4. $(x + 1)(x - 3) > 0$



5. $x^2 - 3x + 2 \geq 0$



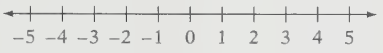
6. $x^2 + 5x + 6 > 0$



7. $x^2 - x - 12 < 0$



8. $x^2 + x - 20 < 0$



9. $(x - 1)(x + 2)(x - 3) < 0$



10. $(x + 4)(x - 2)(x + 1) > 0$



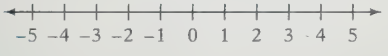
11. $(x + 4)(x - 2)(x - 1) \geq 0$



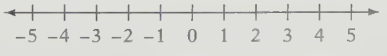
12. $(x - 1)(x + 5)(x - 2) \leq 0$



13. $\frac{x - 4}{x + 2} > 0$



14. $\frac{x + 2}{x - 3} > 0$



15. $\frac{x - 3}{x + 1} \leq 0$



16. $\frac{x - 1}{x} > 0$



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17. $\dfrac{(x-1)(x+2)}{x-3} \le 0$



18. $\dfrac{(x+3)(x-1)}{x-2} \ge 0$



Solve.

19. $x^2 - 16 > 0$

20. $x^2 - 4 \ge 0$

21. $x^2 - 9x \le 36$

22. $x^2 + 4x > 21$

23. $4x^2 - 8x + 3 < 0$

24. $2x^2 + 11x + 12 \ge 0$

25. $\dfrac{3}{x-1} < 2$

26. $\dfrac{x}{(x-1)(x+2)} \ge 0$

27. $\dfrac{x-2}{(x+1)(x-1)} \le 0$

28. $\dfrac{1}{x} < 2$

29. $\dfrac{x}{2x-1} \ge 1$

30. $\dfrac{x}{2x-3} \le 1$

31. $\dfrac{x}{2-x} \le -3$

32. $\dfrac{3}{x-2} > \dfrac{2}{x+2}$

33. $\dfrac{3}{x-5} > \dfrac{1}{x+1}$

APPLYING THE CONCEPTS

Graph the solution set.

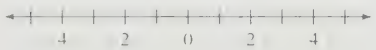
34. $(x+2)(x-3)(x+1)(x+4) > 0$



35. $(x-1)(x+3)(x-2)(x-4) \ge 0$



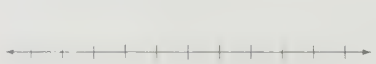
36. $(x^2 + 2x - 8)(x^2 - 2x - 3) < 0$



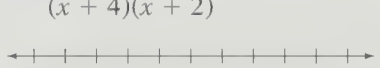
37. $(x^2 + 2x - 3)(x^2 + 3x + 2) \ge 0$



38. $(x^2 + 1)(x^2 - 3x + 2) > 0$



39. $\dfrac{x^2(3-x)(2x+1)}{(x+4)(x+2)} \ge 0$



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Focus on Problem Solving

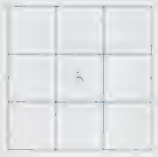
Use a Variety of Problem-Solving Techniques

We have examined several problem-solving strategies throughout the text. See if you can apply those techniques to the following problems.

1. Eight coins look exactly alike, but one is lighter than the others. Explain how the different coin can be found in two weighings on a balance scale.

2. For the sequence of numbers 1, 1, 2, 3, 5, 8, 13, . . . , identify a possible pattern and then use that pattern to determine the next number in the sequence.

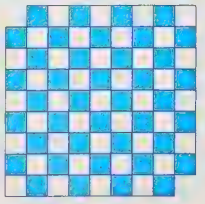
3. Arrange the numbers 1, 2, 3, 4, 5, 6, 7, 8, and 9 in the squares at the right so that the sum of any row, column, or diagonal is 15. (*Suggestion:* Note that 1, 5, 9; 2, 5, 8; 3, 5, 7; and 4, 5, 6 all add to 15. Because 5 is part of each sum, this suggests that 5 be placed in the center of the squares.)



4. A restaurant charges $10.00 for a pizza that has a diameter of 9 in. Determine the selling price of a pizza with a diameter of 18 in. so that the selling price per square inch is the same as for the 9-inch pizza.

5. You have a balance scale and weights of 1 g, 4 g, 8 g, and 16 g. Using only these weights, can you weigh something that weighs 7 g? 9 g? 12 g? 19 g?

6. Can the checkerboard at the right be covered with dominos (which look like ▭) so that every square on the board is covered by a domino? Why or why not? (*Note:* The dominos cannot overlap.)



Projects and Group Activities

Completing the Square

Essentially all of the investigations into mathematics before the Renaissance were geometric. The solutions of quadratic equations were calculated from a construction of a certain area. Proofs of theorems, even theorems about numbers, were based entirely on geometry. In this project, we will examine the geometric solution of a quadratic equation.

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Solve: $x^2 + 6x = 7$

Begin with a line of unknown length, x, and one of length 6, the coefficient of x. Using these lines, construct a rectangle as shown.

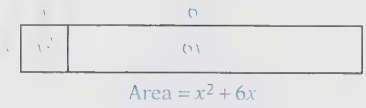


Figure 1

Now draw another area that has exactly the same area as Figure 1 by cutting off one-half of the rectangle of area $6x$ and placing it on the bottom of the square labeled x^2. See Figure 2.

The unshaded area in Figure 2 has exactly the same area as Figure 1. However, when the shaded area is added to Figure 2 to make a square, the total area is 9 square units larger than that of Figure 1. In equation form,

$$(\text{Area of Figure 1}) + 9 = \text{area of Figure 2}$$

or

$$x^2 + 6x + 9 = (x + 3)^2$$

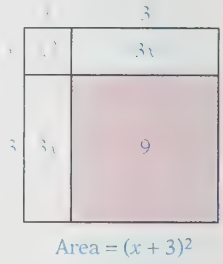


Figure 2

From the original equation, $x^2 + 6x = 7$. Thus,

$$\begin{aligned} x^2 + 6x + 9 &= (x + 3)^2 \\ 7 + 9 &= (x + 3)^2 \quad \bullet\ x^2 + 6x = 7 \\ 16 &= (x + 3)^2 \\ 4 &= x + 3 \quad \bullet\ \text{See note below.} \\ 1 &= x \end{aligned}$$

Note: Although early mathematicians knew that a quadratic equation may have two solutions, both solutions were allowed only if they were positive. After all, a geometric construction could not have a negative length. Therefore, the solution of this equation was 1; the solution -7 would have been dismissed as *fictitious*, the actual word that was frequently used through the 15th century for negative-number solutions of an equation.

Try to solve the quadratic equation $x^2 + 4x = 12$ by geometrically completing the square.

Using a Graphing Calculator to Solve a Quadratic Equation

Recall that an x-intercept of the graph of an equation is a point at which the graph crosses the x-axis. For the graph in Figure 3, the x-intercepts are $(-2, 0)$ and $(3, 0)$.

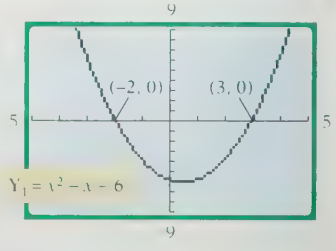


Figure 3

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Recall also that to find the x-intercept of a graph, set $y = 0$ and then solve for x. For the equation in Figure 3, if we set $y = 0$, the resulting equation is $0 = x^2 - x - 6$, which is a quadratic equation. Solving this equation by factoring, we have

$$0 = x^2 - x - 6$$
$$0 = (x + 2)(x - 3)$$

$$x + 2 = 0 \qquad x - 3 = 0$$
$$x = -2 \qquad x = 3$$

Thus the solutions of the equation are the x-coordinates of the x-intercepts of the graph.

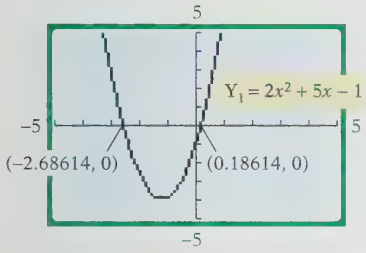


Figure 4

This connection between the solutions of an equation and the x-intercepts of its graph enables us to find approximations of the real number solutions of an equation graphically. For example, to use a TI-83 to approximate the solutions of $2x^2 + 5x - 1 = 0$ graphically, use the following keystrokes to graph $y = 2x^2 + 5x - 1$ and find the decimal approximations of the x-coordinates of the x-intercepts. Use the window shown in the graph in Figure 4.

Y= CLEAR 2 X,T,θ,n x^2 + 5 X,T,θ,n − 1
2nd CALC 2

Use the arrow keys to move to the left of the leftmost x-intercept. Press ENTER. Use the arrow keys to move to a point just to the right of the leftmost x-intercept. Press ENTER twice. The x-coordinate at the bottom of the screen is the approximation of one solution of the equation. To find the other solution, use the same procedure, but move the cursor first to the left and then to the right of the rightmost x-intercept.

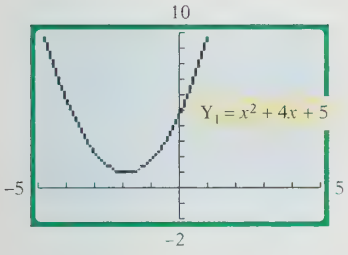


Figure 5

Attempting to find the solutions of an equation graphically will not necessarily find all the solutions. Because the x-coordinates of the x-intercepts of a graph are *real* numbers, only real number solutions can be found. For instance, consider the equation $x^2 + 4x + 5 = 0$. The graph of $y = x^2 + 4x + 5$ is shown in Figure 5. Note that the graph has no x-intercepts and, consequently, no real number solutions. However, $x^2 + 4x + 5 = 0$ does have complex number solutions that can be obtained by using the quadratic formula. They are $-2 + i$ and $-2 - i$.

Chapter Summary

Key Words

A *quadratic equation* is an equation of the form $ax^2 + bx + c = 0$, where a and b are coefficients, c is a constant, and $a \neq 0$. A quadratic equation is also called a *second-degree equation.* [p. 545]

A quadratic equation is in *standard form* when the polynomial is in descending order and equal to zero. [p. 545]

When a quadratic equation has two solutions that are the same number, the solution is called a *double root* of the equation. [p. 545]

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Methods of Solving a Quadratic Equation:

1. **Factoring**
 This method is used only when the polynomial $ax^2 + bx + c$ is factorable.
2. **Taking square roots**
 This method is used only when the value of b in $ax^2 + bx + c$ is 0.
 It is also used when an equation can be written in the form

 Square of a binomial = constant
3. **Completing the square**
 This method can be used to solve any quadratic equation.
4. **Quadratic formula**
 This method can be used to solve any quadratic equation.
 [pp. 545, 547–548, 553–555, 559–561]

Adding to a binomial the constant term that makes it a perfect-square trinomial is called *completing the square.* [p. 553]

For an equation of the form $ax^2 + bx + c = 0$, the quantity $b^2 - 4ac$ is called the *discriminant.* [p. 561]

An equation is *quadratic in form* if it can be written as $au^2 + bu + c = 0$. [p. 565]

A *quadratic inequality* is one that can be written in the form $ax^2 + bx + c > 0$ or $ax^2 + bx + c < 0$, where $a \neq 0$. The symbols $\leq$ or $\geq$ can also be used. [p. 575]

Essential Rules

The Principle of Zero Products [p. 545] If $ab = 0$, then $a = 0$ or $b = 0$.

To Solve a Quadratic Equation by Factoring [p. 545]
1. Write the equation in standard form.
2. Factor.
3. Use the Principle of Zero Products to set each factor equal to 0.
4. Solve each equation.
5. Check the solutions.

To Write a Quadratic Equation Given Its Solutions [p. 547]
Use the equation $(x - r_1)(x - r_2) = 0$. Replace r_1 with one solution and r_2 with the other solution. Then multiply the two factors.

To Complete the Square [p. 553]
Add to a binomial of the form $x^2 + bx$ the square of $\frac{1}{2}$ of the coefficient of x.

To Solve a Quadratic Equation by Completing the Square [p. 555]
1. Write the equation in the form $ax^2 + bx = -c$.
2. Multiply both sides of the equation by $\frac{1}{a}$.
3. Complete the square on $x^2 + \frac{b}{a}x$. Add the number that completes the square to both sides of the equation.
4. Factor the perfect-square trinomial.
5. Take the square root of each side of the equation.
6. Solve the resulting equation for x.
7. Check the solutions.

The Quadratic Formula [p. 559]
The solutions of $ax^2 + bx + c = 0$, $a \neq 0$, are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

The Effect of the Discriminant on the Solutions of a Quadratic Equation [p. 561]
1. If $b^2 - 4ac = 0$, the equation has one real number solution, a double root.
2. If $b^2 - 4ac > 0$, the equation has two unequal real number solutions.
3. If $b^2 - 4ac < 0$, the equation has two complex number solutions.

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Chapter Review

1. Solve by factoring: $2x^2 - 3x = 0$

2. Solve by factoring: $6x^2 + 9cx = 6c^2$

3. Solve by taking square roots:
$x^2 = 48$

4. Solve by taking square roots:
$\left(x + \frac{1}{2}\right)^2 + 4 = 0$

5. Solve by completing the square:
$x^2 + 4x + 3 = 0$

6. Solve by completing the square:
$7x^2 - 14x + 3 = 0$

7. Solve by using the quadratic formula:
$12x^2 - 25x + 12 = 0$

8. Solve by using the quadratic formula:
$x^2 - x + 8 = 0$

9. Write a quadratic equation that has integer coefficients and has solutions 0 and -3.

10. Write a quadratic equation that has integer coefficients and has solutions $\frac{3}{4}$ and $-\frac{2}{3}$.

11. Solve by completing the square:
$x^2 - 2x + 8 = 0$

12. Solve by completing the square:
$(x - 2)(x + 3) = x - 10$

13. Solve by using the quadratic formula:
$3x(x - 3) = 2x - 4$

14. Use the discriminant to determine whether $3x^2 - 5x + 1 = 0$ has one real number solution, two real number solutions, or two complex number solutions.

15. Solve: $(x + 3)(2x - 5) < 0$

16. Solve: $(x - 2)(x + 4)(2x + 3) \leq 0$

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17. Solve: $x^{2/3} + x^{1/3} - 12 = 0$

18. Solve: $2(x - 1) + 3\sqrt{x - 1} - 2 = 0$

19. Solve: $3x = \frac{9}{x - 2}$

20. Solve: $\frac{3x + 7}{x + 2} + x = 3$

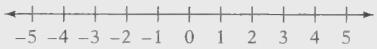
21. Solve and graph the solution set:

$$\frac{x - 2}{2x - 3} \geq 0$$



22. Solve and graph the solution set:

$$\frac{(2x - 1)(x + 3)}{x - 4} \leq 0$$



23. Solve: $x = \sqrt{x} + 2$

24. Solve: $2x = \sqrt{5x + 24} + 3$

25. Solve: $\frac{x - 2}{2x + 3} - \frac{x \quad 4}{x} = 2$

26. Solve: $1 - \frac{x + 4}{2 - x} = \frac{x - 3}{x + 2}$

27. The length of a rectangle is two more centimeters than twice the width. The area of the rectangle is 60 cm^2. Find the length and width of the rectangle.

28. The sum of the squares of three consecutive even integers is fifty-six. Find the three integers.

29. An older computer requires 12 min longer to print the payroll than does a newer computer. Together the computers can print the payroll in 8 min. Find the time for the new computer working alone to print the payroll.

30. A car travels 200 mi. A second car, making the same trip, travels 10 mph faster than the first car and makes the trip in 1 h less time. Find the speed of each car.

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Chapter Test

1. Solve by factoring: $3x^2 + 10x = 8$

2. Solve by factoring: $6x^2 - 5x - 6 = 0$

3. Write a quadratic equation that has integer coefficients and has solutions 3 and -3.

4. Write a quadratic equation that has integer coefficients and has solutions $\frac{1}{2}$ and -4.

5. Solve by taking square roots:
$3(x - 2)^2 - 24 = 0$

6. Solve by completing the square:
$x^2 - 6x - 2 = 0$

7. Solve by completing the square:
$3x^2 - 6x = 2$

8. Solve by using the quadratic formula:
$2x^2 - 2x = 1$

9. Solve by using the quadratic formula:
$x^2 + 4x + 12 = 0$

10. Use the discriminant to determine whether $3x^2 - 4x = 1$ has one real number solution, two real number solutions, or two complex number solutions.

11. Use the discriminant to determine whether $x^2 - 6x = -15$ has one real number solution, two real number solutions, or two complex number solutions.

12. Solve: $2x + 7x^{1/2} - 4 = 0$

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13. Solve: $x^4 - 4x^2 + 3 = 0$

14. Solve: $\sqrt{2x + 1} + 5 = 2x$

15. Solve: $\sqrt{x - 2} = \sqrt{x} - 2$

16. Solve: $\frac{2x}{x - 3} + \frac{5}{x - 1} = 1$

17. Solve and graph the solution set of $(x - 2)(x + 4)(x - 4) < 0$.



18. Solve and graph the solution set of $\frac{2x - 3}{x + 4} \leq 0$.



19. The base of a triangle is 3 ft more than three times the height. The area of the triangle is 30 ft^2. Find the base and height of the triangle.

20. The rate of a river's current is 2 mph. A canoe was rowed 6 mi down the river and back in 4 h. Find the rowing rate in calm water.

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Cumulative Review

1. Evaluate $2a^2 - b^2 \div c^2$ when $a = 3$, $b = -4$, and $c = -2$.

2. Solve: $|3x - 2| < 8$

3. Find the volume of a cylinder with a height of 6 m and a radius of 3 m. Give the exact measure.

4. Given $f(x) = \dfrac{2x - 3}{x^2 - 1}$, find $f(-2)$.

5. Find the slope of the line containing the points $(3, -4)$ and $(-1, 2)$.

6. Find the x- and y-intercepts of the graph of $6x - 5y = 15$.

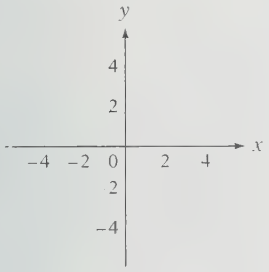
7. Find the equation of the line that contains the point $(1, 2)$ and is parallel to the line $x - y = 1$.

8. Solve the system of equations.

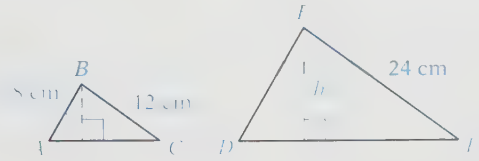
$$\begin{aligned} x + y + z &= 2 \\ -x + 2y - 3z &= -9 \\ x - 2y - 2z &= -1 \end{aligned}$$

9. Graph the solution set:

$$\begin{aligned} x + y &\le 3 \\ 2x - y &< 4 \end{aligned}$$



10. Triangles ABC and DEF are similar. Find the height of triangle DEF.



11. Divide: $(3x^3 - 13x^2 + 10) \div (3x - 4)$

12. Factor: $-3x^3y + 6x^2y^2 - 9xy^3$

13. Factor: $6x^2 - 7x - 20$

14. Multiply: $\dfrac{x^2 + 2x + 1}{8x^2 + 8x} \cdot \dfrac{4x^3 - 4x^2}{x^2 - 1}$

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15. Solve: $\frac{x}{x+2} - \frac{4x}{x+3} = 1$

16. Solve $S = \frac{n}{2}(a + b)$ for b.

17. Multiply: $a^{-1/2}(a^{1/2} - a^{3/2})$

18. Multiply: $-2i(7 - 4i)$

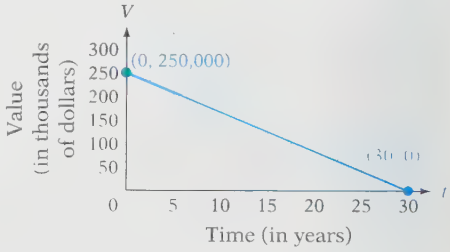
19. Solve: $\sqrt{3x + 1} - 1 = x$

20. Solve: $x^4 - 6x^2 + 8 = 0$

21. A piston rod for an automobile is $9\frac{3}{8}$ in. with a tolerance of $\frac{1}{64}$ in. Find the lower and upper limits of the length of the piston rod.

22. The base of a triangle is $(x + 8)$ ft. The height is $(2x - 4)$ ft. Find the area of the triangle in terms of the variable x.

23. The graph shows the relationship between the value of a building and the time, in years, since depreciation began. Find the slope of the line between the two points shown on the graph. Write a sentence that states the meaning of the slope.



24. How high on a building will a 17-foot ladder reach when the bottom of the ladder is 8 ft from the building?

25. Use the discriminant to determine whether $2x^2 + 4x + 3 = 0$ has one real number solution, two real number solutions, or two complex number solutions.

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Appendix

Calculator Guide for the TI-83 and TI-83Plus

Basic Operations

To evaluate an expression

a. Press the [Y=] key. A menu showing \Y1 = through \Y7 = will be displayed vertically with a blinking cursor to the right of \Y1 =. Press [CLEAR], if necessary, to delete an unwanted expression.

b. Input the expression to be evaluated. For example, to input the expression $-3a^2b - 4c$, use the following keystrokes:

[(−)] 3 [ALPHA] A [X²] [ALPHA] B [−] 4 [ALPHA] C [2nd] QUIT

Note the difference between the keys for a negative sign [(−)] and a minus sign [−].

c. Store the value of each variable that will be used in the expression. For example, to evaluate the expression above when $a = 3$, $b = -2$, and $c = -4$, use the following keystrokes:

3 [STO▷] [ALPHA] A [ENTER] [(−)] 2 [STO▷] [ALPHA] B [ENTER] [(−)] 4 [STO▷] [ALPHA] C [ENTER]

These steps store the value of each variable.

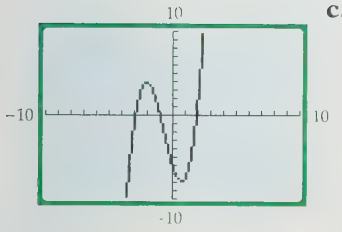
d. Press [VARS] [▷] [1] [1] [ENTER]. The value for the expression, Y1, for the given values is displayed; in this case, Y1 = 70.

To graph a function

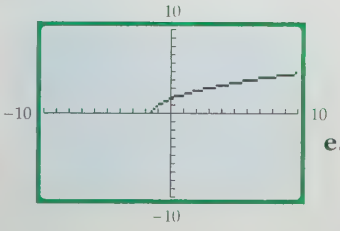
a. Press the [Y=] key. A menu showing \Y1 = through \Y7 = will be displayed vertically with a blinking cursor to the right of \Y1 =. Press [CLEAR], if necessary, to delete an unwanted expression.

b. Input the expression for each function that is to be graphed. Press [X,T,θ,n] to input x. For example, to input $y = x^3 + 2x^2 - 5x - 6$, use the following keystrokes:

[X,T,θ,n] [^] 3 [+] 2 [X,T,θ,n] [X²] [−] 5 [X,T,θ,n] [−] 6



c. Set the viewing window by pressing [WINDOW]. Enter the values for the minimum x-value (Xmin), the maximum x-value (Xmax), the distance between tick marks on the x-axis (Xscl), the minimum y-value (Ymin), the maximum y-value (Ymax), and the distance between tick marks on the y-axis (Yscl). Now press [GRAPH]. For the graph shown at the left, Xmin = −10, Xmax = 10, Xscl = 1, Ymin = −10, Ymax = 10, and Yscl = 1. This is called the standard viewing window. Pressing [ZOOM] [6] is a quick way to set the calculator to the standard viewing window. *Note:* This will also immediately graph the function in that window.



d. Press the [Y=] key. The equal sign has a black rectangle around it. This indicates that the function is active and will be graphed when the [GRAPH] key is pressed. A function is deactivated by using the arrow keys. Move the cursor over the equal sign and press [ENTER]. When the cursor is moved to the right, the black rectangle will not be present and that equation will not be active.

e. Graphing some radical equations requires special care. To graph the equation $y = \sqrt{2x + 3}$ shown at the left, enter the following keystrokes:

[Y=] [CLEAR] [2nd] [√] 2 [X,T,θ,n] [+] 3 [)] [GRAPH]

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To display the *x*-coordinates of rectangular coordinates as integers

a. Set the viewing window as follows: Xmin = −47, Xmax = 47, Xscl = 10, Ymin = −31, Ymax = 31, Yscl = 10.

b. Graph the function and use the TRACE feature. Press [TRACE] and then move the cursor with the [◁] and [▷] keys. The values of x and $y = f(x)$ displayed on the bottom of the screen are the coordinates of a point on the graph.

To display the *x*-coordinates of rectangular coordinates in tenths

a. Set the viewing window as follows: [ZOOM] [4]

b. Graph the function. Press [TRACE] and then move the cursor with the [◁] and [▷] keys. The values of x and $y = f(x)$ displayed on the bottom of the screen are the coordinates of a point on the graph.

To evaluate a function for a given value of *x*, or to produce ordered pairs of a function

a. Input the equation; for example, input $Y_1 = 2x^3 - 3x + 2$.

b. Press [2nd] QUIT.

c. To evaluate the function when $x = 3$, press [VARS] [▷] [1] [1] [(] 3 [)] [ENTER]. The value for the function for the given x-value is displayed, in this case, $Y_1(3) = 47$. An ordered pair of the function is (3, 47).

d. Repeat step **c.** to produce as many pairs as desired.

The TABLE feature can also be used to determine ordered pairs.

To use a table

a. Press [2nd] TBLSET to activate the table setup menu.

b. TblStart is the beginning number for the table; ΔTbl is the difference between any two successive x-values in the table.

c. The portion of the table that appears as Indpnt: **Auto** Ask / Depend: **Auto** Ask allows you to choose between having the calculator automatically produce the results (Auto) or having the calculator ask you for values of x. You can choose Ask by using the arrow keys.

d. Once a table has been set up, enter an expression for Y_1. Now select TABLE by pressing [2nd] TABLE. A table showing ordered pair solutions of the equation will be displayed on the screen.

Zoom Features

To zoom in or out on a graph

Here are two methods of using ZOOM.

a. The first method uses the built-in features of the calculator. Press [TRACE] and then move the cursor to a point on the graph that is of interest. Press [ZOOM]. The ZOOM menu will appear. Press [2] [ENTER] to zoom in on the graph by the amount shown under the SET FACTORS menu. The center of the new graph is the location at which you placed the cursor. Press [ZOOM] [3] [ENTER] to zoom out on the graph by the amount under the SET FACTORS menu. (The SET FACTORS menu is accessed by pressing [ZOOM] [▷] [4].)

b. The second method uses the ZBOX option under the ZOOM menu. To use this method, press [ZOOM] [1]. A cursor will appear on the graph. Use the arrow keys to move the cursor to a portion of the graph that is of interest. Press [ENTER]. Now

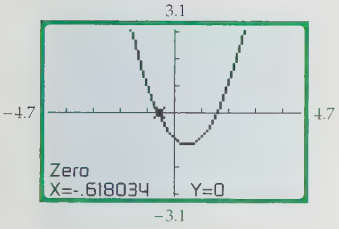
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use the arrow keys to draw a box around the portion of the graph you wish to zoom in on. Press [ENTER]. The portion of the graph defined by the box will be drawn.

c. Pressing [ZOOM] [6] resets the window to the standard viewing window.

Solving Equations

This discussion is based on the fact that the real number solutions of an equation are related to the x-intercepts of a graph. For instance, the real solutions of the equation $x^2 = x + 1$ are the x-intercepts of the graph of $f(x) = x^2 - x - 1$, which are the zeros of f.



To solve $x^2 = x + 1$, rewrite the equation with all terms on one side: $x^2 - x - 1 = 0$. Think of this equation as $Y_1 = x^2 - x - 1$. The x-intercepts of the graph of Y_1 are the solutions of the equation $x^2 = x + 1$.

a. Enter $x^2 - x - 1$ into Y_1.

b. Graph the equation. You may need to adjust the viewing window so that the x-intercepts are visible.

c. Press [2nd] CALC [2].

d. Move the cursor to a point on the curve that is to the left of an x-intercept. Press [ENTER].

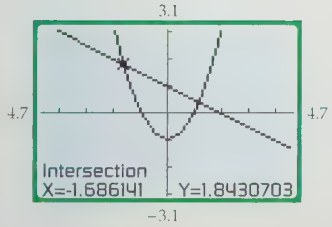
e. Move the cursor to a point on the curve that is to the right of the same x-intercept. Press [ENTER] [ENTER].

f. The root is shown as the x-coordinate on the bottom of the screen; in this case, the root is approximately -0.618034. To find the next intercept, repeat steps **c.** through **e.** The SOLVER feature under the MATH menu can also be used to find solutions of equations.

Solving Systems of Equations in Two Variables

To solve a system of equations

To solve
$$y = x^2 - 1$$
$$\frac{1}{2}x + y = 1,$$



a. If necessary, solve one or both of the equations for y.

b. Enter the first equation as Y_1: $Y_1 = x^2 - 1$.

c. Enter the second equation as Y_2: $Y_2 = 1 - .5x$.

d. Graph both equations. (*Note:* The point of intersection must appear on the screen. It may be necessary to adjust the viewing window so that the points of intersection are displayed.)

e. Press [2nd] CALC [5].

f. Move the cursor close to the first point of intersection. Press [ENTER] [ENTER] [ENTER].

g. The first point of intersection is $(-1.686141, 1.8430703)$.

h. Repeat steps **e.** and **f.** for each point of intersection.

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Finding Minimum or Maximum Values of a Function

a. Enter the function into Y1. The equation $y = x^2 - x - 1$ is used here.

b. Graph the equation. You may need to adjust the viewing window so that the maximum or minimum points are visible.

c. Press [2nd] CALC [3] to determine a minimum value or press [2nd] CALC [4] to determine a maximum value.

d. Move the cursor to a point on the curve that is to the left of the minimum (maximum). Press [ENTER].

e. Move the cursor to a point on the curve that is to the right of the minimum (maximum). Press [ENTER] [ENTER].

f. The minimum (maximum) is shown as the y-coordinate on the bottom of the screen; in this case the minimum value is -1.25.

Statistics

To calculate a linear regression equation

a. Press [STAT] to access the statistics menu. Press 1 to Edit or enter a new list of data. To delete data already in a list, press the up arrow to highlight the list name. For instance, to delete data in L1, highlight L1. Then press [CLEAR] and [ENTER]. Now enter each value of the independent variable in L1. Enter each value of the dependent variable in L2.

b. When all the data has been entered, press [STAT] [▷] 4 [ENTER]. The values of the slope and y-intercept of the linear regression equation will be displayed on the screen.

c. To graph the linear regression equation, modify step **b.** with these keystrokes: [STAT] [▷] 4 [VARS] [▷] 1 1 [ENTER]. This will store the regression equation in Y1. Now press [GRAPH]. It may be necessary to adjust the viewing window.

d. To evaluate the regression equation for a value of x, complete step **c.** but do not graph the equation. Now press [VARS] [▷] 1 1 [(] A [)] [ENTER], where A is replaced by the number at which you want to evaluate the expression.

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Proofs and Tables

Proofs of Logarithmic Properties

In each of the following proofs of logarithmic properties, it is assumed that the Properties of Exponents are true for all real number exponents.

The Logarithm Property of the Product of Two Numbers

For any positive real numbers x, y, and b, $b \neq 1$, $\log_b xy = \log_b x + \log_b y$.

Proof: Let $\log_b x = m$ and $\log_b y = n$.

Write each equation in its equivalent exponential form.	$x = b^m \qquad y = b^n$
Use substitution and the Properties of Exponents.	$xy = b^m b^n$
	$xy = b^{m+n}$
Write the equation in its equivalent logarithmic form.	$\log_b xy = m + n$
Substitute $\log_b x$ for m and $\log_b y$ for n.	$\log_b xy = \log_b x + \log_b y$

The Logarithm Property of the Quotient of Two Numbers

For any positive real numbers x, y, and b, $b \neq 1$, $\log_b \frac{x}{y} = \log_b x - \log_b y$.

Proof: Let $\log_b x = m$ and $\log_b y = n$.

Write each equation in its equivalent exponential form.	$x = b^m \qquad y = b^n$
Use substitution and the Properties of Exponents.	$\frac{x}{y} = \frac{b^m}{b^n}$
	$\frac{x}{y} = b^{m-n}$
Write the equation in its equivalent logarithmic form.	$\log_b \frac{x}{y} = m - n$
Substitute $\log_b x$ for m and $\log_b y$ for n.	$\log_b \frac{x}{y} = \log_b x - \log_b y$

The Logarithm Property of the Power of a Number

For any real numbers x, r, and b, $b \neq 1$, $\log_b x^r = r \log_b x$.

Proof: Let $\log_b x = m$.

Write the equation in its equivalent exponential form.	$x = b^m$
Raise both sides to the r power.	$x^r = (b^m)^r$
	$x^r = b^{mr}$
Write the equation in its equivalent logarithmic form.	$\log_b x^r = mr$
Substitute $\log_b x$ for m.	$\log_b x^r = r \log_b x$

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Table of Symbols

$+$	add
$-$	subtract
$\cdot, \times, (a)(b)$	multiply
$\frac{a}{b}, \div, a\overline{)b}$	divide
$()$	parentheses, a grouping symbol
$[]$	brackets, a grouping symbol
π	pi, a number approximately equal to $\frac{22}{7}$ or 3.14
$-a$	the opposite, or additive inverse, of a
$\frac{1}{a}$	the reciprocal, or multiplicative inverse, of a
$=$	is equal to
$\approx$	is approximately equal to
$\neq$	is not equal to
$<$	is less than
$\leq$	is less than or equal to
$>$	is greater than
$\geq$	is greater than or equal to
(a, b)	an ordered pair whose first component is a and whose second component is b
$^\circ$	degree (for angles)
$\sqrt{a}$	the principal square root of a
$\varnothing, \{\}$	the empty set
$\lvert a \rvert$	the absolute value of a
$\cup$	union of two sets
$\cap$	intersection of two sets
$\in$	is an element of (for sets)
$\notin$	is not an element of (for sets)

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Solutions to Chapter 6 "You Try It"

SECTION 6.1

You Try It 1

$$\begin{aligned}(-3a^2b^4)(-2ab^3)^4 &= (-3a^2b^4)[(-2)^4a^4b^{12}]\\ &= (-3a^2b^4)(16a^4b^{12})\\ &= -48a^6b^{16}\end{aligned}$$

You Try It 2 $(y^{n-3})^2 = y^{(n-3)2} = y^{2n-6}$

You Try It 3 $[(ab^3)^3]^4 = [a^3b^9]^4 = a^{12}b^{36}$

You Try It 4

$$\begin{aligned}\frac{20r^{-2}t^{-5}}{-16r^{-3}s^{-2}} &= -\frac{4\cdot 5r^{-2-(-3)}s^2t^{-5}}{4\cdot 4}\\ &= -\frac{5rs^2}{4t^5}\end{aligned}$$

You Try It 5

$$\begin{aligned}\frac{(9u^{-6}v^4)^{-1}}{(6u^{3}v^{2})^{2}} &= \frac{9^{-1}u^6v^{-4}}{6^{2}u^6v^4}\\ &= 9^{-1}\cdot 6^2u^0v^{-8}\\ &= \frac{36}{9v^8}\\ &= \frac{4}{v^8}\end{aligned}$$

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You Try It 6 $\dfrac{a^{2n+1}}{a^{n+3}} = a^{2n+1-(n+3)} = a^{2n+1-n-3} = a^{n-2}$

You Try It 7 $942{,}000{,}000 = 9.42 \times 10^8$

You Try It 8 $2.7 \times 10^{-5} = 0.000027$

You Try It 9

$$\frac{5{,}600{,}000 \times 0.000000081}{900 \times 0.000000028}$$
$$= \frac{5.6 \times 10^6 \times 8.1 \times 10^{-8}}{9 \times 10^2 \times 2.8 \times 10^{-8}}$$
$$= \frac{(5.6)(8.1) \times 10^{6+(-8)-2-(-8)}}{(9)(2.8)}$$
$$= 1.8 \times 10^4 = 18{,}000$$

You Try It 10

Strategy To find the number of arithmetic operations:

- Find the reciprocal of 1×10^{-7}, which is the number of operations performed in one second.
- Write the number of seconds in one minute (60) in scientific notation.
- Multiply the number of arithmetic operations per second by the number of seconds in one minute.

Solution $\dfrac{1}{1 \times 10^{-7}} = 10^7$

$60 = 6 \times 10$

$6 \times 10 \times 10^7 = 6 \times 10^8$

The computer can perform 6×10^8 operations in one minute.

SECTION 6.2

You Try It 1

$$R(x) = -2x^4 - 5x^3 + 2x - 8$$
$$R(2) = -2(2)^4 - 5(2)^3 + 2(2) - 8$$
$$= -2(16) - 5(8) + 2(2) - 8$$
$$= -32 - 40 + 4 - 8$$
$$= -76$$

You Try It 2 The leading coefficient is -3, the constant term is -12, and the degree is 4.

You Try It 3

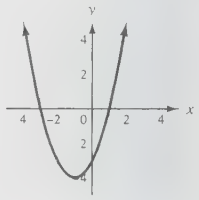
a. This is a polynomial function.

b. This is not a polynomial function. A polynomial function does not have a variable expression raised to a negative power.

c. This is not a polynomial function. A polynomial function does not have a variable expression within a radical.

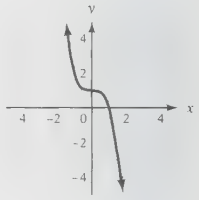
You Try It 4

x	$y = f(x)$
-3	0
-2	-3
-1	-4
0	-3
1	0
2	5



You Try It 5

x	$y = f(x)$
-3	28
-2	9
-1	2
0	1
1	0
2	-7
3	-26



You Try It 6

$$\begin{array}{r} -3x^2 - 4x + 9 \\ -5x^2 - 7x + 1 \\ \hline -8x^2 - 11x + 10 \end{array}$$

You Try It 7 Add the additive inverse of $6x^2 + 3x - 7$ to $-5x^2 + 2x - 3$.

$$\begin{array}{r} -5x^2 + 2x - 3 \\ -6x^2 - 3x + 7 \\ \hline -11x^2 - x + 4 \end{array}$$

You Try It 8

$$S(x) = P(x) + R(x)$$
$$= (4x^3 - 3x^2 + 2) + (-2x^2 + 2x - 3)$$
$$= 4x^3 - 5x^2 + 2x - 1$$

$$S(-1) = 4(-1)^3 - 5(-1)^2 + 2(-1) - 1$$
$$= 4(-1) - 5(1) + 2(-1) - 1$$
$$= -4 - 5 - 2 - 1$$
$$= -12$$

You Try It 9

$$D(x) = P(x) - R(x)$$

$$D(x) = (5x^{2n} - 3x^n - 7) - (-2x^{2n} - 5x^n + 8)$$
$$= (5x^{2n} - 3x^n - 7) + (2x^{2n} + 5x^n - 8)$$
$$= 7x^{2n} + 2x^n - 15$$

SECTION 6.3

You Try It 1 $(-2y + 3)(-4y) = 8y^2 - 12y$

You Try It 2 $-a^2(3a^2 + 2a - 7) = -3a^4 - 2a^3 + 7a^2$

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You Try It 3

$$\begin{array}{r} 2y^3 + 2y^2 \quad - 3 \\ \hline 3y - 1 \\ \hline -2y^3 - 2y^2 \quad + 3 \\ 6y^4 + 6y^3 \quad - 9y \\ \hline 6y^4 + 4y^3 - 2y^2 - 9y + 3 \end{array}$$

You Try It 4

$$\begin{aligned}(4y - 5)(2y - 3) &= 8y^2 - 12y - 10y + 15 \\ &= 8y^2 - 22y + 15\end{aligned}$$

You Try It 5

$$\begin{aligned}(3b + 2)(3b - 5) &= 9b^2 - 15b + 6b - 10 \\ &= 9b^2 - 9b - 10\end{aligned}$$

You Try It 6 $(2a + 5c)(2a - 5c) = 4a^2 - 25c^2$

You Try It 7 $(3x + 2y)^2 = 9x^2 + 12xy + 4y^2$

You Try It 8

Strategy To find the area, replace the variables b and h in the equation $A = \frac{1}{2}bh$ by the given values and solve for A.

Solution $A = \frac{1}{2}bh$

$A = \frac{1}{2}(2x + 6)(x - 4)$

$A = (x + 3)(x - 4)$

$A = x^2 - 4x + 3x - 12$

$A = x^2 - x - 12$

The area is $(x^2 - x - 12)$ ft^2.

SECTION 6.4

You Try It 1

$$\begin{array}{r} 5x - 1 \\ 3x + 4\overline{)15x^2 + 17x - 20} \\ \underline{15x^2 + 20x} \\ -3x - 20 \\ \underline{-3x - 4} \\ -16 \end{array}$$

$$\frac{15x^2 + 17x - 20}{3x + 4} = 5x - 1 - \frac{16}{3x + 4}$$

You Try It 2

$$\begin{array}{r} x^2 + 3x - 1 \\ 3x - 1\overline{)3x^3 + 8x^2 - 6x + 2} \\ \underline{3x^3 - x^2} \\ 9x^2 - 6x \\ \underline{9x^2 - 3x} \\ -3x + 2 \\ \underline{-3x + 1} \\ 1 \end{array}$$

$$\frac{3x^3 + 8x^2 - 6x + 2}{3x - 1} = x^2 + 3x - 1 + \frac{1}{3x - 1}$$

You Try It 3

$$\begin{array}{r} 3x^2 - 2x + 4 \\ x^2 - 3x + 2\overline{)3x^4 - 11x^3 + 16x^2 - 16x + 8} \\ \underline{3x^4 - 9x^3 + 6x^2} \\ -2x^3 + 10x^2 - 16x \\ \underline{-2x^3 + 6x^2 - 4x} \\ 4x^2 - 12x + 8 \\ \underline{4x^2 - 12x + 8} \\ 0 \end{array}$$

$$\frac{3x^4 - 11x^3 + 16x^2 - 16x + 8}{x^2 - 3x + 2} = 3x^2 - 2x + 4$$

You Try It 4

$$\begin{array}{r|rrr} -2 & 6 & 8 & -5 \\ & & -12 & 8 \\ \hline & 6 & -4 & 3 \end{array}$$

$$(6x^2 + 8x - 5) \div (x + 2) = 6x - 4 + \frac{3}{x + 2}$$

You Try It 5

$$\begin{array}{r|rrrr} 2 & 5 & -12 & -8 & 16 \\ & & 10 & -4 & -24 \\ \hline & 5 & -2 & -12 & -8 \end{array}$$

$$\begin{aligned}(5x^3 - 12x^2 - 8x + 16) &\div (x - 2) \\ &= 5x^2 - 2x - 12 - \frac{8}{x - 2}\end{aligned}$$

You Try It 6

$$\begin{array}{r|rrrrr} 3 & 2 & -3 & -8 & 0 & -2 \\ & & 6 & 9 & 3 & 9 \\ \hline & 2 & 3 & 1 & 3 & 7 \end{array}$$

$$\begin{aligned}(2x^4 - 3x^3 - 8x^2 - 2) &\div (x - 3) \\ &= 2x^3 + 3x^2 + x + 3 + \frac{7}{x - 3}\end{aligned}$$

You Try It 7

$$\begin{array}{r|rrrr} -3 & 2 & -5 & 0 & 7 \\ & & -6 & 33 & -99 \\ \hline & 2 & -11 & 33 & -92 \end{array}$$

$P(-3) = -92$

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Solutions to Chapter 7 "You Try It"

SECTION 7.1

You Try It 1
The GCF is $7a^2$.

$$14a^2 - 21a^4b = 7a^2(2) + 7a^2(-3a^2b)$$
$$= 7a^2(2 - 3a^2b)$$

You Try It 2
The GCF is 9.

$$27b^2 + 18b + 9 = 9(3b^2) + 9(2b) + 9(1)$$
$$= 9(3b^2 + 2b + 1)$$

You Try It 3
The GCF is $3x^2y^2$.

$$6x^4y^2 - 9x^3y^2 + 12x^2y^4$$
$$= 3x^2y^2(2x^2) + 3x^2y^2(-3x) + 3x^2y^2(4y^2)$$
$$= 3x^2y^2(2x^2 - 3x + 4y^2)$$

You Try It 4
The GCF is a^2.

$$a^{n+4} + a^2 = a^2(a^{n+2}) + a^2(1)$$
$$= a^2(a^{n+2} + 1)$$

You Try It 5
$$2y(5x - 2) - 3(2 - 5x)$$
$$= 2y(5x - 2) + 3(5x - 2)$$
$$= (5x - 2)(2y + 3)$$

You Try It 6
$$a^2 - 3a + 2ab - 6b$$
$$= (a^2 - 3a) + (2ab - 6b)$$
$$= a(a - 3) + 2b(a - 3)$$
$$= (a - 3)(a + 2b)$$

You Try It 7
$$2mn^2 - n + 8mn - 4 = (2mn^2 - n) + (8mn - 4)$$
$$= n(2mn - 1) + 4(2mn - 1)$$
$$= (2mn - 1)(n + 4)$$

You Try It 8
$$3xy - 9y - 12 + 4x$$
$$= (3xy - 6y) - (12 - 4x)$$
$$= 3y(x - 3) - 4(3 - x)$$
$$= 3y(x - 3) + 4(x - 3)$$
$$= (x - 3)(3y + 4)$$

SECTION 7.2

You Try It 1

Factors	*Sum*
1, 20	21
2, 10	12
4, 5	9

• **Find the positive factors of 20 whose sum is 9.**

$$x^2 + 9x + 20 = (x + 4)(x + 5)$$

You Try It 2

Factors	*Sum*
+1, −18	−17
−1, +18	17
+2, −9	−7
−2, +9	7
+3, −6	−3
−3, +6	+3

• **Find the factors of −18 whose sum is 7.**

$$x^2 + 7x - 18 = (x + 9)(x - 2)$$

You Try It 3
The GCF is $-2x$.

$$-2x^3 + 14x^2 - 12x = -2x(x^2 - 7x + 6)$$

Factor the trinomial $x^2 - 7x + 6$. Find two negative factors of 6 whose sum is −7.

Factors	*Sum*
−1, −6	−7
−2, −3	−5

$$-2x^3 + 14x^2 - 12x = -2x(x - 6)(x - 1)$$

You Try It 4
The GCF is 3.

$$3x^2 - 9xy - 12y^2 = 3(x^2 - 3xy - 4y^2)$$

Factor the trinomial. Find the factors of −4 whose sum is −3.

Factors	*Sum*
+1, −4	−3
−1, +4	3
+2, −2	0

$$3x^2 - 9xy - 12y^2 = 3(x + y)(x - 4y)$$

SECTION 7.3

You Try It 1
Factor the trinomial $2x^2 - x - 3$.

Positive factors of 2: 1, 2

Factors of −3: 1, −3
−1, 3

Trial Factors	*Middle Term*
$(x + 1)(2x - 3)$	$-3x + 2x = -x$
$(x - 3)(2x + 1)$	$x - 6x = -5x$
$(x - 1)(2x + 3)$	$3x - 2x = x$
$(x + 3)(2x - 1)$	$-x + 6x = 5x$

$$2x^2 - x - 3 = (x + 1)(2x - 3)$$

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You Try It 2
The GCF is $-3y$.

$$-45y^3 + 12y^2 + 12y = -3y(15y^2 - 4y - 4)$$

Factor the trinomial $15y^2 - 4y - 4$.

Positive factors of 15:	Factors of -4:
1, 15	1, -4
	-1, 4
3, 5	2, -2

Trial Factors	*Middle Term*
$(y + 1)(15y - 4)$	$-4y + 15y = 11y$
$(y - 4)(15y + 1)$	$y - 60y = -59y$
$(y - 1)(15y + 4)$	$4y - 15y = -11y$
$(y + 4)(15y - 1)$	$-y + 60y = 59y$
$(y + 2)(15y - 2)$	$-2y + 30y = 28y$
$(y - 2)(15y + 2)$	$2y - 30y = -28y$
$(3y + 1)(5y - 4)$	$-12y + 5y = -7y$
$(3y - 4)(5y + 1)$	$3y - 20y = -17y$
$(3y - 1)(5y + 4)$	$12y - 5y = 7y$
$(3y + 4)(5y - 1)$	$-3y + 20y = 17y$
$(3y + 2)(5y - 2)$	$-6y + 10y = 4y$
$(3y - 2)(5y + 2)$	$6y - 10y = -4y$

$$-45y^3 + 12y^2 + 12y = -3y(3y - 2)(5y + 2)$$

You Try It 3

Factors of -14 $[2(-7)]$	*Sum*
1, -14	-13
-1, 14	13
2, -7	-5
-2, 7	5

$$\begin{aligned} 2a^2 + 13a - 7 &= 2a^2 - a + 14a - 7 \\ &= (2a^2 - a) + (14a - 7) \\ &= a(2a - 1) + 7(2a - 1) \\ &= (2a - 1)(a + 7) \end{aligned}$$

$$2a^2 + 13a - 7 = (2a - 1)(a + 7)$$

You Try It 4
The GCF is $5x$.

$$15x^3 + 40x^2 - 80x = 5x(3x^2 + 8x - 16)$$

Factors of -48 $[3(-16)]$	*Sum*
1, -48	-47
-1, 48	47
2, -24	-22
-2, 24	22
3, -16	-13
-3, 16	13
4, -12	-8
-4, 12	8

$$\begin{aligned} 3x^2 + 8x - 16 &= 3x^2 - 4x + 12x - 16 \\ &= (3x^2 - 4x) + (12x - 16) \\ &= x(3x - 4) + 4(3x - 4) \\ &= (3x - 4)(x + 4) \end{aligned}$$

$$\begin{aligned} 15x^3 + 40x^2 - 80x &= 5x(3x^2 + 8x - 16) \\ &= 5x(3x - 4)(x + 4) \end{aligned}$$

SECTION 7.4

You Try It 1
$$\begin{aligned} x^2 - 36y^4 &= x^2 - (6y^2)^2 \\ &= (x + 6y^2)(x - 6y^2) \end{aligned}$$

You Try It 2
$$9x^2 + 12x + 4 = (3x + 2)^2$$

You Try It 3
$$\begin{aligned} &(a + b)^2 - (a - b)^2 \\ &= [(a + b) + (a - b)][(a + b) - (a - b)] \\ &= (a + b + a - b)(a + b - a + b) \\ &= (2a)(2b) = 4ab \end{aligned}$$

You Try It 4
$$\begin{aligned} 8x^3 + y^3z^3 &= (2x)^3 + (yz)^3 \\ &= (2x + yz)(4x^2 - 2xyz + y^2z^2) \end{aligned}$$

You Try It 5
$$\begin{aligned} &(x - y)^3 + (x + y)^3 \\ &= [(x - y) + (x + y)][(x - y)^2 - (x - y)(x + y) + (x + y)^2] \\ &= 2x[x^2 - 2xy + y^2 - (x^2 - y^2) + x^2 + 2xy + y^2] \\ &= 2x(x^2 - 2xy + y^2 - x^2 + y^2 + x^2 + 2xy + y^2) \\ &= 2x(x^2 + 3y^2) \end{aligned}$$

You Try It 6
Let $u = xy$.

$$\begin{aligned} 6x^2y^2 - 19xy + 10 &= 6u^2 - 19u + 10 \\ &= (2u - 5)(3u - 2) \\ &= (2xy - 5)(3xy - 2) \end{aligned}$$

You Try It 7
Let $u = x^2$.

$$\begin{aligned} 3x^4 + 4x^2 - 4 &= 3u^2 + 4u - 4 \\ &= (u + 2)(3u - 2) \\ &= (x^2 + 2)(3x^2 - 2) \end{aligned}$$

You Try It 8
Let $u = a^2b^2$.

$$\begin{aligned} a^4b^4 + 6a^2b^2 - 7 &= u^2 + 6u - 7 \\ &= (u + 7)(u - 1) \\ &= (a^2b^2 + 7)(a^2b^2 - 1) \end{aligned}$$

You Try It 9
$$\begin{aligned} 18x^3 - 6x^2 - 60x &= 6x(3x^2 - x - 10) \\ &= 6x(3x + 5)(x - 2) \end{aligned}$$

You Try It 10
$$\begin{aligned} &4x - 4y - x^3 + x^2y \\ &= (4x - 4y) - (x^3 - x^2y) \\ &= 4(x - y) - x^2(x - y) \\ &= (x - y)(4 - x^2) \\ &= (x - y)(2 + x)(2 - x) \end{aligned}$$

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You Try It 11

$$\begin{aligned} x^{4n} - x^{2n}y^{2n} &= x^{2n+2n} - x^{2n}y^{2n} \\ &= x^{2n}(x^{2n} - y^{2n}) \\ &= x^{2n}[(x^n)^2 - (y^n)^2] \\ &= x^{2n}(x^n + y^n)(x^n - y^n) \end{aligned}$$

You Try It 12

$$\begin{aligned} ax^5 - ax^2y^6 &= ax^2(x^3 - y^6) \\ &= ax^2(x - y^2)(x^2 + xy^2 + y^4) \end{aligned}$$

SECTION 7.5

You Try It 1

$$2x(x + 7) = 0$$

$$2x = 0 \qquad x + 7 = 0$$

$$x = 0 \qquad x = -7$$

The solutions are 0 and -7.

You Try It 2

$$4x^2 - 9 = 0$$

$$(2x - 3)(2x + 3) = 0$$

$$2x - 3 = 0 \qquad 2x + 3 = 0$$

$$2x = 3 \qquad 2x = -3$$

$$x = \frac{3}{2} \qquad x = -\frac{3}{2}$$

The solutions are $\frac{3}{2}$ and $-\frac{3}{2}$.

You Try It 3

$$(x + 2)(x - 7) = 52$$

$$x^2 - 5x - 14 = 52$$

$$x^2 - 5x - 66 = 0$$

$$(x + 6)(x - 11) = 0$$

$$x + 6 = 0 \qquad x - 11 = 0$$

$$x = -6 \qquad x = 11$$

The solutions are -6 and 11.

You Try It 4

Strategy First positive consecutive integer: n
Second positive consecutive integer: $n + 1$

The sum of the squares of the two positive consecutive integers is 61.

Solution

$$n^2 + (n + 1)^2 = 61$$

$$n^2 + n^2 + 2n + 1 = 61$$

$$2n^2 + 2n + 1 = 61$$

$$2n^2 + 2n - 60 = 0$$

$$2(n^2 + n - 30) = 0$$

$$n^2 + n - 30 = 0$$

$$(n - 5)(n + 6) = 0$$

$$n - 5 = 0 \qquad n + 6 = 0$$

$$n = 5 \qquad n = -6$$

Because -6 is not a positive integer, it is not a solution.

$$n = 5$$

$$n + 1 = 5 + 1 = 6$$

The two integers are 5 and 6.

You Try It 5

Strategy Width $= x$
Length $= 2x + 4$

The area of the rectangle is 96 in^2. Use the equation $A = L \cdot W$.

Solution

$$A = L \cdot W$$

$$96 = (2x + 4)x$$

$$96 = 2x^2 + 4x$$

$$0 = 2x^2 + 4x - 96$$

$$0 = 2(x^2 + 2x - 48)$$

$$0 = x^2 + 2x - 48$$

$$0 = (x + 8)(x - 6)$$

$$x + 8 = 0 \qquad x - 6 = 0$$

$$x = -8 \qquad x = 6$$

Because the width cannot be a negative number, -8 is not a solution.

$$x = 6$$

$$2x + 4 = 2(6) + 4 = 12 + 4 = 16$$

The width is 6 in.
The length is 16 in.

Solutions to Chapter 8 "You Try It"

SECTION 8.1

You Try It 1

$$\frac{6x^5y}{12x^2y^3} = \frac{\overset{1}{2} \cdot \overset{1}{3} \cdot x^5y}{2 \cdot 2 \cdot 3 \cdot x^2y^3} = \frac{x^3}{2y^2}$$

You Try It 2

$$\frac{x^2 + 2x - 24}{16 - x^2} = \frac{x^2 + 2x - 24}{-(x^2 - 16)} = \frac{\overset{1}{(x - 4)}(x + 6)}{-(x - 4)(x + 4)}$$

$$= -\frac{x + 6}{x + 4}$$

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You Try It 3 $\dfrac{x^2+4x-12}{x^2-3x+2}=\dfrac{\overset{1}{\cancel{(x-2)}}(x+6)}{(x-1)\underset{1}{\cancel{(x-2)}}}$

$$=\frac{x+6}{x-1}$$

You Try It 4

$$\frac{12x^2+3x}{10x-15}\cdot\frac{8x-12}{9x+18}=\frac{3x(4x+1)}{5(2x-3)}\cdot\frac{4(2x-3)}{9(x+2)}$$

$$=\frac{\overset{1}{\cancel{3x}}(4x+1)\cdot 2\cdot 2\overset{1}{\cancel{(2x-3)}}}{5\underset{1}{\cancel{(2x-3)}}\cdot\underset{1}{\cancel{3}}\cdot 3(x+2)}$$

$$=\frac{4x(4x+1)}{15(x+2)}$$

You Try It 5

$$\frac{x^2+2x-15}{9-x^2}\cdot\frac{x^2-3x-18}{x^2-7x+6}$$

$$=\frac{(x-3)(x+5)}{(3-x)(3+x)}\cdot\frac{(x+3)(x-6)}{(x-1)(x-6)}$$

$$=\frac{\overset{-1}{\cancel{(x-3)}}(x+5)\overset{1}{\cancel{(x+3)}}\overset{1}{\cancel{(x-6)}}}{\underset{1}{\cancel{(3-x)}}\underset{1}{\cancel{(3+x)}}(x-1)\underset{1}{\cancel{(x-6)}}}=-\frac{x+5}{x-1}$$

You Try It 6

$$\frac{a^2}{4bc^2-2b^2c}\div\frac{a}{6bc-3b^2}=\frac{a^2}{4bc^2-2b^2c}\cdot\frac{6bc-3b^2}{a}$$

$$=\frac{a^2\cdot 3b\overset{1}{\cancel{(2c-b)}}}{2bc\underset{1}{\cancel{(2c-b)}}\cdot a}=\frac{3a}{2c}$$

You Try It 7

$$\frac{3x^2+26x+16}{3x^2-7x-6}\div\frac{2x^2+9x-5}{x^2+2x-15}$$

$$=\frac{3x^2+26x+16}{3x^2-7x-6}\cdot\frac{x^2+2x-15}{2x^2+9x-5}$$

$$=\frac{\overset{1}{\cancel{(3x+2)}}(x+8)\cdot\overset{1}{\cancel{(x+5)}}\overset{1}{\cancel{(x-3)}}}{\underset{1}{\cancel{(3x+2)}}\underset{1}{\cancel{(x-3)}}\cdot(2x-1)\underset{1}{\cancel{(x+5)}}}=\frac{x+8}{2x-1}$$

SECTION 8.2

You Try It 1

$8uv^2=2\cdot 2\cdot 2\cdot u\cdot v\cdot v$

$12uw=2\cdot 2\cdot 3\cdot u\cdot w$

LCM $=2\cdot 2\cdot 2\cdot 3\cdot u\cdot v\cdot v\cdot w=24uv^2w$

You Try It 2 $m^2-6m+9=(m-3)(m-3)$

$m^2-2m-3=(m+1)(m-3)$

LCM $=(m-3)(m-3)(m+1)$

You Try It 3

The LCM is $36xy^2z$.

$$\frac{x-3}{4xy^2}=\frac{x-3}{4xy^2}\cdot\frac{9z}{9z}=\frac{9xz-27z}{36xy^2z}$$

$$\frac{2x+1}{9y^2z}=\frac{2x+1}{9y^2z}\cdot\frac{4x}{4x}=\frac{8x^2+4x}{36xy^2z}$$

You Try It 4

The LCM is $(x+2)(x-5)(x+5)$

$$\frac{x+4}{x^2-3x-10}=\frac{x+4}{(x+2)(x-5)}\cdot\frac{x+5}{x+5}$$

$$=\frac{x^2+9x+20}{(x+2)(x-5)(x+5)}$$

$$\frac{2x}{25-x^2}=\frac{2x}{-(x^2-25)}=-\frac{2x}{(x-5)(x+5)}\cdot\frac{x+2}{x+2}$$

$$=-\frac{2x^2+4x}{(x+2)(x-5)(x+5)}$$

You Try It 5 $\dfrac{3}{xy}+\dfrac{12}{xy}=\dfrac{3+12}{xy}=\dfrac{15}{xy}$

You Try It 6

$$\frac{2x^2}{x^2-x-12}-\frac{7x+4}{x^2-x-12}$$

$$=\frac{2x^2-(7x+4)}{x^2-x-12}=\frac{2x^2-7x-4}{x^2-x-12}$$

$$=\frac{(2x+1)\overset{1}{\cancel{(x-4)}}}{(x+3)\underset{1}{\cancel{(x-4)}}}=\frac{2x+1}{x+3}$$

You Try It 7

$$\frac{x^2-1}{x^2-8x+12}-\frac{2x+1}{x^2-8x+12}+\frac{x}{x^2-8x+12}$$

$$=\frac{(x^2-1)-(2x+1)+x}{x^2-8x+12}=\frac{x^2-1-2x-1+x}{x^2-8x+12}$$

$$=\frac{x^2-x-2}{x^2-8x+12}=\frac{(x+1)\overset{1}{\cancel{(x-2)}}}{\underset{1}{\cancel{(x-2)}}(x-6)}=\frac{x+1}{x-6}$$

You Try It 8

The LCM of the denominators is $24y$.

$$\frac{z}{8y}-\frac{4z}{3y}+\frac{5z}{4y}=\frac{z}{8y}\cdot\frac{3}{3}-\frac{4z}{3y}\cdot\frac{8}{8}+\frac{5z}{4y}\cdot\frac{6}{6}$$

$$=\frac{3z}{24y}-\frac{32z}{24y}+\frac{30z}{24y}$$

$$=\frac{3z-32z+30z}{24y}=\frac{z}{24y}$$

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You Try It 9

$2 - x = -(x - 2)$

Therefore, $\dfrac{3}{2-x} = \dfrac{-3}{x-2}$.

The LCM is x 2.

$$\frac{5x}{x-2} - \frac{3}{2-x} = \frac{5x}{x-2} - \frac{-3}{x-2}$$

$$= \frac{5x-(-3)}{x-2} = \frac{5x+3}{x-2}$$

You Try It 10

The LCM is $(3x - 1)(x + 4)$.

$$\frac{4x}{3x-1} - \frac{9}{x+4} = \frac{4x}{3x-1}\cdot\frac{x+4}{x+4} - \frac{9}{x+4}\cdot\frac{3x-1}{3x-1}$$

$$= \frac{4x^2+16x}{(3x-1)(x+4)} - \frac{27x-9}{(3x-1)(x+4)}$$

$$= \frac{(4x^2+16x)-(27x-9)}{(3x-1)(x+4)}$$

$$= \frac{4x^2+16x-27x+9}{(3x-1)(x+4)}$$

$$= \frac{4x^2-11x+9}{(3x-1)(x+4)}$$

You Try It 11

The LCM is $x - 3$.

$$2 - \frac{1}{x-3} = \frac{2}{1}\cdot\frac{x-3}{x-3} - \frac{1}{x-3}$$

$$= \frac{2x-6}{x-3} - \frac{1}{x-3}$$

$$= \frac{2x-6-1}{x-3}$$

$$= \frac{2x-7}{x-3}$$

You Try It 12

The LCM is $a(a - 5)(a + 5)$.

$$\frac{a-3}{a^2-5a} + \frac{a-9}{a^2-25}$$

$$= \frac{a-3}{a(a-5)}\cdot\frac{a+5}{a+5} + \frac{a-9}{(a-5)(a+5)}\cdot\frac{a}{a}$$

$$= \frac{(a-3)(a+5)+a(a-9)}{a(a-5)(a+5)}$$

$$= \frac{(a^2+2a-15)+(a^2-9a)}{a(a-5)(a+5)}$$

$$= \frac{a^2+2a-15+a^2-9a}{a(a-5)(a+5)}$$

$$= \frac{2a^2-7a-15}{a(a-5)(a+5)} = \frac{(2a+3)(a-5)}{a(a-5)(a+5)}$$

$$= \frac{(2a+3)\cancel{(a-5)}}{a\cancel{(a-5)}(a+5)} = \frac{2a+3}{a(a+5)}$$

You Try It 13

The LCM is $(3x + 2)(x - 1)$.

$$\frac{2x-3}{3x^2-x-2} + \frac{5}{3x+2} - \frac{1}{x-1}$$

$$= \frac{2x-3}{(3x+2)(x-1)} + \frac{5}{3x+2}\cdot\frac{x-1}{x-1} - \frac{1}{x-1}\cdot\frac{3x+2}{3x+2}$$

$$= \frac{2x-3}{(3x+2)(x-1)} + \frac{5x-5}{(3x+2)(x-1)} - \frac{3x+2}{(3x+2)(x-1)}$$

$$= \frac{(2x-3)+(5x-5)-(3x+2)}{(3x+2)(x-1)}$$

$$= \frac{2x-3+5x-5-3x-2}{(3x+2)(x-1)}$$

$$= \frac{4x-10}{(3x+2)(x-1)} = \frac{2(2x-5)}{(3x+2)(x-1)}$$

SECTION 8.3

You Try It 1

The LCM of 3, x, 9, and x^2 is $9x^2$.

$$\frac{\frac{1}{3}-\frac{1}{x}}{\frac{1}{9}-\frac{1}{x^2}} = \frac{\frac{1}{3}-\frac{1}{x}}{\frac{1}{9}-\frac{1}{x^2}}\cdot\frac{9x^2}{9x^2} = \frac{\frac{1}{3}\cdot 9x^2 - \frac{1}{x}\cdot 9x^2}{\frac{1}{9}\cdot 9x^2 - \frac{1}{x^2}\cdot 9x^2}$$

$$= \frac{3x^2-9x}{x^2-9} = \frac{3x\cancel{(x-3)}}{\cancel{(x-3)}(x+3)} = \frac{3x}{x+3}$$

You Try It 2

The LCM is $x - 5$.

$$\frac{x+3-\dfrac{20}{x-5}}{x+8+\dfrac{30}{x+5}} = \frac{x+3-\dfrac{20}{x-5}}{x+8+\dfrac{30}{x-5}}\cdot\frac{x-5}{x-5}$$

$$= \frac{x(x-5)+3(x-5)-\dfrac{20}{x-5}\cdot(x-5)}{x(x-5)+8(x-5)+\dfrac{30}{x-5}\cdot(x-5)}$$

$$= \frac{x^2-5x+3x-15-20}{x^2-5x+8x-40+30}$$

$$= \frac{x^2-2x-35}{x^2+3x-10}$$

$$= \frac{\cancel{(x+5)}(x-7)}{(x-2)\cancel{(x+5)}} = \frac{x-7}{x-2}$$

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SECTION 8.4

You Try It 1

$$\frac{x}{x+6} = \frac{3}{x} \quad \text{The LCM is } x(x+6).$$

$$\frac{\cancel{x(x+6)}}{1} \cdot \frac{x}{\cancel{x+6}} = \frac{x(x+6)}{1} \cdot \frac{3}{x}$$

$$x^2 = (x+6)3$$
$$x^2 = 3x + 18$$
$$x^2 - 3x - 18 = 0$$
$$(x+3)(x-6) = 0$$

$$x + 3 = 0 \qquad x - 6 = 0$$
$$x = -3 \qquad x = 6$$

Both -3 and 6 check as solutions.
The solutions are -3 and 6.

You Try It 2

$$\frac{5x}{x+2} = 3 - \frac{10}{x+2} \quad \text{The LCM is } x+2.$$

$$\frac{(x+2)}{1} \cdot \frac{5x}{x+2} = \frac{(x+2)}{1}\left(3 - \frac{10}{x+2}\right)$$

$$\frac{\cancel{x+2}}{1} \cdot \frac{5x}{\cancel{x+2}} = (x+2)3 - \frac{\cancel{(x+2)}}{1} \cdot \frac{10}{\cancel{x+2}}$$

$$5x = (x+2)3 - 10$$
$$5x = 3x + 6 - 10$$
$$5x = 3x - 4$$
$$2x = -4$$
$$x = -2$$

-2 does not check as a solution.
The equation has no solution.

You Try It 3

$$\frac{2}{x+3} = \frac{6}{5x+5}$$

$$\frac{(x+3)(5x+5)}{1} \cdot \frac{2}{x+3} = \frac{(x+3)(5x+5)}{1} \cdot \frac{6}{5x+5}$$

$$\frac{\cancel{(x+3)}(5x+5)}{1} \cdot \frac{2}{\cancel{x+3}} = \frac{(x+3)\cancel{(5x+5)}}{1} \cdot \frac{6}{\cancel{5x+5}}$$

$$(5x+5)2 = (x+3)6$$
$$10x + 10 = 6x + 18$$
$$4x + 10 = 18$$
$$4x = 8$$
$$x = 2$$

The solution is 2.

You Try It 4

Strategy To find the area of triangle AOB:

- Solve a proportion to find the length of AO (the height of triangle AOB).
- Use the formula for the area of a triangle. AB is the base and AO is the height.

Solution

$$\frac{CD}{AB} = \frac{DO}{AO}$$
$$\frac{4}{10} = \frac{3}{AO}$$
$$10 \cdot AO \cdot \frac{4}{10} = 10 \cdot AO \cdot \frac{3}{AO}$$
$$4(AO) = 30$$
$$AO = 7.5$$

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(10)(7.5)$$
$$= 37.5$$

The area of triangle AOB is 37.5 cm^2.

You Try It 5

Strategy To find the total area that 256 ceramic tiles will cover, write and solve a proportion, using x to represent the number of square feet that 256 tiles will cover.

Solution

$$\frac{9}{16} = \frac{x}{256}$$
$$256\left(\frac{9}{16}\right) = 256\left(\frac{x}{256}\right)$$
$$144 = x$$

A 144-square-foot area can be tiled using 256 ceramic tiles.

You Try It 6

Strategy To find the additional amount of medication required for a 200-pound adult, write and solve a proportion, using x to represent the additional medication. Then $3 + x$ is the total amount required for a 200-pound adult.

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Solution

$$\frac{150}{3} = \frac{200}{3 + x}$$

$$\frac{50}{1} = \frac{200}{3 + x}$$

$$(3 + x) \cdot 50 = (3 + x) \cdot \frac{200}{3 + x}$$

$$150 + 50x = 200$$

$$50x = 50$$

$$x = 1$$

One additional ounce is required for a 200-pound adult.

SECTION 8.5

You Try It 1

$$A = P + Prt$$

$$A - P = P - P + Prt$$

$$A - P = Prt$$

$$\frac{A - P}{Pt} = \frac{Prt}{Pt}$$

$$\frac{A - P}{Pt} = r$$

You Try It 2

$$s = \frac{A + L}{2}$$

$$2 \cdot s = 2\left(\frac{A + L}{2}\right)$$

$$2s = A + L$$

$$2s - A = A - A + L$$

$$2s - A = L$$

You Try It 3

$$s = a + (n - 1)d$$

$$s = a + nd - d$$

$$s - a = a - a + nd - d$$

$$s - a = nd - d$$

$$s - a + d = nd - d + d$$

$$s - a + d = nd$$

$$\frac{s - a + d}{d} = \frac{nd}{d}$$

$$\frac{s - a + d}{d} = n$$

You Try It 4

$$S = C + rC$$

$$S = (1 + r)C$$

$$\frac{S}{1 + r} = \frac{(1 + r)C}{1 + r}$$

$$\frac{S}{1 + r} = C$$

SECTION 8.6

You Try It 1

Strategy

- Time for one printer to complete the job: t

	Rate	*Time*	*Part*
1st printer	$\frac{1}{t}$	2	$\frac{2}{t}$
2nd printer	$\frac{1}{t}$	5	$\frac{5}{t}$

The sum of the parts of the task completed must equal 1.

Solution

$$\frac{2}{t} + \frac{5}{t} = 1$$

$$t\left(\frac{2}{t} + \frac{5}{t}\right) = t \cdot 1$$

$$2 + 5 = t$$

$$7 = t$$

Working alone, one printer takes 7 h to print the payroll.

You Try It 2

Strategy

- Rate sailing across the lake: r
 Rate sailing back: $3r$

	Distance	*Rate*	*Time*
Across	6	r	$\frac{6}{r}$
Back	6	$3r$	$\frac{6}{3r}$

- The total time for the trip was 2 h.

Solution

$$\frac{6}{r} + \frac{6}{3r} = 2$$

$$3r\left(\frac{6}{r} + \frac{6}{3r}\right) = 3r(2)$$

$$3r \cdot \frac{6}{r} + 3r \cdot \frac{6}{3r} = 6r$$

$$18 + 6 = 6r$$

$$24 = 6r$$

$$4 = r$$

The rate across the lake was 4 km/h.

SECTION 8.7

You Try It 1

Strategy To find the distance:

- Write the basic direct variation equation, replace the variables by the given values, and solve for k.

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- Write the direct variation equation, replacing k by its value. Substitute 5 for t and solve for s.

Solution

$$s = kt^2$$
$$64 = k(2)^2$$
$$64 = k \cdot 4$$
$$16 = k$$
$$s = 16t^2 = 16(5)^2 = 400$$

The object will fall 400 ft in 5 s.

You Try It 2

Strategy To find the resistance:

- Write the basic inverse variation equation, replace the variables by the given values, and solve for k.
- Write the inverse variation equation, replacing k by its value. Substitute 0.02 for d and solve for R.

Solution

$$R = \frac{k}{d^2}$$
$$0.5 = \frac{k}{(0.01)^2}$$
$$0.5 = \frac{k}{0.0001}$$
$$0.00005 = k$$
$$R = \frac{0.00005}{d^2} = \frac{0.00005}{(0.02)^2} = 0.125$$

The resistance is 0.125 ohm.

You Try It 3

Strategy To find the strength:

- Write the basic combined variation equation, replace the variables by the given values, and solve for k.
- Write the combined variation equation, replacing k by its value. Substitute 4 for w, 8 for d, and 16 for L, and solve for s.

Solution

$$s = \frac{kwd^2}{L}$$
$$1200 = \frac{k \cdot 2 \cdot (12)^2}{12}$$
$$1200 = \frac{k \cdot 2 \cdot 144}{12}$$
$$1200 = 24k$$
$$50 = k$$
$$s = \frac{50wd^2}{L} = \frac{50 \cdot 4 \cdot (8)^2}{16} = 800$$

The strength is 800 lb.

Solutions to Chapter 9 "You Try It"

SECTION 9.1

You Try It 1
$$16^{-3/4} = (2^4)^{-3/4}$$
$$= 2^{-3}$$
$$= \frac{1}{2^3} = \frac{1}{8}$$

You Try It 2 $(-81)^{3/4}$

The base of the exponential expression is negative, and the denominator of the exponent is a positive even number.

Therefore, $(-81)^{3/4}$ is not a real number.

You Try It 3
$$(x^{3/4}y^{1/2}z^{-2/3})^{-4/3} = x^{-1}y^{-2/3}z^{8/9}$$
$$= \frac{z^{8/9}}{xy^{2/3}}$$

You Try It 4
$$\left(\frac{16a^{-2}b^{4/3}}{9a^4b^{-2/3}}\right)^{-1/2} = \left(\frac{2^4a^{-6}b^2}{3^2}\right)^{-1/2}$$
$$= \frac{2^{-2}a^3b^{-1}}{3^{-1}}$$
$$= \frac{3a^3}{2^2b} = \frac{3a^3}{4b}$$

You Try It 5
$$(2x^3)^{3/4} = \sqrt[4]{(2x^3)^3}$$
$$= \sqrt[4]{8x^9}$$

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You Try It 6 $-5a^{5/6} = -5(a^5)^{1/6}$
$= -5\sqrt[6]{a^5}$

You Try It 7 $\sqrt[3]{3ab} = (3ab)^{1/3}$

You Try It 8 $\sqrt[4]{x^4 + y^4} = (x^4 + y^4)^{1/4}$

You Try It 9 $\sqrt[3]{-8x^{12}y^3} = -2x^4y$

You Try It 10 $-\sqrt[4]{81x^{12}y^8} = -3x^3y^2$

SECTION 9.2

You Try It 1
$$\sqrt[5]{x^7} = \sqrt[5]{x^5 \cdot x^2} = \sqrt[5]{x^5}\,\sqrt[5]{x^2} = x\sqrt[5]{x^2}$$

You Try It 2
$$\sqrt[3]{-64x^8y^{18}} = \sqrt[3]{(-4)^3x^8y^{18}} = \sqrt[3]{(-4)^3x^6y^{18}(x^2)} = \sqrt[3]{(-4)^3x^6y^{18}}\,\sqrt[3]{x^2} = -4x^2y^6\sqrt[3]{x^2}$$

You Try It 3
$$3xy\sqrt[3]{81x^5y} - \sqrt[3]{192x^8y^4} = 3xy\sqrt[3]{27x^3}\,\sqrt[3]{3x^2y} - \sqrt[3]{64x^6y^3}\,\sqrt[3]{3x^2y} = 3xy \cdot 3x\,\sqrt[3]{3x^2y} - 4x^2y\sqrt[3]{3x^2y} = 9x^2y\sqrt[3]{3x^2y} - 4x^2y\sqrt[3]{3x^2y} = 5x^2y\sqrt[3]{3x^2y}$$

You Try It 4
$$\sqrt{5b}(\sqrt{3b} - \sqrt{10}) = \sqrt{15b^2} - \sqrt{50b} = \sqrt{15b^2} - \sqrt{25 \cdot 2b} = \sqrt{b^2}\sqrt{15} - \sqrt{25}\sqrt{2b} = b\sqrt{15} - 5\sqrt{2b}$$

You Try It 5
$$(2\sqrt[3]{2x} - 3)(\sqrt[3]{2x} - 5) = 2\sqrt[3]{4x^2} - 10\sqrt[3]{2x} - 3\sqrt[3]{2x} + 15 = 2\sqrt[3]{4x^2} - 13\sqrt[3]{2x} + 15$$

You Try It 6
$$(\sqrt{a} - 3\sqrt{y})(\sqrt{a} + 3\sqrt{y}) = (\sqrt{a})^2 - (3\sqrt{y})^2 = a - 9y$$

You Try It 7
$$\frac{y}{\sqrt{3y}} = \frac{y}{\sqrt{3y}} \cdot \frac{\sqrt{3y}}{\sqrt{3y}} = \frac{y\sqrt{3y}}{(\sqrt{3y})^2} = \frac{y\sqrt{3y}}{3y} = \frac{\sqrt{3y}}{3}$$

You Try It 8
$$\frac{3x}{\sqrt[3]{3x^2}} = \frac{3x}{\sqrt[3]{3x^2}} \cdot \frac{\sqrt[3]{9x}}{\sqrt[3]{9x}} = \frac{3x\sqrt[3]{9x}}{\sqrt[3]{27x^3}} = \frac{3x\sqrt[3]{9x}}{3} = \sqrt[3]{9x}$$

You Try It 9
$$\frac{3 + \sqrt{6}}{2 - \sqrt{6}} = \frac{3 + \sqrt{6}}{2 - \sqrt{6}} \cdot \frac{2 + \sqrt{6}}{2 + \sqrt{6}} = \frac{6 + 3\sqrt{6} + 2\sqrt{6} + (\sqrt{6})^2}{2^2 - (\sqrt{6})^2} = \frac{6 + 5\sqrt{6} + 6}{4 - 6} = \frac{12 + 5\sqrt{6}}{-2} = -\frac{12 + 5\sqrt{6}}{2}$$

SECTION 9.3

You Try It 1 $\sqrt{-45} = i\sqrt{45} = i\sqrt{9 \cdot 5} = 3i\sqrt{5}$

You Try It 2
$$\sqrt{98} - \sqrt{-60} = \sqrt{98} - i\sqrt{60} = \sqrt{49 \cdot 2} - i\sqrt{4 \cdot 15} = 7\sqrt{2} - 2i\sqrt{15}$$

You Try It 3 $(-4 + 2i) - (6 - 8i) = -10 + 10i$

You Try It 4
$$(16 - \sqrt{-45}) - (3 + \sqrt{-20}) = (16 - i\sqrt{45}) - (3 + i\sqrt{20}) = (16 - i\sqrt{9 \cdot 5}) - (3 + i\sqrt{4 \cdot 5}) = (16 - 3i\sqrt{5}) - (3 + 2i\sqrt{5}) = 13 - 5i\sqrt{5}$$

You Try It 5 $(3 - 2i) + (-3 + 2i) = 0 + 0i = 0$

You Try It 6 $(-3i)(-10i) = 30i^2 = 30(-1) = -30$

You Try It 7
$$-\sqrt{-8} \cdot \sqrt{-5} = -i\sqrt{8} \cdot i\sqrt{5} = -i^2\sqrt{40} = -(-1)\sqrt{40} = \sqrt{4 \cdot 10} = 2\sqrt{10}$$

You Try It 8
$$-6i(3 + 4i) = -18i - 24i^2 = -18i - 24(-1) = 24 - 18i$$

You Try It 9
$$\sqrt{-3}(\sqrt{27} - \sqrt{-6}) = i\sqrt{3}(\sqrt{27} - i\sqrt{6}) = i\sqrt{81} - i^2\sqrt{18} = i\sqrt{81} - (-1)\sqrt{9 \cdot 2} = 9i + 3\sqrt{2} = 3\sqrt{2} + 9i$$

You Try It 10
$$(4 - 3i)(2 - i) = 8 - 4i - 6i + 3i^2 = 8 - 10i + 3i^2 = 8 - 10i + 3(-1) = 5 - 10i$$

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You Try It 11 $(3 + 6i)(3 - 6i) = 3^2 + 6^2$
$= 9 + 36$
$= 45$

You Try It 12

$$(3 - i)\left(\frac{3}{10} + \frac{1}{10}i\right) = \frac{9}{10} + \frac{3}{10}i - \frac{3}{10}i - \frac{1}{10}i^2$$
$$= \frac{9}{10} - \frac{1}{10}i^2 = \frac{9}{10} - \frac{1}{10}(-1)$$
$$= \frac{9}{10} + \frac{1}{10} = 1$$

You Try It 13

$$\frac{2 - 3i}{4i} = \frac{2 - 3i}{4i} \cdot \frac{i}{i}$$
$$= \frac{2i - 3i^2}{4i^2}$$
$$= \frac{2i - 3(-1)}{4(-1)}$$
$$= \frac{3 + 2i}{-4} = -\frac{3}{4} - \frac{1}{2}i$$

You Try It 14

$$\frac{2 + 5i}{3 - 2i} = \frac{2 + 5i}{3 - 2i} \cdot \frac{3 + 2i}{3 + 2i} = \frac{6 + 4i + 15i + 10i^2}{3^2 + 2^2}$$
$$= \frac{6 + 19i + 10(-1)}{13} = \frac{-4 + 19i}{13}$$
$$= -\frac{4}{13} + \frac{19}{13}i$$

SECTION 9.4

You Try It 1

$\sqrt[4]{x - 8} = 3$
$(\sqrt[4]{x - 8})^4 = 3^4$
$x - 8 = 81$
$x = 89$

Check:

$\sqrt[4]{x - 8} = 3$

$\sqrt[4]{89 - 8}$	3
$\sqrt[4]{81}$	3
$3 = 3$	

The solution is 89.

You Try It 2

$\sqrt{x} - \sqrt{x + 5} = 1$
$\sqrt{x} = 1 + \sqrt{x + 5}$
$(\sqrt{x})^2 = (1 + \sqrt{x + 5})^2$
$x = 1 + 2\sqrt{x + 5} + x + 5$
$0 = 6 + 2\sqrt{x + 5}$
$-6 = 2\sqrt{x + 5}$
$-3 = \sqrt{x + 5}$
$(-3)^2 = (\sqrt{x + 5})^2$
$9 = x + 5$
$4 = x$

4 does not check as a solution. The equation has no solution.

You Try It 3

Strategy To find the diagonal, use the Pythagorean Theorem. One leg is the length of the rectangle. The second leg is the width of the rectangle. The hypotenuse is the diagonal of the rectangle.

Solution
$c^2 = a^2 + b^2$
$c^2 = (6)^2 + (3)^2$
$c^2 = 36 + 9$
$c^2 = 45$
$(c^2)^{1/2} = (45)^{1/2}$
$c = \sqrt{45}$
$c \approx 6.7$

The diagonal is 6.7 cm.

You Try It 4

Strategy To find the height, replace d in the equation with the given value and solve for h.

Solution
$d = \sqrt{1.5h}$
$5.5 = \sqrt{1.5h}$
$(5.5)^2 = (\sqrt{1.5h})^2$
$30.25 = 1.5h$
$20.17 \approx h$

The periscope must be approximately 20.17 ft above the water.

You Try It 5

Strategy To find the distance, replace the variables v and a in the equation by their given values and solve for s.

Solution
$v = \sqrt{2as}$
$88 = \sqrt{2 \cdot 22s}$
$88 = \sqrt{44s}$
$(88)^2 = (\sqrt{44s})^2$
$7744 = 44s$
$176 = s$

The distance required is 176 ft.

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Solutions to Chapter 10 "You Try It"

SECTION 10.1

You Try It 1

$$2x^2 = 7x - 3$$
$$2x^2 - 7x + 3 = 0$$
$$(2x - 1)(x - 3) = 0$$

$$2x - 1 = 0 \qquad x - 3 = 0$$
$$2x = 1 \qquad x = 3$$
$$x = \frac{1}{2}$$

The solutions are $\frac{1}{2}$ and 3.

You Try It 2

$$x^2 - 3ax - 4a^2 = 0$$
$$(x + a)(x - 4a) = 0$$

$$x + a = 0 \qquad x - 4a = 0$$
$$x = -a \qquad x = 4a$$

The solutions are $-a$ and $4a$.

You Try It 3

$$(x - r_1)(x - r_2) = 0$$
$$(x - 3)\left[x - \left(-\frac{1}{2}\right)\right] = 0$$
$$(x - 3)\left(x + \frac{1}{2}\right) = 0$$
$$x^2 - \frac{5}{2}x - \frac{3}{2} = 0$$
$$2\left(x^2 - \frac{5}{2}x - \frac{3}{2}\right) = 2 \cdot 0$$
$$2x^2 - 5x - 3 = 0$$

You Try It 4

$$2(x + 1)^2 - 24 = 0$$
$$2(x + 1)^2 = 24$$
$$(x + 1)^2 = 12$$
$$\sqrt{(x + 1)^2} = \sqrt{12}$$
$$x + 1 = \pm 2\sqrt{3}$$

$$x + 1 = 2\sqrt{3} \qquad x + 1 = -2\sqrt{3}$$
$$x = -1 + 2\sqrt{3} \qquad x = -1 - 2\sqrt{3}$$

The solutions are $-1 + 2\sqrt{3}$ and $-1 - 2\sqrt{3}$.

SECTION 10.2

You Try It 1

$$4x^2 - 4x - 1 = 0$$
$$4x^2 - 4x = 1$$
$$\frac{1}{4}(4x^2 - 4x) = \frac{1}{4} \cdot 1$$
$$x^2 - x = \frac{1}{4}$$

Complete the square.

$$x^2 - x + \frac{1}{4} = \frac{1}{4} + \frac{1}{4}$$
$$\left(x - \frac{1}{2}\right)^2 = \frac{2}{4}$$
$$\sqrt{\left(x - \frac{1}{2}\right)^2} = \sqrt{\frac{2}{4}}$$
$$x - \frac{1}{2} = \pm\frac{\sqrt{2}}{2}$$
$$x - \frac{1}{2} = \frac{\sqrt{2}}{2} \qquad x - \frac{1}{2} = -\frac{\sqrt{2}}{2}$$
$$x = \frac{1}{2} + \frac{\sqrt{2}}{2} \qquad x = \frac{1}{2} - \frac{\sqrt{2}}{2}$$

The solutions are $\frac{1 + \sqrt{2}}{2}$ and $\frac{1 - \sqrt{2}}{2}$.

You Try It 2

$$2x^2 + x - 5 = 0$$
$$2x^2 + x = 5$$
$$\frac{1}{2}(2x^2 + x) = \frac{1}{2} \cdot 5$$
$$x^2 + \frac{1}{2}x = \frac{5}{2}$$

Complete the square.

$$x^2 + \frac{1}{2}x + \frac{1}{16} = \frac{5}{2} + \frac{1}{16}$$
$$\left(x + \frac{1}{4}\right)^2 = \frac{41}{16}$$
$$\sqrt{\left(x + \frac{1}{4}\right)^2} = \sqrt{\frac{41}{16}}$$
$$x + \frac{1}{4} = \pm\frac{\sqrt{41}}{4}$$
$$x + \frac{1}{4} = \frac{\sqrt{41}}{4} \qquad x + \frac{1}{4} = -\frac{\sqrt{41}}{4}$$
$$x = -\frac{1}{4} + \frac{\sqrt{41}}{4} \qquad x = -\frac{1}{4} - \frac{\sqrt{41}}{4}$$

The solutions are $\frac{-1 + \sqrt{41}}{4}$ and $\frac{-1 - \sqrt{41}}{4}$.

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SECTION 10.3

You Try It 1

$$x^2 - 2x + 10 = 0$$
$$a = 1, b = -2, c = 10$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(10)}}{2 \cdot 1}$$
$$= \frac{2 \pm \sqrt{4 - 40}}{2} = \frac{2 \pm \sqrt{-36}}{2}$$
$$= \frac{2 \pm 6i}{2} = 1 \pm 3i$$

The solutions are $1 + 3i$ and $1 - 3i$.

You Try It 2

$$4x^2 = 4x - 1$$
$$4x^2 - 4x + 1 = 0$$
$$a = 4, b = -4, c = 1$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(1)}}{2 \cdot 4}$$
$$= \frac{4 \pm \sqrt{16 - 16}}{8} = \frac{4 \pm \sqrt{0}}{8}$$
$$= \frac{4}{8} = \frac{1}{2}$$

The solution is $\frac{1}{2}$.

You Try It 3

$$3x^2 - x - 1 = 0$$
$$a = 3, b = -1, c = -1$$
$$b^2 - 4ac = (-1)^2 - 4(3)(-1)$$
$$= 1 + 12 = 13$$
$$13 > 0$$

Because the discriminant is greater than zero, the equation has two real number solutions.

SECTION 10.4

You Try It 1

$$x - 5x^{1/2} + 6 = 0$$
$$(x^{1/2})^2 - 5(x^{1/2}) + 6 = 0$$
$$u^2 - 5u + 6 = 0$$
$$(u - 2)(u - 3) = 0$$
$$u - 2 = 0 \qquad u - 3 = 0$$
$$u = 2 \qquad u = 3$$

Replace u by $x^{1/2}$.

$$x^{1/2} = 2 \qquad x^{1/2} = 3$$
$$\sqrt{x} = 2 \qquad \sqrt{x} = 3$$
$$(\sqrt{x})^2 = 2^2 \qquad (\sqrt{x})^2 = 3^2$$
$$x = 4 \qquad x = 9$$

The solutions are 4 and 9.

You Try It 2

$$\sqrt{2x + 1} + x = 7$$
$$\sqrt{2x + 1} = 7 - x$$
$$(\sqrt{2x + 1})^2 = (7 - x)^2$$
$$2x + 1 = 49 - 14x + x^2$$
$$0 = x^2 - 16x + 48$$
$$0 = (x - 4)(x - 12)$$
$$x - 4 = 0 \qquad x - 12 = 0$$
$$x = 4 \qquad x = 12$$

4 checks as a solution.
12 does not check as a solution.

The solution is 4.

You Try It 3

$$\sqrt{2x - 1} + \sqrt{x} = 2$$

Solve for one of the radical expressions.

$$\sqrt{2x - 1} = 2 - \sqrt{x}$$
$$(\sqrt{2x - 1})^2 = (2 - \sqrt{x})^2$$
$$2x - 1 = 4 - 4\sqrt{x} + x$$
$$x - 5 = -4\sqrt{x}$$

Square each side of the equation.

$$(x - 5)^2 = (-4\sqrt{x})^2$$
$$x^2 - 10x + 25 = 16x$$
$$x^2 - 26x + 25 = 0$$
$$(x - 1)(x - 25) = 0$$
$$x - 1 = 0 \qquad x - 25 = 0$$
$$x = 1 \qquad x = 25$$

1 checks as a solution.
25 does not check as a solution.

The solution is 1.

You Try It 4

$$3y + \frac{25}{3y - 2} = -8$$
$$(3y - 2)\left(3y + \frac{25}{3y - 2}\right) = (3y - 2)(-8)$$
$$(3y - 2)(3y) + (3y - 2)\left(\frac{25}{3y - 2}\right) = (3y - 2)(-8)$$
$$9y^2 - 6y + 25 = -24y + 16$$
$$9y^2 + 18y + 9 = 0$$
$$9(y^2 + 2y + 1) = 0$$
$$y^2 + 2y + 1 = 0$$
$$(y + 1)(y + 1) = 0$$
$$y + 1 = 0 \qquad y + 1 = 0$$
$$y = -1 \qquad y = -1$$

The solution is -1.

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SECTION 10.5

You Try It 1

Strategy

- This is a geometry problem.
- Width of the rectangle: W
 Length of the rectangle: $W + 3$
- Use the equation $A = L \cdot W$.

Solution

$$A = L \cdot W$$
$$54 = (W + 3)(W)$$
$$54 = W^2 + 3W$$
$$0 = W^2 + 3W - 54$$
$$0 = (W + 9)(W - 6)$$

$$W + 9 = 0 \qquad W - 6 = 0$$
$$W = -9 \qquad W = 6$$

The solution -9 is not possible.

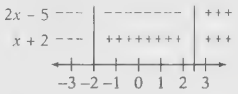
$$W + 3 = 6 + 3 = 9$$

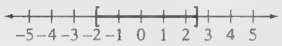
The length is 9 m.

SECTION 10.6

You Try It 1

$$2x^2 - x - 10 \le 0$$
$$(2x - 5)(x + 2) \le 0$$





$$\left\{x \mid -2 \le x \le \frac{5}{2}\right\}$$

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Answers to Chapter 6 Selected Exercises

PREP TEST

1. 1 [1.1C] **2.** -18 [1.1D] **3.** $\frac{2}{3}$ [1.1D] **4.** $-12y$ [1.4C] **5.** -8 [1.2E] **6.** $3a - 8b$ [1.4B]

7. $11x - 2y - 2$ [1.4D] **8.** 0 [1.4B] **9.** No [1.4B]

SECTION 6.1

1. **a** is the multiplication of two exponential expressions. **b** is the power of an exponential expression. **3.** a^4b^4

5. $-18x^3y^4$ **7.** x^8y^{16} **9.** $81x^8y^{12}$ **11.** $729a^{10}b^6$ **13.** x^5y^{11} **15.** $729x^6$ **17.** $a^{18}b^{18}$ **19.** $4096x^{12}y^{12}$

21. $64a^{24}b^{18}$ **23.** x^{2n+1} **25.** y^{6n-2} **27.** a^{2n^2-6n} **29.** x^{15n+10} **31.** $-6x^5y^5z^4$ **33.** $-12a^2b^9c^2$

35. $-6x^4y^4z^5$ **37.** $-432a^7b^{11}$ **39.** $54a^{13}b^{17}$ **41.** 243 **43.** y^3 **45.** $\frac{a^3b^2}{4}$ **47.** $\frac{x}{y^4}$ **49.** $\frac{1}{2}$ **51.** $-\frac{1}{9}$ **53.** $\frac{1}{y^8}$

55. $\frac{1}{x^6y^{10}}$ **57.** x^5y^5 **59.** $\frac{a^8}{b^9}$ **61.** $\frac{1}{2187a}$ **63.** $\frac{y^6}{x^3}$ **65.** $\frac{y^4}{x^3}$ **67.** $-\frac{1}{2x^2}$ **69.** $-\frac{1}{243a^5b^{10}}$ **71.** $\frac{16x^8}{81y^4z^{12}}$

73. $\frac{3a^4}{4b^3}$ **75.** $-\frac{9a}{8b^6}$ **77.** $\frac{16x^2}{y^6}$ **79.** $\frac{1}{b^{4n}}$ **81.** $-\frac{1}{y^{6n}}$ **83.** y^{n-2} **85.** $\frac{1}{y^{2n}}$ **87.** $\frac{x^{n-5}}{y^6}$ **89.** $\frac{8b^{15}}{3a^{18}}$

91. 4.67×10^{-6} **93.** 1.7×10^{-10} **95.** 2×10^{11} **97.** 0.000000123 **99.** 8,200,000,000,000,000 **101.** 0.039

103. 150,000 **105.** 20,800,000 **107.** 0.000000015 **109.** 0.0000000000178 **111.** 140,000,000 **113.** 11,000,000

115. 0.000008 **117.** Light travels 2.592×10^{10} km in one day. **119.** The signals traveled at $1.0\overline{81} \times 10^7$ mi/min.

121. The centrifuge makes one revolution in $6.\overline{6} \times 10^{-7}$ s. **123.** The sun is 3.38983×10^5 times heavier than Earth.

125. One orchid seed weighs 3.225806×10^{-8} oz. **127.** The Coma cluster is 1.6423949×10^{21} mi from Earth.

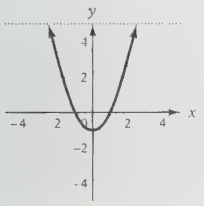
SECTION 6.2

1. $P(3) = 13$ **3.** $R(2) = 10$ **5.** $f(-1) = -11$ **7.a.** -1 **b.** 8 **c.** 2

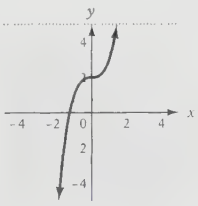
9. The expression is not a polynomial function. **11.** The expression is not a polynomial function.

13.a. 3 **b.** π **c.** 5 **15.a.** -5 **b.** 2 **c.** 3 **17.a.** 14 **b.** 14 **c.** 0

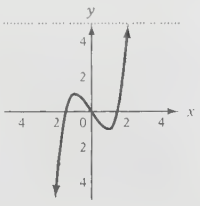
19.



21.



23.



25. $6x^2 - 6x + 5$

27. $-x^2 + 1$ **29.** $5y^2 - 15y + 2$ **31.** $7a^2 - a + 2$ **33.** $P(x) + R(x) = 3x^2 - 3x - 2$

35. $P(x) - R(x) = 8x^2 - 2x + 5$ **37.** $S(x) = 3x^4 - 8x^2 + 2x$; $S(2) = 20$ **39.a.** $k = 8$ **b.** $k = -4$

SECTION 6.3

1. $x^2 - 2x$ **3.** $-x^2 - 7x$ **5.** $3a^3 - 6a^2$ **7.** $-5x^4 + 5x^3$ **9.** $-3x^5 + 7x^3$ **11.** $12x^3 - 6x^2$ **13.** $6x^2 - 12x$

15. $3x^2 + 4x$ **17.** $-x^3y + xy^3$ **19.** $2x^4 - 3x^2 + 2x$ **21.** $2a^3 + 3a^2 + 2a$ **23.** $3x^6 - 3x^4 - 2x^2$

25. $-6y^4 - 12y^3 + 14y^2$ **27.** $-2a^3 - 6a^2 + 8a$ **29.** $6y^4 - 3y^3 + 6y^2$ **31.** $x^3y - 3x^2y^2 + xy^3$

33. $x^3 + 4x^2 + 5x + 2$ **35.** $a^3 - 6a^2 + 13a - 12$ **37.** $-2b^3 + 7b^2 + 19b - 20$ **39.** $-6x^3 + 31x^2 - 41x + 10$

41. $x^3 - 3x^2 + 5x - 15$ **43.** $x^4 - 4x^3 - 3x^2 + 14x - 8$ **45.** $15y^3 - 16y^2 - 70y + 16$

47. $5a^4 - 20a^3 - 5a^2 + 22a - 8$ **49.** $y^4 + 4y^3 + y^2 - 5y + 2$ **51.** $x^2 + 4x + 3$ **53.** $a^2 + a - 12$ **55.** $y^2 - 5y - 24$

57. $y^2 - 10y + 21$ **59.** $2x^2 + 15x + 7$ **61.** $3x^2 + 11x - 4$ **63.** $4x^2 - 31x + 21$ **65.** $3y^2 - 2y - 16$

67. $9x^2 + 54x + 77$ **69.** $21a^2 - 83a + 80$ **71.** $6a^2 - 25ab + 14b^2$ **73.** $2a^2 - 11ab - 63b^2$

75. $100a^2 - 100ab + 21b^2$ **77.** $15x^2 + 56xy + 48y^2$ **79.** $14x^2 - 97xy - 60y^2$ **81.** $56x^2 - 61xy + 15y^2$

85. $y^2 - 25$ **87.** $4x^2 - 9$ **89.** $x^2 + 2x + 1$ **91.** $9a^2 - 30a + 25$ **93.** $9x^2 - 49$ **95.** $4a^2 + 4ab + b^2$

97. $16 - 9y^2$ **99.** $25x^2 + 20xy + 4y^2$ **101.** The area is $(10x^2 - 35x)$ ft^2. **103.** The area is $(4x^2 + 4x + 1)$ km^2.

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105. The area is $(18x^2 + 12x + 2)$ in^2. **107.** The area is $(4x^2 + 10x)$ m^2. **109.** The total area is $(3000 + 60w)$ yd^2.
111. $4ab$ **113.** $a^3 + 9a^2 + 27a + 27$ **115.a.** $7x^2 - 11x - 8$ **b.** $x^3 - 7x^2 - 7$

SECTION 6.4

1. $x + 8$ **3.** $x^2 + \frac{2}{x-3}$ **5.** $3x + 5 + \frac{3}{2x+1}$ **7.** $5x + 7 + \frac{2}{2x-1}$ **9.** $4x^2 + 6x + 9 + \frac{18}{2x-3}$

11. $3x^2 + 1 + \frac{1}{2x^2-5}$ **13.** $x^2 - 3x - 10$ **15.** $x^2 - 2x + 1 - \frac{1}{x-3}$ **17.** $2x^3 - 3x^2 + x - 4$

19. $2x + \frac{x+2}{x^2+2x-1}$ **21.** $x^2 + 4x + 6 + \frac{10x+8}{x^2-2x-1}$ **23.** $x^2 - 2x + 4 + \frac{-6x-7}{x^2+2x+3}$ **25.** $2x - 8$ **27.** $3x - 8$

29. $3x + 3 - \frac{1}{x-1}$ **31.** $2x^2 - 3x + 9$ **33.** $4x^2 + 8x + 15 + \frac{12}{x-2}$ **35.** $2x^2 - 3x + 7 - \frac{8}{x+4}$

37. $3x^3 + 2x^2 + 12x + 19 + \frac{33}{x-2}$ **39.** $3x^3 - x + 4 - \frac{2}{x+1}$ **41.** $2x^3 + 6x^2 + 17x + 51 + \frac{155}{x-3}$ **43.** $f(3) = 0$

45. $P(3) = 8$ **47.** $R(4) = 43$ **49.** $P(-2) = -39$ **51.** $Z(-3) = -60$ **53.** $Q(2) = 31$ **55.** $F(-3) = 178$
57. $P(5) = 122$ **59.** $R(-3) = 302$ **61.** $Q(2) = 0$ **63.** $R(-2) = -65$ **65.a.** $a^2 - ab + b^2$

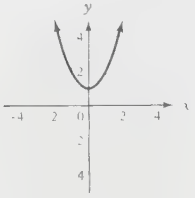
b. $x^4 - x^3y + x^2y^2 - xy^3 + y^4$ **c.** $x^5 - x^4y + x^3y^2 - x^2y^3 + xy^4 - y^5$ **67.a.** $\frac{1}{2}x^2 + \frac{3}{4}x - 1$

b. $\frac{2}{3}x^2 + \frac{7}{2}x + \frac{10}{3} + \frac{64}{6x-12}$ **c.** $2x^2 - x + 1 + \frac{3x-11}{x^2-x+1}$ **d.** $x^2 + 6x + 17 + \frac{51x+56}{x^2-2x-3}$

CHAPTER REVIEW

1. $-6a^3b^6$ [6.1A] **2.** $9x^4y^6$ [6.1A] **3.** $\frac{c^{10}}{2b^{17}}$ [6.1B] **4.** 0.00254 [6.1C]

5. -7 [6.2A] **6.** $4x^2 - 8xy + 5y^2$ [6.2B] **7.** $\frac{18x^5y^4}{z}$ [6.1B] **8.** $-54a^{13}b^5c^7$ [6.1A]

9. 

[6.2A] **10.a.** 3 **b.** 8 **c.** 5 [6.2A] **11.** 1 [6.4C] **12.** $-8x^3 - 14x^2 + 18x$ [6.3A]

13. $8a^3b^3 - 4a^2b^4 + 6ab^5$ [6.3A] **14.** $6y^3 + 17y^2 - 2y - 21$ [6.3B] **15.** $10a^2 + 31a - 63$ [6.3C]

16. 1.27×10^{-7} [6.1C] **17.** $25y^2 - 70y + 49$ [6.3D] **18.** $5x + 4 + \frac{6}{3x-2}$ [6.4A]

19. $4x^2 + 3x - 8 + \frac{50}{x+6}$ [6.4B] **20.** $x^3 + 4x^2 + 16x + 64 + \frac{252}{x-4}$ [6.4B] **21.** 8,100,000,000 [6.1C]

22. $-\frac{2a^3}{3b^3}$ [6.1B] **23.** $25a^2 - 4b^2$ [6.3D] **24.** 68 [6.4C] **25.** $21y^2 + 4y - 1$ [6.2B]

26. $12b^5 - 4b^4 - 6b^3 - 8b^2 + 5$ [6.3B] **27.** $a^2 - 49$ [6.3D] **28.** 7.65×10^{11} [6.1C]
29. The area of the Ping-Pong table is $(2w^2 - w)$ ft^2. [6.3E] **30.** The Great Galaxy of Andromeda is 1.291224×10^{19} mi from Earth. [6.1D] **31.** There are 6.048×10^5 s in one week. [6.1D] **32.** The area is $(9x^2 - 12x + 4)$ in^2. [6.3E]
33. The area is $(10x^2 - 29x - 21)$ cm^2. [6.3E]

CHAPTER TEST

1. $4x^3 - 6x^2$ [6.3A] **2.** -8 [6.4C] **3.** $-\frac{4}{x^6}$ [6.1B] **4.** $-6x^3y^6$ [6.1A] **5.** $x - 1 + \frac{2}{x+1}$ [6.4A/6.4B]

6. $x^3 - 7x^2 + 17x - 15$ [6.3B] **7.** $-8a^6b^3$ [6.1A] **8.** $\frac{9y^{10}}{x^{10}}$ [6.1B] **9.** $a^2 + 3ab - 10b^2$ [6.3B] **10.** -3 [6.2A]

11. $x + 7$ [6.4A/6.4B] **12.** $6y^4 - 9y^3 + 18y^2$ [6.3A] **13.** $-4x^4 + 8x^3 - 3x^2 - 14x + 21$ [6.3B]

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14. $16y^2 - 9$ [6.3D] **15.** a^4b^7 [6.1A] **16.** $8ab^4$ [6.1B] **17.** $\frac{2b^7}{a^{10}}$ [6.1B] **18.** $-5a^3 + 3a^2 - 4a + 3$ [6.2B]

19. $4x^2 - 20x + 25$ [6.3D] **20.** $x^2 - 5x + 10 - \frac{23}{x+3}$ [6.4A/6.4B] **21.** $10x^2 - 43xy + 28y^2$ [6.3C]

22. $3x^3 + 6x^2 - 8x + 3$ [6.2B] **23.** 3.02×10^{-9} [6.1C] **24.** The mass of the moon is 8.103×10^{19} tons. [6.1D]

25. The area of the circle is $(\pi x^2 - 10\pi x + 25\pi)$ m^2. [6.3E]

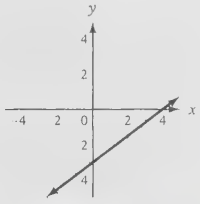
CUMULATIVE REVIEW

1. -3 and 3 [1.1A] **2.** -83 [1.1B] **3.** 6 [1.3A] **4.** $-\frac{5}{4}$ [1.4A] **5.** $-50\sqrt{3}$ [1.2F] **6.** The Inverse Property of Addition [1.4B] **7.** $-186x + 8$ [1.4D] **8.** $-\frac{1}{6}$ [2.2A] **9.** $-\frac{11}{4}$ [2.2B] **10.** -1 and $\frac{7}{3}$ [2.5A]

11. 18 [4.2A] **12.** Yes [4.2A] **13.** $-\frac{1}{6}$ [4.4A] **14.** $y = -\frac{3}{2}x + \frac{1}{2}$ [4.5A] **15.** $y = \frac{2}{3}x + \frac{16}{3}$ [4.6A]

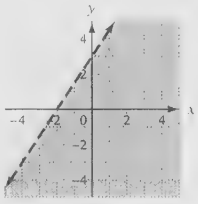
16. $\left(-\frac{7}{5}, -\frac{8}{5}\right)$ [5.3B] **17.** $\left(-\frac{9}{7}, \frac{2}{7}, \frac{11}{7}\right)$ [5.2B] **18.** $5x - 3xy$ [1.4B] **19.** $4x^3 - 7x + 3$ [6.3B]

20. 5.01×10^{-6} [6.1C] **21.**



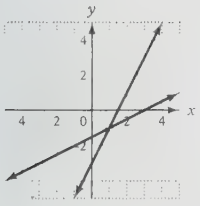
[4.3B]

22.



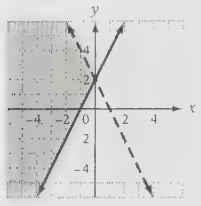
[4.7A]

23.



[5.1A]

24.



[5.5A]

25. $\frac{b^5}{a^8}$ [6.1B] **26.** $\frac{y^2}{25x^6}$ [6.1B]

The solution is $(1, -1)$.

27. The two integers are 9 and 15. [2.2D] **28.** 40 oz of pure gold must be used. [2.3A] **29.** The cyclists are traveling at 5 mph and 7.5 mph. [2.3D] **30.** \$4500 must be invested at an annual simple interest rate of 10%. [2.3C]

31. The slope is 50. The slope represents the average speed in miles per hour. [4.4A] **32.** The length is 15 m. The width is 6 m. [3.2A] **33.** The area is $(4x^2 + 12x + 9)$ m^2. [6.3E]

Answers to Chapter 7 Selected Exercises

PREP TEST

1. $2 \cdot 3 \cdot 5$ [1.2D] **2.** $-12y + 15$ [1.4D] **3.** $-a + b$ [1.4D] **4.** $-3a + 3b$ [1.4D] **5.** 0 [2.1B]

6. $-\frac{1}{2}$ [2.2A] **7.** $x^2 - 2x - 24$ [6.3C] **8.** $6x^2 - 11x - 10$ [6.3C] **9.** x^3 [6.1B] **10.** $3x^3y$ [6.1B]

SECTION 7.1

1. $5(a + 1)$ **3.** $8(2 - a^2)$ **5.** $4(2x + 3)$ **7.** $6(5a - 1)$ **9.** $x(7x - 3)$ **11.** $a^2(3 + 5a^3)$ **13.** $y(14y + 11)$

15. $2x(x^3 - 2)$ **17.** $2x^2(5x^2 - 6)$ **19.** $4a^5(2a^3 - 1)$ **21.** $xy(xy - 1)$ **23.** $3xy(xy^3 - 2)$ **25.** $xy(x - y^2)$

27. $5y(y^2 - 4y + 2)$ **29.** $3y^2(y^2 - 3y - 2)$ **31.** $3y(y^2 - 3y + 8)$ **33.** $a^2(6a^3 - 3a - 2)$ **35.** $ab(2a - 5ab + 7b)$

37. $2b(2b^4 + 3b^2 - 6)$ **39.** $x^2(8y^2 - 4y + 1)$ **41.** $a^{2n}(2a^{3n} + 1)$ **43.** $y^{2n}(y^{2n} + 1)$ **45.** $b^5(b^n - 1)$

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47. $(a + z)(y + 7)$ **49.** $(a - b)(3r + s)$ **51.** $(m - 7)(t - 7)$ **53.** $(4a - b)(2y + 1)$ **55.** $(x + 2)(x + 2y)$
57. $(p - 2)(p - 3r)$ **59.** $(a + 6)(b - 4)$ **61.** $(2z - 1)(z + y)$ **63.** $(4v + 7)(2v - 3y)$ **65.** $(2x - 5)(x - 3y)$
67. $(y - 2)(3y - a)$ **69.** $(3x - y)(y + 1)$ **71.** $(3s + t)(t - 2)$ **73a.** 28 **b.** 496

SECTION 7.2

1. $(x + 1)(x + 2)$ **3.** $(x + 1)(x - 2)$ **5.** $(a + 4)(a - 3)$ **7.** $(a - 1)(a - 2)$ **9.** $(a + 2)(a - 1)$ **11.** $(b - 3)(b - 3)$
13. $(b + 8)(b - 1)$ **15.** $(y + 11)(y - 5)$ **17.** $(y - 2)(y - 3)$ **19.** $(z - 5)(z - 9)$ **21.** $(z + 8)(z - 20)$
23. $(p + 3)(p + 9)$ **25.** $(x + 10)(x + 10)$ **27.** $(b + 4)(b + 5)$ **29.** $(x + 3)(x - 14)$ **31.** $(b + 4)(b - 5)$
33. $(y + 3)(y - 17)$ **35.** $(p + 3)(p - 7)$ **37.** nonfactorable over the integers **39.** $(x - 5)(x - 15)$ **41.** $(x - 7)(x - 8)$
43. $(x + 8)(x - 7)$ **45.** $(a + 3)(a - 24)$ **47.** $(a - 3)(a - 12)$ **49.** $(z + 8)(z - 17)$ **51.** $(c + 9)(c - 10)$
53. $(z + 4)(z + 11)$ **55.** $(c + 2)(c + 17)$ **57.** $(x + 8)(x - 12)$ **59.** $(x - 8)(x - 14)$ **61.** $(b + 15)(b - 7)$
63. $(a + 3)(a - 12)$ **65.** $(b - 6)(b - 17)$ **67.** $(a + 3)(a + 24)$ **69.** $(x + 12)(x + 13)$ **71.** $(x + 6)(x - 16)$
73. $2(x + 1)(x + 2)$ **75.** $-(x + 2)(x - 9)$ **77.** $a(b + 5)(b - 3)$ **79.** $x(y - 2)(y - 3)$ **81.** $z(z - 3)(z - 4)$
83. $-3y(y - 2)(y - 3)$ **85.** $3(x + 4)(x - 3)$ **87.** $5(z + 4)(z - 7)$ **89.** $2a(a + 8)(a - 4)$ **91.** $(x - 2y)(x - 3y)$
93. $(a - 4b)(a - 5b)$ **95.** $(x + 4y)(x - 7y)$ **97.** nonfactorable over the integers **99.** $z^2(z - 5)(z - 7)$
101. $b^2(b - 10)(b - 12)$ **103.** $2y^2(y + 3)(y - 16)$ **105.** $-x^2(x + 8)(x - 1)$ **107.** $4y(x + 7)(x - 2)$
109. $c(c + 20)(c - 2)$ **111.** $-4x(x + 3)(x - 2)$ **113.** $(y + x)(y - 8x)$ **115.** $(y + 7z)(y - 3z)$ **117.** $(y - z)(y - 15z)$
119. $4y(x + 1)(x - 18)$ **121.** $4x(x + 8)(x - 5)$ **123.** $5z(z + 2)(z - 12)$ **125.** $5x(x + 2)(x + 4)$ **127.** $4(p + 8)(p - 15)$
129. $p^2(p + 8)(p - 7)$ **131.** $(a - 5b)(a - 5b)$ **133.** $(x + 10y)(x - 6y)$ **135.** $6x(x - 5)(x + 4)$ **137.** $y(x + 6)(x - 9)$
139. $k = -36, 36, -12, 12$ **141.** $k = 22, -22, 10, -10$ **143.** $k = 6, 10, 12$ **145.** $k = 6, 10, 12$ **147.** $k = 4, 6$

SECTION 7.3

1. $(x + 1)(2x + 1)$ **3.** $(y + 3)(2y + 1)$ **5.** $(a - 1)(2a - 1)$ **7.** $(b - 5)(2b - 1)$ **9.** $(x + 1)(2x - 1)$
11. $(x - 3)(2x + 1)$ **13.** $(t + 2)(2t - 5)$ **15.** $(p - 5)(3p - 1)$ **17.** $(3y - 1)(4y - 1)$ **19.** nonfactorable over the integers **21.** $(2t - 1)(3t - 4)$ **23.** $(x + 4)(8x + 1)$ **25.** nonfactorable over the integers **27.** $(3y + 1)(4y + 5)$
29. $(a + 7)(7a - 2)$ **31.** $(b - 4)(3b - 4)$ **33.** $(z - 14)(2z + 1)$ **35.** $(p + 8)(3p - 2)$ **37.** $2(x + 1)(2x + 1)$
39. $5(y - 1)(3y - 7)$ **41.** $x(x - 5)(2x - 1)$ **43.** $b(a - 4)(3a - 4)$ **45.** nonfactorable over the integers
47. $-3x(x - 3)(x + 4)$ **49.** $4(4y - 1)(5y - 1)$ **51.** $z(2z + 3)(4z + 1)$ **53.** $y(2x - 5)(3x + 2)$ **55.** $5(t + 2)(2t - 5)$
57. $p(p - 5)(3p - 1)$ **59.** $2(z + 4)(13z - 3)$ **61.** $2y(y - 4)(5y - 2)$ **63.** $yz(z + 2)(4z - 3)$ **65.** $3a(2a + 3)(7a - 3)$
67. $y(3x - 5y)(3x - 5y)$ **69.** $xy(3x - 4y)(3x - 4y)$ **71.** $(2x - 3)(3x - 4)$ **73.** $(b + 7)(5b - 2)$ **75.** $(3a + 8)(2a - 3)$
77. $(z + 2)(4z + 3)$ **79.** $(2p + 5)(11p - 2)$ **81.** $(y + 1)(8y + 9)$ **83.** $(6t - 5)(3t + 1)$ **85.** $(b + 12)(6b - 1)$
87. $(3x + 2)(3x + 2)$ **89.** $(2b - 3)(3b - 2)$ **91.** $(3b + 5)(11b - 7)$ **93.** $(3y - 4)(6y - 5)$ **95.** $(3a + 7)(5a - 3)$
97. $(2y - 5)(4y - 3)$ **99.** $(2z + 3)(4z - 5)$ **101.** nonfactorable over the integers **103.** $(2z - 5)(5z - 2)$
105. $(6z + 5)(6z + 7)$ **107.** $(x + y)(3x - 2y)$ **109.** $(a + 2b)(3a - b)$ **111.** $(y - 2z)(4y - 3z)$ **113.** $-(z - 7)(z + 4)$
115. $-(x - 1)(x + 8)$ **117.** $3(x + 5)(3x - 4)$ **119.** $4(2x - 3)(3x - 2)$ **121.** $a^2(5a + 2)(7a - 1)$ **123.** $5(b - 7)(3b - 2)$
125. $(x - 7y)(3x - 5y)$ **127.** $3(8y - 1)(9y + 1)$ **129.** $-(x - 1)(x + 21)$ **131.** $(5a + 7b)(3a - 2b)$
133. $-z(z + 11)(z - 3)$ **135.** $(3a + 2)(a + 3)$ **137.** $k = 7, -7, 5, -5$ **139.** $k = 7, -7, 5, -5$

SECTION 7.4

1. 4; $25x^6$; $100x^4y^4$ **3.** $4z^4$ **5.** $9a^2b^3$ **7.** $(x + 4)(x - 4)$ **9.** $(2x + 1)(2x - 1)$ **11.** $(4x + 11)(4x - 11)$
13. $(1 + 3a)(1 - 3a)$ **15.** $(xy + 10)(xy - 10)$ **17.** nonfactorable over the integers **19.** $(5 + ab)(5 - ab)$
21. $(a^n + 1)(a^n - 1)$ **23.** $(x - 6)^2$ **25.** $(b - 1)^2$ **27.** $(4x - 5)^2$ **29.** nonfactorable over the integers
31. nonfactorable over the integers **33.** $(x + 3y)^2$ **35.** $(5a - 4b)^2$ **37.** $(x^n + 3)^2$ **39.** $(x - 7)(x - 1)$
41. $(x - y + a + b)(x - y - a - b)$ **43.** 8; x^9; $27c^{15}d^{18}$ **45.** $2x^3$ **47.** $4a^2b^6$ **49.** $(x - 3)(x^2 + 3x + 9)$
51. $(2x - 1)(4x^2 + 2x + 1)$ **53.** $(x - y)(x^2 + xy + y^2)$ **55.** $(m + n)(m^2 - mn + n^2)$ **57.** $(4x + 1)(16x^2 - 4x + 1)$
59. $(3x - 2y)(9x^2 + 6xy + 4y^2)$ **61.** $(xy + 4)(x^2y^2 - 4xy + 16)$ **63.** nonfactorable over the integers
65. nonfactorable over the integers **67.** $(a - 2b)(a^2 - ab + b^2)$ **69.** $(x^{2n} + y^n)(x^{4n} - x^{2n}y^n + y^{2n})$

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71. $(x^n + 2)(x^{2n} - 2x^n + 4)$ **73.** $(xy - 3)(xy - 5)$ **75.** $(xy - 5)(xy - 12)$ **77.** $(x^2 - 3)(x^2 - 6)$
79. $(b^2 + 5)(b^2 - 18)$ **81.** $(x^2y^2 - 2)(x^2y^2 - 6)$ **83.** $(x^n + 1)(x^n + 2)$ **85.** $(3xy - 5)(xy - 3)$ **87.** $(2ab - 3)(3ab - 7)$
89. $(2x^2 - 15)(x^2 + 1)$ **91.** $(2x^n - 1)(x^n - 3)$ **93.** $(2a^n + 5)(3a^n + 2)$ **95.** $3(2x - 3)^2$ **97.** $a(3a - 1)(9a^2 + 3a + 1)$
99. $5(2x + 1)(2x - 1)$ **101.** $y^3(y + 11)(y - 5)$ **103.** $(4x^2 + 9)(2x + 3)(2x - 3)$ **105.** $2a(2 - a)(4 + 2a + a^2)$
107. $b^3(ab - 1)(a^2b^2 + ab + 1)$ **109.** $2x^2(2x - 5)^2$ **111.** $(x^2 + y^2)(x + y)(x - y)$ **113.** $(x^2 + y^2)(x^4 - x^2y^2 + y^4)$
115. nonfactorable over the integers **117.** $2a(2a - 1)(4a^2 + 2a + 1)$ **119.** $2b^2(3a + 5b)(4a - 9b)$ **121.** $(x - 2)^2(x + 2)$
123. $(x + y)(x - y)(2x + 1)(2x - 1)$ **125.** $x(x^n + 1)^2$ **127.** $b^n(3b - 2)(b + 2)$ **129.** The dimensions are $(4x + 3)$ m by $(4x + 3)$ m. Yes, x can equal 0. The possible values of x are $x > -\frac{3}{4}$.

SECTION 7.5

3. 3, 5 **5.** $-8, 9$ **7.** $0, -2$ **9.** $0, -12$ **11.** $0, \frac{7}{4}$ **13.** $0, -\frac{5}{2}$ **15.** $8, -3$ **17.** $11, -11$ **19.** $\frac{1}{4}, -\frac{1}{4}$
21. $\frac{7}{4}, -\frac{7}{4}$ **23.** 3, 5 **25.** $8, -9$ **27.** $-\frac{2}{3}, -4$ **29.** $\frac{5}{3}, \frac{3}{2}$ **31.** 0, 5 **33.** 0, 4 **35.** $-3, 8$ **37.** $-1, 8$
39. $\frac{5}{3}, -2$ **41.** $\frac{6}{5}, 2$ **43.** $12, -1$ **45.** $-3, -5$ **47.** $5, -4$ **49.** 3, 5 **51.** $15, -1$ **53.** $-6, 3$ **55.** 1, 3
57. $-12, 5$ **59.** $-\frac{3}{2}, -2$ **61.** The number is -3. **63.** The numbers are 3 and 5. **65.** The integers are 8 and 10.
67. The length is 38 cm, and the width is 10 cm. **69.** 18 consecutive natural numbers beginning with 1 will give a sum of 171. **71.** There are 10 teams in the league. **73.** The object will hit the ground 4 s later. **75.** The ball will be 64 ft above the ground 2 s later. **77.** The height of the triangle is 14 m. **79.** The width of the border is 1.5 ft.
81. The radius of the original circle is 3.81 in. **83.** The length is 20 in., and the width is 10 in. **85.** 1, 18

CHAPTER REVIEW

1. $5x(x^2 + 2x + 7)$ [7.1A] **2.** $3ab(4a + b)$ [7.1A] **3.** $7y^3(2y^6 - 7y^3 + 1)$ [7.1A] **4.** $(x - 3)(4x + 5)$ [7.1B]
5. $(2x + 5)(5x + 2y)$ [7.1B] **6.** $(3a - 5b)(7x + 2y)$ [7.1B] **7.** $(b - 3)(b - 10)$ [7.2A] **8.** $(c + 6)(c + 2)$ [7.2A]
9. $(y - 4)(y + 9)$ [7.2A] **10.** $3(a + 2)(a - 7)$ [7.2B] **11.** $4x(x - 6)(x + 1)$ [7.2B] **12.** $n^2(n + 1)(n - 3)$ [7.2B]
13. $(2x - 7)(3x - 4)$ [7.3A] **14.** $(6y - 1)(2y + 3)$ [7.3A] **15.** nonfactorable over the integers [7.3A]
16. $(3x - 2)(x - 5)$ [7.3B] **17.** $(2a + 5)(a - 12)$ [7.3B] **18.** $(6a - 5)(3a + 2)$ [7.3B]
19. $(xy + 3)(xy - 3)$ [7.4A] **20.** $(2x + 3y)^2$ [7.4A] **21.** $(x^n - 6)^2$ [7.4A] **22.** $(4a - 3b)(16a^2 + 12ab + 9b^2)$ [7.4B]
23. $(3x^2 + 2)(5x^2 - 3)$ [7.4C] **24.** $(7x^2y^2 + 3)(3x^2y^2 + 2)$ [7.4C] **25.** $3a^2(a^2 + 1)(a^2 - 6)$ [7.4D]
26. $\frac{1}{4}$ and -7 [7.5A] **27.** -3 and 7 [7.5A] **28.** The length is 100 yd. The width is 60 yd. [7.5B]
29. The width of the frame is $\frac{3}{2}$ in. or 1.5 in. or $1\frac{1}{2}$ in. [7.5B] **30.** The length of a side is 20 ft. [7.5B]

CHAPTER TEST

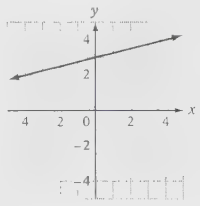
1. $(b + 6)(a - 3)$ [7.1B] **2.** $2y^2(y + 1)(y - 8)$ [7.2B] **3.** $4(x + 4)(2x - 3)$ [7.3B] **4.** $(2x + 1)(3x + 8)$ [7.3A]
5. $(a - 3)(a - 16)$ [7.2A] **6.** $2x(3x^2 - 4x + 5)$ [7.1A] **7.** $(x + 5)(x - 3)$ [7.2A] **8.** $\frac{1}{2}, -\frac{1}{2}$ [7.5A]
9. $5(x^2 - 9x - 3)$ [7.1A] **10.** $(p + 6)^2$ [7.4A] **11.** 3 and 5 [7.5A] **12.** $3(x + 2y)^2$ [7.4D]
13. $(3x - 2)(9x^2 + 6x + 4)$ [7.4B] **14.** $3y^2(2x + 1)(x + 1)$ [7.3B] **15.** $(2a^2 - 5)(3a^2 + 1)$ [7.4C]
16. $(x - 2)(a + b)$ [7.1B] **17.** $(p + 1)(x - 1)$ [7.1B] **18.** $3(a + 5)(a - 5)$ [7.4D] **19.** nonfactorable over the integers [7.3A] **20.** $(x + 3)(x - 12)$ [7.2A] **21.** $(2a - 3b)^2$ [7.4A] **22.** $(2x + 7y)(2x - 7y)$ [7.4A]
23. $\frac{3}{2}$ and -7 [7.5A] **24.** The length of the rectangle is 15 cm, and the width is 6 cm. [7.5B] **25.** The numbers are 3 and 7. [7.5B]

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CUMULATIVE REVIEW

1. 7 [1.1C] **2.** 4 [1.3A] **3.** -7 [1.4A] **4.** $15x^2$ [1.4C] **5.** 12 [1.4D] **6.** $\frac{2}{3}$ [2.1C] **7.** $\frac{7}{4}$ [2.2B]

8. 3 [2.2C] **9.** 45 [2.1D] **10.** 1 [4.2A] **11.**



[4.3A] **12.**



[4.3B]

13. $y = \frac{2}{3}x + 6$ [4.5A] **14.** (1, 6) [5.1B] **15.** $(-1, -2)$ [5.2A] **16.** $9a^6b^4$ [6.1A] **17.** $x^3 - 3x^2 - 6x + 8$ [6.3B]

18. $4x + 8 + \frac{21}{2x - 3}$ [6.4A] **19.** $\frac{y^6}{x^8}$ [6.1B] **20.** $(a - b)(3 - x)$ [7.1B] **21.** $5xy^2(3 - 4y^2)$ [7.1A]

22. $(x - 7y)(x + 2y)$ [7.2A] **23.** $\frac{2}{3}$ and -7 [7.5A] **24.** $\frac{5}{2}$ and 4 [7.5A] **25.** The third angle measures 59°. [3.1C]

26. The width is 15 ft. [3.2A] **27.** The pieces are 4 ft long and 6 ft long. [2.2D] **28.** $6500 was invested at 11%. [2.3C] **29.** The distance to the resort is 168 mi. [2.3D] **30.** The length of the base of the triangle is 12 in. [7.5B]

Answers to Chapter 8 Selected Exercises

PREP TEST

1. 50 [1.2B] **2.** $-\frac{1}{6}$ [1.2D] **3.** $-\frac{3}{2}$ [1.2D] **4.** $\frac{1}{24}$ [1.2C] **5.** $\frac{5}{24}$ [1.2C] **6.** $\frac{1}{3}$ [1.4A] **7.** -2 [2.2C]

8. $\frac{10}{7}$ [2.2C] **9.** 110 mph, 130 mph [2.3D]

SECTION 8.1

3. $\frac{3}{4x}$ **5.** $\frac{1}{x + 3}$ **7.** -1 **9.** $\frac{2}{3y}$ **11.** $-\frac{3}{4x}$ **13.** $\frac{a}{b}$ **15.** $-\frac{2}{x}$ **17.** $\frac{y - 2}{y - 3}$ **19.** $\frac{x + 5}{x + 4}$ **21.** $\frac{x + 4}{x - 3}$

23. $-\frac{x + 2}{x + 5}$ **25.** $\frac{2(x + 2)}{x + 3}$ **27.** $\frac{2x - 1}{2x + 3}$ **29.** $-\frac{x + 7}{x + 6}$ **31.** $\frac{35ab^2}{24x^2y}$ **33.** $\frac{4x^3y^3}{3a^2}$ **35.** $\frac{3}{4}$ **37.** ab^2

39. $\frac{x^2(x - 1)}{y(x + 3)}$ **41.** $\frac{y(x - 1)}{x^2(x + 10)}$ **43.** $-ab^2$ **45.** $\frac{x + 5}{x + 4}$ **47.** 1 **49.** $-\frac{n - 10}{n - 7}$ **51.** $\frac{x(x + 2)}{2(x - 1)}$ **53.** $-\frac{x + 2}{x - 6}$

55. $\frac{x + 5}{x - 12}$ **57.** $\frac{7a^3y^2}{40bx}$ **59.** $\frac{4}{3}$ **61.** $\frac{3a}{2}$ **63.** $\frac{x^2(x + 4)}{y^2(x + 2)}$ **65.** $\frac{x(x - 2)}{y(x - 6)}$ **67.** $-\frac{3by}{ax}$ **69.** $\frac{(x - 3)(x + 6)}{(x + 7)(x - 6)}$

71. 1 **73.** $-\frac{x + 8}{x - 4}$ **75.** $\frac{2n + 1}{2n - 3}$ **79.** $\frac{n - 2}{n + 3}$ **81.** $\frac{8}{3y}$ **83.** 1

SECTION 8.2

1. $24x^3y^2$ **3.** $30x^4y^2$ **5.** $8x^2(x + 2)$ **7.** $6x^2y(x + 4)$ **9.** $36x(x + 2)^2$ **11.** $6(x + 1)^2$ **13.** $(x - 1)(x + 2)(x + 3)$

15. $(2x + 3)^2(x - 5)$ **17.** $(x - 1)(x - 2)$ **19.** $(x - 3)(x + 2)(x + 4)$ **21.** $(x + 4)(x + 1)(x - 7)$

23. $(x - 6)(x + 6)(x + 4)$ **25.** $(x - 10)(x - 8)(x + 3)$ **27.** $(3x - 2)(x - 3)(x + 2)$ **29.** $(x + 2)(x - 3)$

31. $(x - 5)(x + 1)$ **33.** $(x - 3)(x - 2)(x - 1)(x - 6)$ **35.** $\frac{5}{ab^2}$; $\frac{6b}{ab^2}$ **37.** $\frac{15y^2}{18x^2y}$; $\frac{14x}{18x^2y}$

39. $\frac{ay + 5a}{y^2(y + 5)}$; $\frac{6y}{y^2(y + 5)}$ **41.** $\frac{a^2y + 7a^2}{y(y + 7)^2}$; $\frac{ay}{y(y + 7)^2}$ **43.** $\frac{b}{y(y - 4)}$; $\frac{-b^2y}{y(y - 4)}$ **45.** $-\frac{3y - 21}{(y - 7)^2}$; $\frac{2}{(y - 7)^2}$

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47. $\frac{2y^2}{y^2(y-3)}$; $\frac{3}{y^2(y-3)}$ **49.** $\frac{x^3+4x^2}{(2x-1)(x+4)}$; $\frac{2x^2+x-1}{(2x-1)(x+4)}$ **51.** $\frac{3x^2+15x}{(x-5)(x+5)}$; $\frac{4}{(x-5)(x+5)}$

53. $\frac{x-3}{(3x-2)(x+2)}$; $\frac{6x-4}{(3x-2)(x+2)}$ **55.** $\frac{x^2-1}{(x+5)(x-3)(x+1)}$; $\frac{x^2-3x}{(x+5)(x-3)(x+1)}$ **57.** $\frac{11}{y^2}$ **59.** $-\frac{7}{x+4}$

61. $\frac{8x}{2x+3}$ **63.** $\frac{5x+7}{x-3}$ **65.** $\frac{2x-5}{x+9}$ **67.** $\frac{-3x-4}{2x+7}$ **69.** $\frac{1}{x+5}$ **71.** $\frac{1}{x-6}$ **73.** $\frac{3}{2y-1}$ **75.** $\frac{1}{x-5}$

79. $\frac{7b+5a}{ab}$ **81.** $\frac{11}{12a}$ **83.** $\frac{11}{12y}$ **85.** $\frac{120+7y}{20y^2}$ **87.** $\frac{21b+4a}{12ab}$ **89.** $\frac{14x+3}{12x}$ **91.** $\frac{8x-3}{6x}$ **93.** $\frac{7}{36}$

95. $\frac{-3x^2-11x-2}{3x^2}$ **97.** $\frac{6x^2+5x+10}{6x^2}$ **99.** $\frac{2x^2+1}{x}$ **101.** $\frac{4x+2}{x+1}$ **103.** $\frac{x^2-x+2}{x^2y}$

105. $\frac{16xy-12y+6x^2+3x}{12x^2y^2}$ **107.** $\frac{3xy-6y-2x^2-14x}{24x^2y}$ **109.** $\frac{9x+2}{(x-2)(x+3)}$ **111.** $\frac{2(x+23)}{(x-7)(x+3)}$

113. $\frac{2x^2-5x+1}{(x+1)(x-3)}$ **115.** $\frac{4x^2-34x+5}{(2x-1)(x-6)}$ **117.** $\frac{2a-5}{a-7}$ **119.** $\frac{4x+9}{(x+3)(x-3)}$ **121.** $\frac{-x+9}{(x+2)(x-3)}$

123. $\frac{8y}{(y+2)^2}$ **125.** $\frac{3x-y}{x-y}$ **127.** $\frac{11}{(a-3)(a-4)}$ **129.** $\frac{-x-1}{(x+2)(x-5)}$ **131.** $\frac{x+3}{x+5}$ **133.** $\frac{x-2}{x+4}$

135. $\frac{4(2x-1)}{(x+5)(x-2)}$ **137a.** $\frac{3}{y}+\frac{6}{x}$ **b.** $\frac{4}{b^2}+\frac{3}{ab}$ **c.** $\frac{1}{4mn}+\frac{1}{6m^2}$ **139a.** $\frac{19{,}200}{x}$ dollars **b.** $\frac{96{,}500}{x(x+5)}$ dollars

c. \$128

SECTION 8.3

1. $\frac{x}{x-3}$ **3.** $\frac{2}{3}$ **5.** $\frac{y+3}{y-4}$ **7.** $\frac{2(2x+13)}{5x+36}$ **9.** $\frac{x+2}{x+3}$ **11.** $\frac{x-6}{x+5}$ **13.** $-\frac{x-2}{x+1}$ **15.** $x-1$

17. $\frac{1}{2x-1}$ **19.** $\frac{x-3}{x+5}$ **21.** $\frac{x-7}{x-8}$ **23.** $\frac{2y-1}{2y+1}$ **25.** $\frac{x-2}{2x-5}$ **27.** $-\frac{x+1}{4x-3}$ **29.** $\frac{x+1}{2(5x-2)}$

31. $\frac{5}{3}$ **33.** $-\frac{1}{x-1}$ **35.** $\frac{y+4}{2(y-2)}$

SECTION 8.4

1. 3 **3.** 1 **5.** 9 **7.** 1 **9.** $\frac{1}{4}$ **11.** 1 **13.** -3 **15.** $\frac{1}{2}$ **17.** 8 **19.** 5 **21.** -1 **23.** 5

25. The equation has no solution. **27.** 2, 4 **29.** $-\frac{3}{2}$, 4 **31.** 3 **33.** 4 **37.** 9 **39.** 36 **41.** 10 **43.** 113

45. -2 **47.** 15 **49.** 4 **51.** 6.7 cm **53.** 2.9 m **55.** 22.5 ft **57.** 48 m² **59.** 6.25 cm **61.** 6 in.
63. 13 cm **65.** The height of the flagpole is 14.375 ft. **67.** The width of the ravine is 82.5 ft. **69.** 20,000 voters voted in favor of the amendment. **71.** There are 175 mi between the two cities. **73.** There are 160 ml of water in 280 ml of soft drink. **75.** It would take 18 min to print a document 45 pages long. **77.** The window should be 40 ft². **79.** The person's height is 67.5 in. **81.** The first person's share of the winnings is \$1.25 million. **83.** The basketball player made 210 shots.

SECTION 8.5

1. $h=\frac{2A}{b}$ **3.** $t=\frac{d}{r}$ **5.** $T=\frac{PV}{nR}$ **7.** $L=\frac{P-2W}{2}$ **9.** $b_1=\frac{2A-hb_2}{h}$ **11.** $h=\frac{3V}{A}$ **13.** $S=C-Rt$

15. $P=\frac{A}{1+rt}$ **17.** $w=\frac{A}{S+1}$ **19a.** $h=\frac{S-2\pi r^2}{2\pi r}$ **b.** The height is 5 in. **c.** The height is 4 in.

21a. $S=\frac{F+BV}{B}$ **b.** The required selling price is \$180. **c.** The required selling price is \$75.

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SECTION 8.6

1. It would take 2 h with both sprinklers working. **3.** It would take both skiploaders 3 h working together. **5.** It would take 30 h with both computers working. **7.** It would take 30 min with both air conditioners working. **9.** It would take the second pipeline 90 min to fill the tank. **11.** It would take the apprentice 15 h to construct the wall. **13.** It would take the second technician 3 h to complete the wiring. **15.** It would have taken one welder 40 h to complete the welds. **17.** It would take one machine 28 h working alone. **19.** The rate through the congested traffic is 20 mph. **21.** The rate of the jogger is 8 mph, and the rate of the cyclist is 20 mph. **23.** The rate of the jet is 360 mph. **25.** The rate of the runner is 8 mph. **27.** The rate of the car is 48 mph. **29.** The rate of the wind is 20 mph. **31.** The rate of the gulf current is 6 mph. **33.** The rate of the trucker for the first 330 mi was 55 mph. **35.** The less-experienced helper can complete the job in 30 h. **37.** The bus usually travels 60 mph.

SECTION 8.7

3. The profit is \$80,000. **5.** The pressure is 6.75 lb/in^2. **7.** The object will fall 1600 ft. **9.** The ball will roll 54 ft in 3 s. **11.** At a rate of 65 mph, the time to travel between the two cities is 4.2 h. **13.** The pressure is 112.5 lb/in^2. **15.** The repulsive force is 80 lb. **17.** y is doubled. **19.** inversely **21.** inversely

CHAPTER REVIEW

1. $\frac{b^3y}{10ax}$ [8.1C] **2.** $\frac{7x+22}{60x}$ [8.2D] **3.** $\frac{x-2}{3x-10}$ [8.3A] **4.** $-\frac{x+6}{x+3}$ [8.1A] **5.** $\frac{2x^4}{3y^7}$ [8.1A]

6. 62 [8.4B] **7.** $\frac{(3y-2)^2}{(y-1)(y-2)}$ [8.1C] **8.** $\frac{by^3}{6ax^2}$ [8.1B] **9.** $\frac{x}{x-7}$ [8.3A]

10. $\frac{3x^2-x}{(2x+3)(6x-1)(3x-1)}, \frac{24x^3-4x^2}{(2x+3)(6x-1)(3x-1)}$ [8.2B] **11.** $a=\frac{T-2bc}{2b+2c}$ [8.5A] **12.** 2 [8.4A]

13. $c=\frac{100m}{i}$ [8.5A] **14.** The equation has no solution. [8.4A] **15.** $\frac{1}{x^2}$ [8.1C] **16.** $\frac{2y-3}{5y-7}$ [8.2D]

17. $\frac{1}{x+3}$ [8.2C] **18.** $(5x-3)(2x-1)(4x-1)$ [8.2A] **19.** $y=-\frac{4}{9}x+2$ [8.5A] **20.** $\frac{8x+5}{3x-4}$ [8.1B]

21. 5 [8.4A] **22.** $\frac{3x-1}{x-5}$ [8.2D] **23.** 10 [8.4B] **24.** 12 [8.4B] **25.** The perimeter of triangle ABC is 24 in. [8.4C] **26.** It would take 6 h to fill the pool using both hoses. [8.6A] **27.** The rate of the car is 45 mph. [8.6B] **28.** The rate of the wind is 20 mph. [8.6B] **29.** The pitcher's ERA is 1.35. [8.4D] **30.** The current is 2 amps. [8.7A]

CHAPTER TEST

1. $\frac{2x^3}{3y^3}$ [8.1A] **2.** $-\frac{x+5}{x+1}$ [8.1A] **3.** $\frac{x+1}{x^3(x-2)}$ [8.1B] **4.** $\frac{(x-5)(2x-1)}{(x+3)(2x+5)}$ [8.1B] **5.** $\frac{x+5}{x+4}$ [8.1C]

6. $3(2x-1)(x+1)$ [8.2A] **7.** $\frac{3(x+2)}{x(x+2)(x-2)}; \frac{x^2}{x(x+2)(x-2)}$ [8.2B] **8.** $\frac{2}{x+5}$ [8.2C]

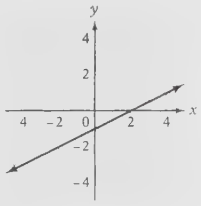
9. $\frac{5}{(2x-1)(3x+1)}$ [8.2D] **10.** $\frac{x^2-4x+5}{(x-2)(x+3)}$ [8.2D] **11.** $\frac{x-3}{x-2}$ [8.3A] **12.** 2 [8.4A] **13.** The equation has no solution. [8.4A] **14.** -1 [8.4B] **15.** The area is 64.8 m^2. [8.4C] **16.** $t=\frac{d-s}{r}$ [8.5A] **17.** 14 rolls of wallpaper are needed. [8.4D] **18.** It would take 10 min with both landscapers working. [8.6A] **19.** The rate of the cyclist is 10 mph. [8.6B] **20.** The resistance is 0.4 ohm. [8.7A]

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CUMULATIVE REVIEW

1. $\frac{31}{30}$ [1.3A] **2.** 21 [1.4A] **3.** $5x - 2y$ [1.4B] **4.** $-8x + 26$ [1.4D] **5.** $-\frac{9}{2}$ [2.2A] **6.** -12 [2.2C]

7. 10 [2.1D] **8.** $\{x|x \le 8\}$ [2.4A] **9.** The volume is 200 ft^3. [3.3A] **10.**



[4.3B]

11. $\frac{3}{7}$ [4.2A] **12.** $y = \frac{3}{2}x + 2$ [4.6A] **13.** -28 [5.3A] **14.** a^3b^7 [6.1A] **15.** $\frac{a^5}{b^6}$ [6.1B]

16. 3.5×10^{-8} [6.1C] **17.** $-4a^4 + 6a^3 - 2a^2$ [6.3A] **18.** $a^2 + ab - 12b^2$ [6.3C] **19.** $x^2 + 2x + 4$ [6.4B]

20. $(y - 6)(y - 1)$ [7.2A] **21.** $(4x + 1)(3x - 1)$ [7.3A/7.3B] **22.** $a(2a - 3)(a + 5)$ [7.3A/7.3B]

23. $4(b + 5)(b - 5)$ [7.4D] **24.** -3 and $\frac{5}{2}$ [7.5A] **25.** $\frac{2x^3}{3y^5}$ [8.1A] **26.** $-\frac{x - 2}{x + 5}$ [8.1A] **27.** 1 [8.1C]

28. $\frac{3}{(2x - 1)(x + 1)}$ [8.2D] **29.** $\frac{x + 3}{x + 5}$ [8.3A] **30.** 4 [8.4A] **31.** The alloy contains 70% silver. [2.3B]

32. It would cost \$160. [8.4D] **33.** It would take 6 min to fill the tank. [8.6A]

Answers to Chapter 9 Selected Exercises

PREP TEST

1. 16 [1.1D] **2.** 32 [1.2E] **3.** 9 [1.2D] **4.** $\frac{1}{12}$ [1.2C] **5.** $-5x - 1$ [1.4D] **6.** $\frac{xy^5}{4}$ [6.1B]

7. $9x^2 - 12x + 4$ [6.3C] **8.** $-12x^2 + 14x + 10$ [6.3C] **9.** $36x^2 - 1$ [6.3D] **10.** $-1, 15$ [7.5A]

SECTION 9.1

1. 2 **3.** 27 **5.** $\frac{1}{9}$ **7.** 4 **9.** $(-25)^{5/2}$ is not a real number. **11.** $\frac{343}{125}$ **13.** x **15.** $y^{1/2}$ **17.** $x^{1/12}$ **19.** $a^{7/12}$

21. $\frac{1}{a}$ **23.** $\frac{1}{y}$ **25.** $y^{3/2}$ **27.** $\frac{1}{x}$ **29.** $\frac{1}{x^4}$ **31.** a **33.** $x^{3/10}$ **35.** a^3 **37.** $\frac{1}{x^{1/2}}$ **39.** $y^{1/9}$ **41.** x^4y

43. $x^6y^3z^9$ **45.** $\frac{x}{y^2}$ **47.** $\frac{x^{3/2}}{y^{1/4}}$ **49.** $\frac{x^2}{y^8}$ **51.** $\frac{1}{x^{11/12}}$ **53.** $\frac{1}{y^{5/2}}$ **55.** $\frac{1}{b^{7/8}}$ **57.** a^5b^{13} **59.** $\frac{m^2}{4n^{3/2}}$ **61.** $\frac{y^{17/2}}{x^3}$

63. $\frac{16b^2}{a^{1/3}}$ **65.** $y^2 - y$ **67.** $a - a^2$ **69.** x^{4n} **71.** $x^{3n/2}$ **73.** $y^{3n/2}$ **75.** x^{2n^2} **77.** $x^{2n}y^n$ **79.** $x^{4n}y^{2n}$

81. $\sqrt[4]{3}$ **83.** $\sqrt{a^3}$ **85.** $\sqrt{32t^5}$ **87.** $-2\sqrt[3]{x^2}$ **89.** $\sqrt[3]{a^4b^2}$ **91.** $\sqrt[5]{a^6b^{12}}$ **93.** $\sqrt[4]{(4x - 3)^3}$ **95.** $\frac{1}{\sqrt[3]{x^2}}$

97. $14^{1/2}$ **99.** $x^{1/3}$ **101.** $x^{4/3}$ **103.** $b^{3/5}$ **105.** $(2x^2)^{1/3}$ **107.** $-(3x^5)^{1/2}$ **109.** $3xy^{2/3}$ **111.** $(a^2 - 2)^{1/2}$ **113.** x^8

115. $-x^4$ **117.** xy^3 **119.** $-x^5y$ **121.** $4a^2b^6$ **123.** $\sqrt{-16x^4y^2}$ is not a real number. **125.** $3x^3$ **127.** $-4x^3y^4$

129. $-x^2y^3$ **131.** x^4y^2 **133.** $3xy^5$ **135.** $2ab^2$ **137a.** false; 2 **b.** true **c.** true **d.** false; $(a^n + b^n)^{1/n}$

e. false; $a + 2a^{1/2}b^{1/2} + b$ **f.** false; $a^{n/m}$

SECTION 9.2

1. $x^2yz^2\sqrt{yz}$ **3.** $2ab^4\sqrt{2a}$ **5.** $3xyz^2\sqrt{5yz}$ **7.** $\sqrt{-9x^3}$ is not a real number. **9.** $a^5b^2\sqrt[3]{ab^2}$ **11.** $-5y\sqrt[3]{x^2y}$

13. $abc^2\sqrt[3]{ab^2}$ **15.** $2x^2y\sqrt[4]{xy}$ **17.** $-6\sqrt{x}$ **19.** $-2\sqrt{2}$ **21.** $3\sqrt{2b} + 5\sqrt{3b}$ **23.** $-2xy\sqrt{2y}$

25. $6ab^2\sqrt{3ab} + 3ab\sqrt{3ab}$ **27.** $-\sqrt[3]{2}$ **29.** $8b\sqrt[3]{2b^2}$ **31.** $3a\sqrt[4]{2a}$ **33.** $17\sqrt{2} - 15\sqrt{5}$ **35.** $5b\sqrt{b}$

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37. $-8xy\sqrt{2x} + 2xy\sqrt{xy}$ **39.** $2y\sqrt[3]{2x}$ **41.** $-4ab\sqrt[4]{2b}$ **43.** 16 **45.** $2\sqrt[3]{4}$ **47.** $xy^3\sqrt{x}$ **49.** $8xy\sqrt{x}$
51. $2x^2y\sqrt[3]{2}$ **53.** $2ab\sqrt[4]{3a^2b}$ **55.** 6 **57.** $x - \sqrt{2x}$ **59.** $4x - 8\sqrt{x}$ **61.** $x - 6\sqrt{x} + 9$ **63.** $84 + 16\sqrt{5}$
65. $672x^2y^2$ **67.** $4a^3b^3\sqrt[3]{a}$ **69.** $-8\sqrt{5}$ **71.** $x - y^2$ **73.** $12x - y$ **77.** $y\sqrt{5y}$ **79.** $b\sqrt{13b}$ **81.** $\frac{\sqrt{2}}{2}$
83. $\frac{2\sqrt{3y}}{3y}$ **85.** $\frac{3\sqrt{3a}}{a}$ **87.** $\frac{\sqrt{2y}}{2}$ **89.** $\frac{5\sqrt[3]{3}}{3}$ **91.** $\frac{5\sqrt[3]{9y^2}}{3y}$ **93.** $\frac{b\sqrt{2a}}{2a^2}$ **95.** $\frac{\sqrt{15x}}{5x}$ **97.** $2 + 2\sqrt{2}$
99. $\frac{-12 - 4\sqrt{2}}{7}$ **101.** $-\frac{10 + 5\sqrt{7}}{3}$ **103.** $-\frac{7\sqrt{x} + 21}{x - 9}$ **105.** $-\sqrt{6} + 3 - 2\sqrt{2} + 2\sqrt{3}$ **107.** $-\frac{17 + 5\sqrt{5}}{4}$
109. $\frac{8a - 10\sqrt{ab} + 3b}{16a - 9b}$ **111.** $\frac{3 - 7\sqrt{y} + 2y}{1 - 4y}$ **113a.** false; $\sqrt[6]{432}$ **b.** true **c.** false; $\sqrt[3]{x^2}$ **d.** false; $\sqrt{x} + \sqrt{y}$
e. false; $\sqrt[2]{2} + \sqrt[3]{3}$ **f.** true **115.** $\sqrt[4]{a+b}$

SECTION 9.3

3. $2i$ **5.** $7i\sqrt{2}$ **7.** $3i\sqrt{3}$ **9.** $4 + 2i$ **11.** $2\sqrt{3} - 3i\sqrt{2}$ **13.** $4\sqrt{10} - 7i\sqrt{3}$ **15.** $8 - i$ **17.** $-8 + 4i$
19. $6 - 6i$ **21.** $19 - 7i\sqrt{2}$ **23.** $6\sqrt{2} - 3i\sqrt{2}$ **25.** 63 **27.** -4 **29.** $-3\sqrt{2}$ **31.** $-4 + 12i$ **33.** $-2 + 4i$
35. $17 - i$ **37.** $8 + 27i$ **39.** 1 **41.** 1 **43.** $-3i$ **45.** $\frac{3}{4} + \frac{1}{2}i$ **47.** $\frac{10}{13} - \frac{2}{13}i$ **49.** $\frac{4}{5} + \frac{2}{5}i$ **51.** $-i$
53. $-\frac{\sqrt{5}}{5} + \frac{2\sqrt{5}}{5}i$ **55.** $\frac{3}{10} - \frac{11}{10}i$ **57.** $\frac{6}{5} + \frac{7}{5}i$ **59a.** yes **b.** yes

SECTION 9.4

1. 25 **3.** 27 **5.** 48 **7.** -2 **9.** The equation has no solution. **11.** 9 **13.** -23 **15.** -16 **17.** $\frac{9}{4}$
19. 14 **21.** 7 **23.** 7 **25.** -122 **27.** 35 **29.** $-5, -2$ **31.** 45 **33.** 23 **35.** -4 **37.** 10 **39.** 1
41. 3 **43.** 21 **45.** $\frac{25}{72}$ **47.** 5 **49.** $-2, 4$ **51.** 6 **53.** 0, 1 **55.** $-2, 2$ **57.** $-2, 1$ **59.** 2, 11
61. The width is 6 ft. **63.** The bottom of the ladder is 10 ft from the wall. **65.** The object has fallen 2500 ft.
67. The periscope must be 8.64 ft above the water. **69.** The length of the pendulum is 4.67 ft. **71.** The distance is 180 m. **73.** The HDTV screen is 7.15 in. wider. **75a.** 2, -2; integer, rational number, real number
b. i, $-i$; imaginary **c.** 16; integer, rational number, real number **77.** $a = \sqrt{c^2 - b^2}$ or $a = -\sqrt{c^2 - b^2}$ **79.** $x = \sqrt{6}$

CHAPTER REVIEW

1. $20x^2y^2$ [9.1A] **2.** 7 [9.4A] **3.** $39 - 2i$ [9.3C] **4.** $7x^{2/3}y$ [9.1B] **5.** $6\sqrt{3} - 13$ [9.2C] **6.** -2 [9.4A]
7. $\frac{1}{x^5}$ [9.1A] **8.** $\frac{8\sqrt{3y}}{3y}$ [9.2D] **9.** $-2a^2b^4$ [9.1C] **10.** $2a^2b\sqrt{2b}$ [9.2B] **11.** $\frac{x\sqrt{x} - x\sqrt{2} + 2\sqrt{x} - 2\sqrt{2}}{x - 2}$ [9.2D]
12. $\frac{2}{3} - \frac{5}{3}i$ [9.3D] **13.** $3ab^3\sqrt{2a}$ [9.2A] **14.** $-4\sqrt{2} + 8i\sqrt{2}$ [9.3B] **15.** $5x^3y^3\sqrt[3]{2x^2y}$ [9.2B]
16. $4xy^2\sqrt[3]{x^2}$ [9.2C] **17.** $7 + 3i$ [9.3C] **18.** $3\sqrt[4]{x^3}$ [9.1B] **19.** $-2ab^2\sqrt[5]{2a^3b^2}$ [9.2A]
20. $-2 + 7i$ [9.3D] **21.** $-6\sqrt{2}$ [9.3C] **22.** 30 [9.4A] **23.** $3a^2b^3$ [9.1C] **24.** $5i\sqrt{2}$ [9.3A]
25. $-12 + 10i$ [9.3B] **26.** $31 - 10\sqrt{6}$ [9.2C] **27.** $6x^2\sqrt{3y}$ [9.2B] **28.** The amount of power generated is 120 watts. [9.4B] **29.** The distance required to reach a velocity of 88 ft/s is 242 ft. [9.4B] **30.** The bottom of the ladder is 6.63 ft from the building. [9.4B]

CHAPTER TEST

1. $\frac{1}{2}x^{3/4}$ [9.1B] **2.** $-2x^2y\sqrt[3]{2x}$ [9.2B] **3.** $3\sqrt[5]{y^2}$ [9.1B] **4.** $18 + 16i$ [9.3C] **5.** $4x + 4\sqrt{xy} + y$ [9.2C]
6. $r^{1/6}$ [9.1A] **7.** 4 [9.4A] **8.** $2xy^2$ [9.1C] **9.** $-4x\sqrt{3}$ [9.2C] **10.** $-3 + 2i$ [9.3B] **11.** $4x^2y^3\sqrt{2y}$ [9.2A]

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12. $14 + 10\sqrt{3}$ [9.2C] **13.** 4 [9.3B] **14.** 2 [9.2D] **15.** $8a\sqrt{2a}$ [9.2B] **16.** $2a - \sqrt{ab} - 15b$ [9.2C]

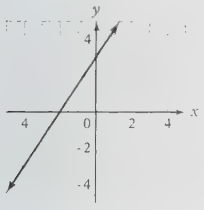
17. $\frac{64x^3}{y^6}$ [9.1A] **18.** $\frac{x + \sqrt{xy}}{x - y}$ [9.2D] **19.** $-\frac{4}{5} + \frac{7}{5}i$ [9.3D] **20.** -3 [9.4A] **21.** $\frac{b^3}{8a^6}$ [9.1A]

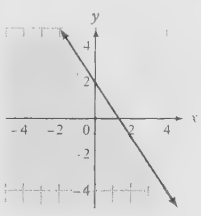
22. $3abc^2\sqrt[3]{ac}$ [9.2A] **23.** $\frac{4x^2}{y}$ [9.2D] **24.** -4 [9.3C] **25.** The object has fallen 576 ft. [9.4B]

CUMULATIVE REVIEW

1. 92 [1.3A] **2.** 56 [1.4A] **3.** $-10x + 1$ [1.4D] **4.** $\frac{3}{2}$ [2.2A] **5.** $\frac{2}{3}$ [2.2C] **6.** $\{x|x > 1\}$ [2.4A]

7. $\frac{1}{3}, \frac{7}{3}$ [2.5A] **8.** $\{x|-6 \le x \le 3\}$ [2.5B] **9.** The area is 187.5 cm^2. [3.2B] **10.** 3 [5.3A]

11.  $m = \frac{3}{2}$, $b = 3$ [4.3B]

12.  [4.7A]

13. $y = \frac{1}{3}x + \frac{7}{3}$ [4.5B]

14. $\left(\frac{17}{4}, \frac{9}{2}\right)$ [5.3B] **15.** $2x^2y^2$ [6.1B] **16.** $(9x + y)(9x - y)$ [7.4A] **17.** $x(x^2 + 3)(x + 1)(x - 1)$ [7.4D]

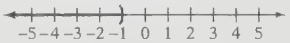
18. $C = R - nP$ [8.5A] **19.** $\frac{y^5}{x^4}$ [9.1A] **20.** $-x\sqrt{10x}$ [9.2B] **21.** $13 - 7\sqrt{3}$ [9.2C] **22.** $\sqrt{6} + \sqrt{2}$ [9.2D]

23. $-\frac{1}{5} + \frac{3}{5}i$ [9.3D] **24.** -20 [9.4A] **25.** The length of side DE is 27 m. [8.4C] **26.** \$4000 must be invested at 8.4%. [2.3C] **27.** The rate of the plane was 250 mph. [8.6B] **28.** It takes 1.25 s for light to travel to Earth from the moon. [6.1D] **29.** The slope is 0.08. The slope represents the simple interest rate on the investment. The interest rate is 8%. [4.4A] **30.** The periscope must be 32.7 ft above the water. [9.4B]

Answers to Chapter 10 Selected Exercises

PREP TEST

1. $3\sqrt{2}$ [1.2F] **2.** $3i$ [9.3A] **3.** $\frac{2x - 1}{x - 1}$ [8.2B] **4.** 8 [1.4A] **5.** yes [7.4A] **6.** $(2x - 1)^2$ [7.4A]

7. $(3x + 2)(3x - 2)$ [7.4A] **8.**  [1.5C] **9.** $-3, 5$ [7.5A] **10.** 4 [8.4B]

SECTION 10.1

3. $2x^2 - 4x - 5 = 0$; $a = 2, b = -4, c = -5$ **5.** $4x^2 - 5x + 6 = 0$; $a = 4, b = -5, c = 6$ **7.** 0, 4 **9.** 5, -5

11. 3, -2 **13.** 3 **15.** 0, 2 **17.** 5, -2 **19.** 2, 5 **21.** 6, $-\frac{3}{2}$ **23.** 2, $\frac{1}{4}$ **25.** $\frac{1}{3}, -4$ **27.** $\frac{9}{2}, -\frac{2}{3}$

29. $\frac{1}{4}, -4$ **31.** 9, -2 **33.** $-2, -\frac{3}{4}$ **35.** 2, -5 **37.** $-4, -\frac{3}{2}$ **39.** $2b, 7b$ **41.** $7c, -c$ **43.** $-\frac{b}{2}, -b$

45. $4a, \frac{2a}{3}$ **47.** $-\frac{a}{3}, 3a$ **49.** $-\frac{3y}{2}, -\frac{y}{2}$ **51.** $-\frac{a}{2}, -\frac{4a}{3}$ **53.** $x^2 - 7x + 10 = 0$ **55.** $x^2 + 6x + 8 = 0$

57. $x^2 - 5x - 6 = 0$ **59.** $x^2 - 9 = 0$ **61.** $x^2 - 8x + 16 = 0$ **63.** $x^2 - 5x = 0$ **65.** $x^2 - 3x = 0$

67. $2x^2 - 7x + 3 = 0$ **69.** $4x^2 - 5x - 6 = 0$ **71.** $3x^2 + 11x + 10 = 0$ **73.** $9x^2 - 4 = 0$ **75.** $6x^2 - 5x + 1 = 0$

77. $10x^2 - 7x - 6 = 0$ **79.** $8x^2 + 6x + 1 = 0$ **81.** $50x^2 - 25x - 3 = 0$ **83.** 7, -7 **85.** $2i, -2i$ **87.** 2, -2

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89. $\frac{9}{2}, -\frac{9}{2}$ **91.** $7i, -7i$ **93.** $4\sqrt{3}, -4\sqrt{3}$ **95.** $5\sqrt{3}, -5\sqrt{3}$ **97.** $3i\sqrt{2}, -3i\sqrt{2}$ **99.** $7, -5$ **101.** $0, -6$

103. $-7, 3$ **105.** $1, 0$ **107.** $-5 + \sqrt{6}, -5 - \sqrt{6}$ **109.** $3 + 3i\sqrt{5}, 3 - 3i\sqrt{5}$ **111.** $\frac{2 - 9\sqrt{2}}{3}, \frac{2 + 9\sqrt{2}}{3}$

113. $x^2 - 2 = 0$ **115.** $x^2 - 18 = 0$ **117.** $\frac{3b}{a}, -\frac{3b}{a}$ **119.** $-\frac{1}{2}$

SECTION 10.2

3. $5, -1$ **5.** $-9, 1$ **7.** 3 **9.** $-2 + \sqrt{11}, -2 - \sqrt{11}$ **11.** $3 + \sqrt{2}, 3 - \sqrt{2}$ **13.** $1 + i, 1 - i$ **15.** $8, -3$

17. $4, -9$ **19.** $\frac{3 + \sqrt{5}}{2}, \frac{3 - \sqrt{5}}{2}$ **21.** $\frac{1 + \sqrt{5}}{2}, \frac{1 - \sqrt{5}}{2}$ **23.** $3 + \sqrt{13}, 3 - \sqrt{13}$ **25.** $5, 3$ **27.** $2 + 3i, 2 - 3i$

29. $-3 + 2i, -3 - 2i$ **31.** $1 + 3\sqrt{2}, 1 - 3\sqrt{2}$ **33.** $\frac{1 + \sqrt{17}}{2}, \frac{1 - \sqrt{17}}{2}$ **35.** $1 + 2i\sqrt{3}, 1 - 2i\sqrt{3}$

37. $\frac{1}{2} + i, \frac{1}{2} - i$ **39.** $\frac{1}{3} + \frac{1}{3}i, \frac{1}{3} - \frac{1}{3}i$ **41.** $\frac{2 + \sqrt{14}}{2}, \frac{2 - \sqrt{14}}{2}$ **43.** $1, -\frac{3}{2}$ **45.** $1 + \sqrt{5}, 1 - \sqrt{5}$ **47.** $\frac{1}{2}, 5$

49. $2 + \sqrt{5}, 2 - \sqrt{5}$ **51.** $1.236, -3.236$ **53.** $1.707, 0.293$ **55.** $0.309, -0.809$ **57.** $x = 2a, x = -a$

59. $x = 2a, x = -5a$

SECTION 10.3

3. $5, -2$ **5.** $4, -9$ **7.** $4 + 2\sqrt{22}, 4 - 2\sqrt{22}$ **9.** $3, -8$ **11.** $\frac{-5 + \sqrt{33}}{4}, \frac{-5 - \sqrt{33}}{4}$ **13.** $\frac{3}{2}, -\frac{1}{4}$

15. $7 + 3\sqrt{5}, 7 - 3\sqrt{5}$ **17.** $\frac{1 + \sqrt{3}}{2}, \frac{1 - \sqrt{3}}{2}$ **19.** $-1 + i, -1 - i$ **21.** $1 + 2i, 1 - 2i$ **23.** $2 + 3i, 2 - 3i$

25. $\frac{1 + \sqrt{11}}{2}, \frac{1 - \sqrt{11}}{2}$ **27.** $-\frac{3}{2} + \frac{1}{2}i, -\frac{3}{2} - \frac{1}{2}i$ **29.** $\frac{3}{4} + \frac{3\sqrt{3}}{4}i, \frac{3}{4} - \frac{3\sqrt{3}}{4}i$ **31.** $7.606, 0.394$ **33.** $0.236, -4.236$

35. $1.851, -1.351$ **37.** two complex number solutions **39.** one real number solution **41.** two real number solutions **43.** $\{p | p < 9\}$ **45.** $\{p | p > 1\}$ **47.** i and $-2i$

SECTION 10.4

1. $3, -3, 2, -2$ **3.** $\sqrt{2}, -\sqrt{2}, 2, -2$ **5.** $1, 4$ **7.** 16 **9.** $2i, -2i, 1, -1$ **11.** $4i, -4i, 2, -2$ **13.** 16

15. $1, 512$ **17.** $\frac{2}{3}, -\frac{2}{3}, 1, -1$ **19.** 3 **21.** 9 **23.** $2, -1$ **25.** $0, 2$ **27.** $2, -\frac{1}{2}$ **29.** -2 **31.** 1 **33.** 1

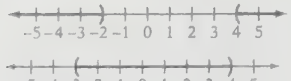
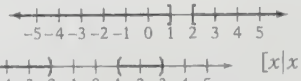
35. -3 **37.** $10, -1$ **39.** $-\frac{1}{2} + \frac{\sqrt{7}}{2}i, -\frac{1}{2} - \frac{\sqrt{7}}{2}i$ **41.** $1, -3$ **43.** $0, -1$ **45.** $\frac{1}{2}, -\frac{1}{3}$ **47.** $-\frac{2}{3}, 6$ **49.** $\frac{4}{3}, 3$

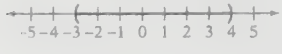
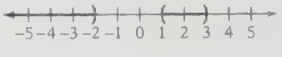
51. $-\frac{1}{4}, 3$ **53.** 4

SECTION 10.5

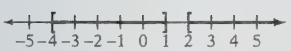
1. The height is 3 cm; the base is 14 cm. **3.** The dimensions of Colorado are approximately 272 mi by 383 mi. **5.** The maximum safe speed is 33 mph. **7.** The time for the projectile to return to Earth is 12.5 s. **9.** The maximum speed is 72.5 km/h. **11.** The smaller pipe requires 12 min. The larger pipe requires 6 min. **13.** The rate of the wind is 108 mph. **15.** The rowing rate of the guide is 6 mph. **17.** The radius of the cone 1.5 in.

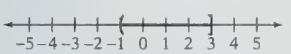
SECTION 10.6

3.  $[x | x < -2 \text{ or } x > 4]$ **5.**  $[x | x \le 1 \text{ or } x \ge 2]$

7.  $[x | -3 < x < 4]$ **9.**  $[x | x < -2 \text{ or } 1 < x < 3]$

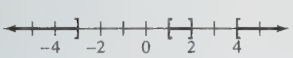
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11.  $\{x|-4 \le x \le 1 \text{ or } x \ge 2\}$ **13.**  $\{x|x < -2 \text{ or } x > 4\}$

15.  $\{x|-1 < x \le 3\}$ **17.**  $\{x|x \le -2 \text{ or } 1 \le x < 3\}$

19. $\{x|x > 4 \text{ or } x < -4\}$ **21.** $\{x|-3 \le x \le 12\}$ **23.** $\left\{x\middle|\frac{1}{2} < x < \frac{3}{2}\right\}$ **25.** $\left\{x\middle|x < 1 \text{ or } x > \frac{5}{2}\right\}$

27. $\{x|x < -1 \text{ or } 1 < x \le 2\}$ **29.** $\left\{x\middle|\frac{1}{2} < x \le 1\right\}$ **31.** $\{x|2 < x \le 3\}$ **33.** $\{x|x > 5 \text{ or } -4 < x < -1\}$

35.  **37.**  **39.** 

CHAPTER REVIEW

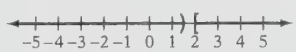
1. $0, \frac{3}{2}$ [10.1A] **2.** $\frac{c}{2}, -2c$ [10.1A] **3.** $4\sqrt{3}, -4\sqrt{3}$ [10.1C] **4.** $-\frac{1}{2} + 2i, -\frac{1}{2} - 2i$ [10.1C]

5. $-3, -1$ [10.2A] **6.** $\frac{7 + 2\sqrt{7}}{7}, \frac{7 - 2\sqrt{7}}{7}$ [10.2A] **7.** $\frac{3}{4}, \frac{4}{3}$ [10.3A] **8.** $\frac{1}{2} + \frac{\sqrt{31}}{2}i, \frac{1}{2} - \frac{\sqrt{31}}{2}i$ [10.3A]

9. $x^2 + 3x = 0$ [10.1B] **10.** $12x^2 - x - 6 = 0$ [10.1B] **11.** $1 + i\sqrt{7}, 1 - i\sqrt{7}$ [10.2A] **12.** $2i, -2i$ [10.2A]

13. $\frac{11 + \sqrt{73}}{6}, \frac{11 - \sqrt{73}}{6}$ [10.3A] **14.** two real number solutions [10.3A] **15.** $\left\{x|-3 < x < \frac{5}{2}\right\}$ [10.6A]

16. $\left\{x|x \le -4 \text{ or } -\frac{3}{2} \le x \le 2\right\}$ [10.6A] **17.** 27, −64 [10.4A] **18.** $\frac{5}{4}$ [10.4B] **19.** $-1, 3$ [10.4C]

20. -1 [10.4C] **21.**  [10.6A] $\left\{x\middle|x < \frac{3}{2} \text{ or } x \ge 2\right\}$

22.  [10.6A] $\left\{x|x \le -3 \text{ or } \frac{1}{2} \le x < 4\right\}$

23. 4 [10.4B] **24.** 5 [10.4B] **25.** $\frac{-3 + \sqrt{249}}{10}, \frac{-3 - \sqrt{249}}{10}$ [10.4C] **26.** $\frac{-11 + \sqrt{129}}{2}, \frac{-11 - \sqrt{129}}{2}$ [10.4C]

27. The length is 12 cm, and the width is 5 cm. [10.5A] **28.** The three integers are 2, 4, and 6 or −6, −4, and −2. [10.5A] **29.** Working alone, the new computer takes 12 min to print the payroll. [10.5A] **30.** The speed of the first car is 40 mph. The speed of the second car is 50 mph. [10.5A]

CHAPTER TEST

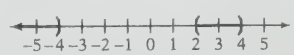
1. $\frac{2}{3}, -4$ [10.1A] **2.** $\frac{3}{2}, -\frac{2}{3}$ [10.1A] **3.** $x^2 - 9 = 0$ [10.1B] **4.** $2x^2 + 7x - 4 = 0$ [10.1B]

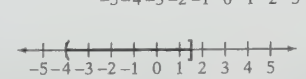
5. $2 + 2\sqrt{2}, 2 - 2\sqrt{2}$ [10.1C] **6.** $3 + \sqrt{11}, 3 - \sqrt{11}$ [10.2A] **7.** $\frac{3 + \sqrt{15}}{3}, \frac{3 - \sqrt{15}}{3}$ [10.2A]

8. $\frac{1 + \sqrt{3}}{2}, \frac{1 - \sqrt{3}}{2}$ [10.3A] **9.** $-2 + 2i\sqrt{2}, -2 - 2i\sqrt{2}$ [10.3A] **10.** two real number solutions [10.3A]

11. two complex number solutions [10.3A] **12.** $\frac{1}{4}$ [10.4A] **13.** $1, -1, \sqrt{3}, -\sqrt{3}$ [10.4A] **14.** 4 [10.4B]

15. The equation has no solution. [10.4B] **16.** 2, −9 [10.4C]

17. $\{x|x < -4 \text{ or } 2 < x < 4\}$  [10.6A]

18. $\left\{x|-4 < x \le \frac{3}{2}\right\}$  [10.6A] **19.** The base is 15 ft. The height is 4 ft. [10.5A]

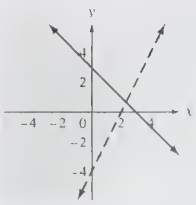
20. The rowing rate of the canoe in calm water is 4 mph. [10.5A]

CUMULATIVE REVIEW

1. 14 [1.4A] **2.** $\left\{x|-2 < x < \frac{10}{3}\right\}$ [2.5B] **3.** The volume is 54π m^3. [3.3A] **4.** $-\frac{7}{3}$ [4.2A] **5.** $-\frac{3}{2}$ [4.4A]

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6. $\left(\frac{5}{2}, 0\right), (0, -3)$ [4.3B] **7.** $y = x + 1$ [4.6A] **8.** $(1, -1, 2)$ [5.2B] **9.**



[5.5A]

10. The height of triangle *DEF* is 16 cm. [8.4C] **11.** $x^2 - 3x - 4 - \frac{6}{3x - 4}$ [6.4A] **12.** $-3xy(x^2 - 2xy + 3y^2)$ [7.1A]

13. $(2x - 5)(3x + 4)$ [7.3A/7.3B] **14.** $\frac{x}{2}$ [8.1B] **15.** $-\frac{3}{2}$ and -1 [8.4A] **16.** $b = \frac{2S - an}{n}$ [8.5A]

17. $1 - a$ [9.1A] **18.** $-8 - 14i$ [9.3C] **19.** 0, 1 [9.4A] **20.** $2, -2, \sqrt{2}, -\sqrt{2}$ [10.4A] **21.** The lower limit is $9\frac{23}{64}$ in. The upper limit is $9\frac{25}{64}$ in. [2.5C] **22.** The area is $(x^2 + 6x - 16)$ ft^2. [6.3E] **23.** The slope is $-\frac{25{,}000}{3}$. The building decreases \$8333.33 in value each year. [4.4A] **24.** The ladder will reach 15 ft up on the building. [9.4B]

25. There are two complex number solutions. [10.3A]

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INTRODUCTORY AND INTERMEDIATE ALGEBRA

STUDENT SOLUTION MANUAL

CHAPTERS 6-10

Introductory and Intermediate Algebra
Student Solutions Manual

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Chapter 6
Polynomials

Prep Test

1. $-2-(-3)=-2+3=1$ **2.** $-3(6)=-18$ **3.** $-\frac{24}{-36}=\frac{2}{3}$ **4.** $-4(3y)=-12y$

5. $(-2)^3=-2\cdot-2\cdot-2=-8$ **6.** $-4a-8b+7a$
$=3a-8b$

7. $3x-2[y-4(x+1)+5]$
$=3x-2[y-4x-4+5]$
$=3x-2[-4x+y+1]$
$=3x+8x-2y-2$
$=11x-2y-2$

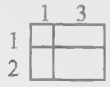
8. $-4y+4y=0$

9. No

Go Figure

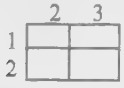
Since the other three smaller rectangles are of different sizes, the possible values for x cannot be the same as any of the three other rectangles.

The first possible value of x is 1. The lengths of the sides of the rectangles are shown below.

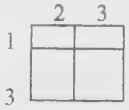


The second possible value of x is 4.

The first possible value of x is 4. The lengths of the sides of the rectangles are shown below.



The third possible value of x is 9. The lengths of the sides of the rectangles are shown below.



Section 6.1

Objective A Exercises

1. a is the multiplication of two exponential expressions.
b is the power of an exponential expression.

3. $(ab^3)(a^3b)=a^4b^4$

5. $(9xy^2)(-2x^2y^2)=-18x^3y^4$

7. $(x^2y^4)^4=x^8y^{16}$

9. $(-3x^2y^3)^4=(-3)^4x^8y^{12}=81x^8y^{12}$

11. $(3^3a^5b^3)^2=3^6a^{10}b^6=729a^{10}b^6$

13. $(x^2y^2)(xy^3)^3=(x^2y^2)(x^3y^9)$
$=x^5y^{11}$

15. $[(3x)^3]^2=(3x)^6=3^6x^6=729x^6$

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17. $\left[(ab)^3\right]^6 = (ab)^{18} = a^{18}b^{18}$

19. $\left[(2xy)^3\right]^4 = (2xy)^{12} = 2^{12}x^{12}y^{12} = 4096x^{12}y^{12}$

21. $\left[\left(2a^4b^3\right)^3\right]^2 = \left(2a^4b^3\right)^6 = 2^6a^{24}b^{18} = 64a^{24}b^{18}$

23. $x^n \cdot x^{n+1} = x^{n+n+1} = x^{2n+1}$

25. $y^{3n} \cdot y^{3n-2} = y^{3n+3n-2^1} = y^{6n-2}$

27. $\left(a^{n-3}\right)^{2n} = a^{(n-3)2n} = a^{2n^2-6n}$

29. $\left(x^{3n+2}\right)^5 = x^{(3n+2)5} = x^{15n+10}$

31. $(2xy)\left(-3x^2yz\right)\left(x^2y^3z^3\right) = -6x^5y^5z^4$

33. $\left(3b^5\right)\left(2ab^2\right)\left(-2ab^2c^2\right) = -12a^2b^9c^2$

35. $\left(-2x^2y^3z\right)\left(3x^2yz^4\right) = -6x^4y^4z^5$

37. $\left(-3ab^3\right)^3\left(-2^2a^2b\right)^2 = \left[(-3)^3a^3b^9\right]\left[\left(-2^2\right)^2a^4b^2\right] = \left(-27a^3b^9\right)\left(16a^4b^2\right) = -432a^7b^{11}$

39. $\left(-2ab^2\right)\left(-3a^4b^5\right)^3 = \left(-2ab^2\right)\left[(-3)^3a^{12}b^{15}\right] = \left(-2ab^2\right)\left(-27a^{12}b^{15}\right) = 54a^{13}b^{17}$

Objective B Exercises

41. $\frac{1}{3^{-5}} = 3^5 = 243$ **43.** $\frac{1}{y^{-3}} = y^3$

45. $\frac{a^3}{4b^{-2}} = \frac{a^3b^2}{4}$

47. $xy^{-4} = \frac{x}{y^4}$

49. $\frac{1}{2x^0} = \frac{1}{2}$

51. $\frac{-3^{-2}}{(2y)^0} = \frac{-1}{3^2} = -\frac{1}{9}$

53. $\frac{y^{-2}}{y^6} = y^{-8} = \frac{1}{y^8}$

55. $\left(x^3y^5\right)^{-2} = x^{-6}y^{-10} = \frac{1}{x^6y^{10}}$

57. $\frac{x^4y^3}{x^{-1}y^{-2}} = x^5y^5$

59. $\frac{a^6b^{-4}}{a^{-2}b^5} = a^8b^{-9} = \frac{a^8}{b^9}$

61. $(3a)^{-3}\left(9a^{-1}\right)^{-2} = (3a)^{-3}\left(3^2a^{-1}\right)^{-2} = \left(3^{-3}a^{-3}\right)\left(3^{-4}a^2\right) = 3^{-7}a^{-1} = \frac{1}{3^7a} = \frac{1}{2187a}$

63. $\left(x^{-1}y^2\right)^{-3}\left(x^2y^{-4}\right)^{-3} = \left(x^3y^{-6}\right)\left(x^{-6}y^{12}\right) = x^{-3}y^6 = \frac{y^6}{x^3}$

65. $\frac{x^3y^6}{x^6y^2} = \frac{y^4}{x^3}$

67. $\frac{-6x^2y}{12x^4y} = -\frac{1}{2x^2}$

69. $\frac{-3ab^2}{\left(9a^2b^4\right)^3} = \frac{-3ab^2}{9^3a^6b^{12}} = \frac{-3ab^2}{729a^6b^{12}} = -\frac{1}{243a^5b^{10}}$

71. $\left(\frac{12x^3y^2z}{18xy^3z^4}\right)^4 = \left(\frac{2x^2}{3yz^3}\right)^4 = \frac{2^4x^8}{3^4y^4z^{12}} = \frac{16x^8}{81y^4z^{12}}$

73. $\frac{\left(3a^2b\right)^3}{\left(-6ab^3\right)^2} = \frac{3^3a^6b^3}{(-6)^2a^2b^6} = \frac{27a^6b^3}{36a^2b^6} = \frac{3a^4}{4b^3}$

75. $\frac{\left(-3a^2b^3\right)^2}{\left(-2ab^4\right)^3} = \frac{(-3)^2a^4b^6}{(-2)^3a^3b^{12}} = \frac{9a^4b^6}{-8a^3b^{12}} = -\frac{9a}{8b^6}$

77. $\frac{\left(-8x^2y^2\right)^4}{\left(16x^3y^7\right)^2} = \frac{(-8)^4x^8y^8}{16^2x^6y^{14}} = \frac{4096x^8y^8}{256x^6y^{14}} = \frac{16x^2}{y^6}$

79. $\frac{b^{6n}}{b^{10n}} = \frac{1}{b^{10n-6n}} = \frac{1}{b^{4n}}$

81. $\frac{y^{2n}}{-y^{8n}} = -\frac{1}{y^{8n-2n}} = -\frac{1}{y^{6n}}$

83. $\frac{y^{3n+2}}{y^{2n+4}} = y^{3n+2-(2n+4)} = y^{3n+2-2n-4} = y^{n-2}$

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85. $\dfrac{x^n y^{3n}}{x^n y^{5n}} = \dfrac{1}{y^{5n-3n}} = \dfrac{1}{y^{2n}}$

87. $\dfrac{x^{2n-1}y^{n-3}}{x^{n+4}y^{n+3}} = x^{2n-1-(n+4)}y^{n-3-(n+3)} = x^{2n-1-n-4}y^{n-3-n-3} = x^{n-5}y^{-6} = \dfrac{x^{n-5}}{y^6}$

89. $\left(\dfrac{9ab^{-2}}{8a^{-2}b}\right)^{-2}\left(\dfrac{3a^{-2}b}{2a^2b^{-2}}\right)^3 = \left(\dfrac{9a^3b^{-3}}{8}\right)^{-2}\left(\dfrac{3a^{-4}b^3}{2}\right) = \dfrac{9^{-2}a^{-6}b^6}{8^{-2}} \cdot \dfrac{3^3a^{-12}b^9}{2^3} = \dfrac{8^2 \cdot 3^2a^{-18}b^{15}}{9^2 \cdot 2^3} = \dfrac{64 \cdot 27b^{15}}{81 \cdot 8a^{18}} = \dfrac{8b^{15}}{3a^{18}}$

Objective C Exercises

91. $0.00000467 = 4.67 \times 10^{-6}$

93. $0.00000000017 = 1.7 \times 10^{-10}$

95. $200{,}000{,}000{,}000 = 2 \times 10^{11}$

97. $1.23 \times 10^{-7} = 0.000000123$

99. $8.2 \times 10^{15} = 8{,}200{,}000{,}000{,}000{,}000$

101. $3.9 \times 10^{-2} = 0.039$

103. $(3 \times 10^{-12})(5 \times 10^{16}) = (3)(5) \times 10^{-12+16} = 15 \times 10^4 = 150{,}000$

105. $(0.0000065)(3{,}200{,}000{,}000{,}000) = (6.5 \times 10^{-6})(3.2 \times 10^{12})$
$= (6.5)(3.2) \times 10^{-6+12}$
$= 20.8 \times 10^6 = 20{,}800{,}000$

107. $\dfrac{9 \times 10^{-3}}{6 \times 10^5} = 1.5 \times 10^{-3-5} = 1.5 \times 10^{-8} = 0.000000015$

109. $\dfrac{0.0089}{500{,}000{,}000} = \dfrac{8.9 \times 10^{-3}}{5 \times 10^8} = 1.78 \times 10^{-3-8} = 1.78 \times 10^{-11}$
$= 0.0000000000178$

111. $\dfrac{0.00056}{0.000000000004} = \dfrac{5.6 \times 10^{-4}}{4 \times 10^{-12}} = 1.4 \times 10^{-4-(-12)} = 1.4 \times 10^8$
$= 140{,}000{,}000$

113. $\dfrac{(3.3 \times 10^{-11})(2.7 \times 10^{15})}{8.1 \times 10^{-3}} = \dfrac{(3.3)(2.7) \times 10^{-11+15-(-3)}}{8.1}$
$= 1.1 \times 10^7 = 11{,}000{,}000$

115. $\dfrac{(0.00000004)(84{,}000)}{(0.0003)(1{,}400{,}000)} = \dfrac{4 \times 10^{-8} \times 8.4 \times 10^4}{3 \times 10^{-4} \times 1.4 \times 10^6}$
$= \dfrac{(4)(8.4) \times 10^{-8+4-(-4)-6}}{(3)(1.4)} = 8 \times 10^{-6}$
$= 0.000008$

Objective D Exercises

117. Strategy

To find the distance traveled:

- Write the speed of light in scientific notation.
- Write the number of seconds in one day in scientific notation.
- Use the equation $d = rt$, where r is the speed of light and t is the number of seconds in one day.

Solution

$300{,}000 = 3 \times 10^5$

$24 \cdot 60 \cdot 60 = 86{,}400 = 8.64 \times 10^4$

$d = rt = (3 \times 10^5)(8.64 \times 10^4)$
$= 3 \times 8.64 \times 10^9 = 25.92 \times 10^9$
$= 2.592 \times 10^{10}$

Light travels 2.592×10^{10} km in one day.

119. Strategy

To find the speed:

- Use the equation $r = \dfrac{d}{t}$, where r is the speed, d is the distance traveled and t is the time.

Solution

$r = \dfrac{d}{t} = \dfrac{119 \times 10^6}{11} = 1.08\overline{1} \times 10^7$

The signals travel at a rate of $1.08\overline{1} \times 10^7$ mi/min.

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121. Strategy

To find the time for one revolution:
- Write the number of seconds in one minute (60) in scientific notation.
- Divide the number of revolutions per minute by the number of seconds in one minute.
- Find the reciprocal of the number of revolutions per second, which is the number of seconds per revolution.

Solution

$$60 = 6 \times 10$$

$$\frac{9 \times 10^7}{6 \times 10} = 1.5 \times 10^6$$

$$\frac{1}{1.5 \times 10^6} = \frac{1}{1.5} \times 10^{-6}$$

$$= 0.6\overline{6} \times 10^{-6} = 6.\overline{6} \times 10^{-7}$$

The high-speed centrifuge can make one revolution in $6.\overline{6} \times 10^{-7}$ s.

123. Strategy

To find the number of times heavier the sun is:
- Divide the mass of the sun by the mass of the earth.

Solution

$$\frac{2 \times 10^{33}}{5.9 \times 10^{27}} = 0.338983 \times 10^6 = 3.38983 \times 10^5$$

The sun is 3.38983×10^5 times heavier than the earth.

125. Strategy

To find the weight of one seed:
- Write the number of seeds per ounce in scientific notation.
- Find the reciprocal of the number of seeds per ounce, which is the number of ounces per seed.

Solution

$$31{,}000{,}000 = 3.1 \times 10^7$$

$$\frac{1}{3.1 \times 10^7} = 0.3225806 \times 10^{-7}$$

$$= 3.225806 \times 10^{-8}$$

The weight of one orchid seed is 3.225806×10^{-8} oz.

127. Strategy

To find the distance:
- Use the equation $d = rt$, where r is the speed of light and t is the time.

Solution

$$60 \cdot 60 \cdot 24 \cdot 365 \cdot 2.8 \times 10^8 = 8.83 \times 10^{15}$$

$$d = rt = (1.86 \times 10^5)(8.83 \times 10^{15}) = 1.6423949 \times 10^{21} \text{ mi}$$

The Coma cluster is 1.6423949×10^{21} mi from earth.

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Section 6.2

Objective A Exercises

1. $P(3) = 3(3)^2 - 2(3) - 8$
$P(3) = 13$

3. $R(2) = 2(2)^3 - 3(2)^2 + 4(2) - 2$
$R(2) = 10$

5. $f(-1) = (-1)^4 - 2(-1)^2 - 10$
$f(-1) = -11$

7. Polynomial: (a) –1; (b) 8; (c) 2

9. Not a polynomial.

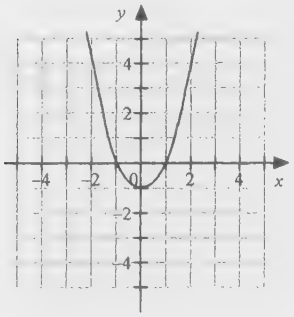
11. Not a polynomial.

13. Polynomial: (a) 3; (b) π; (c) 5

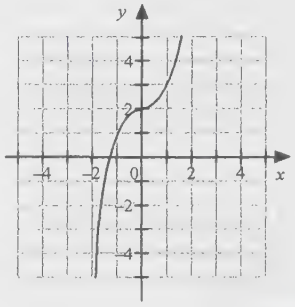
15. Polynomial: (a) –5; (b) 2; (c) 3

17. Polynomial: (a) 14; (b) 14; (c) 0

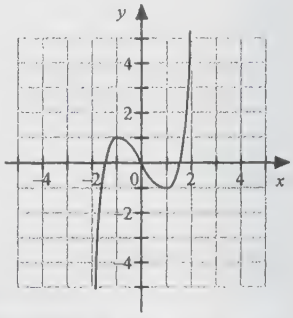
19.



21.



23.



Objective B Exercises

25.
$$\begin{array}{rrrr} 5x^2 & + 2x & - 7 & \\ x^2 & - 8x & + 12 & \\ \hline 6x^2 & - 6x & + 5 & \end{array}$$

27.
$$\begin{array}{rrrr} x^2 & - 3x & + 8 & \\ -2x^2 & + 3x & - 7 & \\ \hline -x^2 & & + 1 & \end{array}$$

29. $(3y^2 - 7y) + (2y^2 - 8y + 2) = (3y^2 + 2y^2) + (-7y - 8y) + 2 = 5y^2 - 15y + 2$

31. $(2a^2 - 3a - 7) - (-5a^2 - 2a - 9) = (2a^2 + 5a^2) + (-3a + 2a) + (-7 + 9) = 7a^2 - a + 2$

33.
$$\begin{aligned} P(x) + R(x) &= (x^2 - 3x + 1) + (2x^2 - 3) \\ &= (x^2 + 2x^2) - 3x + (1 - 3) \\ &= 3x^2 - 3x - 2 \end{aligned}$$

35.
$$\begin{aligned} P(x) - R(x) &= (3x^2 + 2) - (-5x^2 + 2x - 3) \\ &= (3x^2 + 5x^2) - 2x + (2 + 3) \\ &= 8x^2 - 2x + 5 \end{aligned}$$

37.
$$\begin{aligned} S(x) &= (3x^4 - 3x^3 - x^2) + (3x^3 - 7x^2 + 2x) \\ &= 3x^4 + (-3x^3 + 3x^3) + (-x^2 - 7x^2) + 2x \end{aligned}$$
$S(x) = 3x^4 - 8x^2 + 2x$
$S(2) = 3(2)^4 - 8(2)^2 + 2(2)$
$S(2) = 20$

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Applying the Concepts

39. **a.**
$$(2x^3+3x^2+kx+5)-(x^3+2x^2+3x+7)=x^3+x^2+5x-2$$
$$(2x^3-x^3)+(3x^2-2x^2)+(kx-3x)+(5-7)=x^3+x^2+5x-2$$
$$x^3+x^2+(k-3)x-2=x^3+x^2+5x-2$$
$$(k-3)x=5x$$
$$k-3=5$$
$$k=8$$

b.
$$(6x^3+kx^2-2x-1)-(4x^3-3x^2+1)=2x^3-x^2-2x-2$$
$$(6x^3-4x^3)+(kx^2+3x^2)+(-2x)+(-1-1)=2x^3-x^2-2x-2$$
$$2x^3+(k+3)x^2-2x-2=2x^3-x^2-2x-2$$
$$(k+3)x^2=-x^2$$
$$k+3=-1$$
$$k=-4$$

Section 6.3

Objective A Exercises

1. $x(x-2)=x^2-2x$

3. $-x(x+7)=-x^2-7x$

5. $3a^2(a-2)=3a^3-6a^2$

7. $-5x^2(x^2-x)=-5x^4+5x^3$

9. $-x^3(3x^2-7)=-3x^5+7x^3$

11. $2x(6x^2-3x)=12x^3-6x^2$

13. $(2x-4)(3x)=6x^2-12x$

15. $(3x+4)x=3x^2+4x$

17. $-xy(x^2-y^2)=-x^3y+xy^3$

19. $x(2x^3-3x+2)=2x^4-3x^2+2x$

21. $-a(-2a^2-3a-2)=2a^3+3a^2+2a$

23. $x^2(3x^4-3x^2-2)=3x^6-3x^4-2x^2$

25. $2y^2(-3y^2-6y+7)=-6y^4-12y^3+14y^2$

27. $(a^2+3a-4)(-2a)=-2a^3-6a^2+8a$

29. $-3y^2(-2y^2+y-2)=6y^4-3y^3+6y^2$

31. $xy(x^2-3xy+y^2)=x^3y-3x^2y^2+xy^3$

Objective B Exercises

33.
$$\begin{array}{r} x^2+3x+2 \\ \times\quad x+1 \\ \hline x^2+3x+2 \\ x^3+3x^2+2x \\ \hline x^3+4x^2+5x+2 \end{array}$$

35.
$$\begin{array}{r} a^2-3a+4 \\ \times\quad a-3 \\ \hline -3a^2+9a-12 \\ a^3-3a^2+4a \\ \hline a^3-6a^2+13a-12 \end{array}$$

37.
$$\begin{array}{r} -2b^2-3b+4 \\ \times\quad b-5 \\ \hline 10b^2+15b-20 \\ -2b^3-3b^2+4b \\ \hline -2b^3+7b^2+19b-20 \end{array}$$

39.
$$\begin{array}{r} -2x^2+7x-2 \\ \times\quad 3x-5 \\ \hline 10x^2-35x+10 \\ -6x^3+21x^2-6x \\ \hline -6x^3+31x^2-41x+10 \end{array}$$

41.
$$\begin{array}{r} x^2+5 \\ \times\quad x-3 \\ \hline -3x^2-15 \\ x^3+5x \\ \hline x^3-3x^2+5x-15 \end{array}$$

43.
$$\begin{array}{r} x^3-3x+2 \\ \times\quad x-4 \\ \hline -4x^3+12x-8 \\ x^4-3x^2+2x \\ \hline x^4-4x^3-3x^2+14x-8 \end{array}$$

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45.
$$\begin{array}{r} 5y^2 + 8y - 2 \\ \times \quad 3y - 8 \\ \hline -40y^2 - 64y + 16 \\ 15y^3 + 24y^2 - 6y \\ \hline 15y^3 - 16y^2 - 70y + 16 \end{array}$$

47.
$$\begin{array}{r} 5a^3 - 5a + 2 \\ \times \quad a - 4 \\ \hline -20a^3 + 20a - 8 \\ 5a^4 - 5a^2 + 2a \\ \hline 5a^4 - 20a^3 - 5a^2 + 22a - 8 \end{array}$$

49.
$$\begin{array}{r} y^3 + 2y^2 - 3y + 1 \\ \times \quad y + 2 \\ \hline 2y^3 + 4y^2 - 6y + 2 \\ y^4 + 2y^3 - 3y^2 + y \\ \hline y^4 + 4y^3 + y^2 - 5y + 2 \end{array}$$

Objective C Exercises

51. $(x+1)(x+3) = x^2 + 3x + x + 3 = x^2 + 4x + 3$

53. $(a-3)(a+4) = a^2 + 4a - 3a - 12 = a^2 + a - 12$

55. $(y+3)(y-8) = y^2 - 8y + 3y - 24 = y^2 - 5y - 24$

57. $(y-7)(y-3) = y^2 - 3y - 7y + 21 = y^2 - 10y + 21$

59. $(2x+1)(x+7) = 2x^2 + 14x + x + 7 = 2x^2 + 15x + 7$

61. $(3x-1)(x+4) = 3x^2 + 12x - x - 4 = 3x^2 + 11x - 4$

63. $(4x-3)(x-7) = 4x^2 - 28x - 3x + 21 = 4x^2 - 31x + 21$

65. $(3y-8)(y+2) = 3y^2 + 6y - 8y - 16 = 3y^2 - 2y - 16$

67. $(3x+7)(3x+11) = 9x^2 + 33x + 21x + 77 = 9x^2 + 54x + 77$

69. $(7a-16)(3a-5) = 21a^2 - 35a - 48a + 80 = 21a^2 - 83a + 80$

71. $(3a-2b)(2a-7b) = 6a^2 - 21ab - 4ab + 14b^2 = 6a^2 - 25ab + 14b^2$

73. $(a-9b)(2a+7b) = 2a^2 + 7ab - 18ab - 63b^2 = 2a^2 - 11ab - 63b^2$

75. $(10a-3b)(10a-7b) = 100a^2 - 70ab - 30ab + 21b^2 = 100a^2 - 100ab + 21b^2$

77. $(5x+12y)(3x+4y) = 15x^2 + 20xy + 36xy + 48y^2 = 15x^2 + 56xy + 48y^2$

79. $(2x-15y)(7x+4y) = 14x^2 + 8xy - 105xy - 60y^2 = 14x^2 - 97xy - 60y^2$

81. $(8x-3y)(7x-5y) = 56x^2 - 40xy - 21xy + 15y^2 = 56x^2 - 61xy + 15y^2$

Objective D Exercises

85. $(y-5)(y+5) = y^2 - 25$

87. $(2x+3)(2x-3) = 4x^2 - 9$

89. $(x+1)^2 = x^2 + 2x + 1$

91. $(3a-5)^2 = 9a^2 - 30a + 25$

93. $(3x-7)(3x+7) = 9x^2 - 49$

95. $(2a+b)^2 = 4a^2 + 4ab + b^2$

97. $(4-3y)(4+3y) = 16 - 9y^2$

99. $(5x+2y)^2 = 25x^2 + 20xy + 4y^2$

Objective E Exercises

101. Strategy

- To find the area, replace the variables L and W in the equation $A = L \cdot W$ by the given values and solve for A.

Solution

$A = L \cdot W$

$A = (5x)(2x - 7)$

$A = 10x^2 - 35x$

The area is $(10x^2 - 35x)\ \text{ft}^2$.

103. Strategy

- To find the area, replace the variable s in the equation $A = s^2$ by the given values and solve for A.

Solution

$A = s^2$

$A = (2x+1)^2$

$A = (2x+1)(2x+1)$

$A = 4x^2 + 4x + 1$

The area is $(4x^2 + 4x + 1)\ \text{km}^2$.

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105. Strategy

- To find the area, replace the variables L and W in the equation $A = L \cdot W$ by the given values and solve for A.
 $W = (3x + 1)$, $L = 2W = 2(3x + 1)$.

Solution

$A = L \cdot W$
$A = 2(3x + 1)(3x + 1)$
$A = (6x + 2)(3x + 1)$
$A = 18x^2 + 12x + 2$
The area is $(18x^2 + 12x + 2)\ \text{in}^2$.

107. Strategy

- To find the area, replace the variables b and h in the equation $A = \frac{1}{2}bh$ by the given values and solve for A.

Solution

$A = \frac{1}{2}bh$
$A = \frac{1}{2}(4x)(2x + 5)$
$A = 2x(2x + 5)$
$A = 4x^2 + 10x$
The area is $(4x^2 + 10x)\ \text{m}^2$.

109. Strategy

- One side: 30
 Second side: $100 + 2w$
- Use the equation for the area of a rectangle.

Solution

$A = LW$
$A = 30(100 + 2w)$
$A = 3000 + 60w$
The area is $(3000 + 60w)\ \text{yd}^2$.

Applying the Concepts

111. $(a+b)^2 - (a-b)^2 = (a^2 + 2ab + b^2) - (a^2 - 2ab + b^2) = a^2 + 2ab + b^2 - a^2 + 2ab - b^2 = 4ab$

113. $(a+3)^3 = (a+3)^2(a+3) = (a^2 + 6a + 9)(a+3)$
$= a^3 + 3a^2 + 6a^2 + 18a + 9a + 27$
$= a^3 + 9a^2 + 27a + 27$

115. a. $(2x-5)(3x+1) + (x^2 + 2x - 3) = 6x^2 + 2x - 15x - 5 + x^2 + 2x - 3 = 7x^2 - 11x - 8$

b. $(x^2 + x + 3)(x-4) - (4x^2 - x - 5) = x^3 - 4x^2 + x^2 - 4x + 3x - 12 - 4x^2 + x + 5 = x^3 - 7x^2 - 7$

Section 6.4

Objective A Exercises

1.

$$x-5\overline{)\,x^2+3x-40}\qquad x+8$$
$$x^2-5x$$
$$8x-40$$
$$8x-40$$
$$0$$

$(x^2 + 3x - 40) \div (x-5) = x + 8$

3.

$$x-3\overline{)\,x^3-3x^2+0+2}\qquad x^2$$
$$x^3-3x^2$$
$$2$$

$(x^3 - 3x^2 + 2) \div (x-3) = x^2 + \frac{2}{x-3}$

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5.
$$\begin{array}{r} 3x+5 \\ 2x+1\overline{)6x^2+13x+8} \\ \underline{6x^2+\ 3x} \\ 10x+8 \\ \underline{10x+5} \\ 3 \end{array}$$

$$(6x^2+13x+8)\div(2x+1)=3x+5+\frac{3}{2x+1}$$

7.
$$\begin{array}{r} 5x+7 \\ 2x-1\overline{)10x^2+9x-5} \\ \underline{10x^2-5x} \\ 14x-5 \\ \underline{14x-7} \\ 2 \end{array}$$

$$(10x^2+9x-5)\div(2x-1)=5x+7+\frac{2}{2x-1}$$

9.
$$\begin{array}{r} 4x^2+6x+9 \\ 2x-3\overline{)8x^3+\ \ 0+\ 0-9} \\ \underline{8x^3-12x^2} \\ 12x^2+\ 0 \\ \underline{12x^2-18x} \\ 18x-\ 9 \\ \underline{18x-27} \\ 18 \end{array}$$

$$(8x^3-9)\div(2x-3)=4x^2+6x+9+\frac{18}{2x-3}$$

11.
$$\begin{array}{r} 3x^2\ \ \ +1 \\ 2x^2-5\overline{)6x^4+\ \ 0-13x^2+0-4} \\ \underline{6x^4\ \ \ \ -15x^2} \\ 2x^2+0-4 \\ \underline{2x^2\ \ \ -5} \\ 1 \end{array}$$

$$(6x^4-13x^2-4)\div(2x^2-5)=3x^2+1+\frac{1}{2x^2-5}$$

13.
$$\begin{array}{r} x^2-\ 3x-10 \\ 3x+1\overline{)3x^3-8x^2-33x-10} \\ \underline{3x^3+\ x^2} \\ -9x^2-33x \\ \underline{-9x^2-\ 3x} \\ -30x-10 \\ \underline{-30x-10} \\ 0 \end{array}$$

$$\frac{3x^3-8x^2-33x-10}{3x+1}=x^2-3x-10$$

15.
$$\begin{array}{r} x^2-2x+1 \\ x-3\overline{)x^3-5x^2+7x-4} \\ \underline{x^3-3x^2} \\ -2x^2+7x \\ \underline{-2x^2+6x} \\ x-4 \\ \underline{x-3} \\ -1 \end{array}$$

$$\frac{x^3-5x^2+7x-4}{x-3}=x^2-2x+1-\frac{1}{x-3}$$

17.
$$\begin{array}{r} 2x^3\ -3x^2\ +x-\ 4 \\ x-5\overline{)2x^4-13x^3+16x^2-9x+20} \\ \underline{2x^4-10x^3} \\ -3x^3+16x^2 \\ \underline{-3x^3+15x^2} \\ x^2-9x \\ \underline{x^2-5x} \\ -4x+20 \\ \underline{-4x+20} \\ 0 \end{array}$$

$$\frac{2x^4-13x^3+16x^2-9x+20}{x-5}=2x^3-3x^2+x-4$$

19.
$$\begin{array}{r} 2x \\ x^2+2x-1\overline{)2x^3+4x^2-\ x+2} \\ \underline{2x^3+4x^2-2x} \\ x+2 \end{array}$$

$$\frac{2x^3+4x^2-x+2}{x^2+2x-1}=2x+\frac{x+2}{x^2+2x-1}$$

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21.
$$\begin{array}{r} x^2+4x+6 \\ x^2-2x-1\overline{)\,x^4+2x^3-3x^2-6x+2} \\ \underline{x^4-2x^3-\ x^2} \\ 4x^3-2x^2-6x \\ \underline{4x^3-8x^2-4x} \\ 6x^2-\ 2x+2 \\ \underline{6x^2-12x-6} \\ 10x+8 \end{array}$$

$$\frac{x^4+2x^3-3x^2-6x+2}{x^2-2x-1}=x^2+4x+6+\frac{10x+8}{x^2-2x-1}$$

23.
$$\begin{array}{r} x^2-2x+4 \\ x^2+2x+3\overline{)\,x^4+\ 0\ +3x^2-4x+5} \\ \underline{x^4+2x^3+3x^2} \\ -2x^3+0x^2-4x \\ \underline{-2x^3-4x^2-6x} \\ 4x^2+2x+\ 5 \\ \underline{4x^2+8x+12} \\ -6x-\ 7 \end{array}$$

$$\frac{x^4+3x^2-4x+5}{x^2+2x+3}=x^2-2x+4+\frac{-6x-7}{x^2+2x+3}$$

Objective B Exercises

25.
$$\begin{array}{r|rrr} -1 & 2 & -6 & -8 \\ & & -2 & 8 \\ \hline & 2 & -8 & 0 \end{array}$$

$(2x^2-6x-8)\div(x+1)=2x-8$

27.
$$\begin{array}{r|rrr} 2 & 3 & -14 & 16 \\ & & 6 & -16 \\ \hline & 3 & -8 & 0 \end{array}$$

$(3x^2-14x+16)\div(x-2)=3x-8$

29.
$$\begin{array}{r|rrr} 1 & 3 & 0 & -4 \\ & & 3 & 3 \\ \hline & 3 & 3 & -1 \end{array}$$

$(3x^2-4)\div(x-1)=3x+3-\frac{1}{x-1}$

31.
$$\begin{array}{r|rrrr} -1 & 2 & -1 & 6 & 9 \\ & & -2 & 3 & -9 \\ \hline & 2 & -3 & 9 & 0 \end{array}$$

$(2x^3-x^2+6x+9)\div(x+1)=2x^2-3x+9$

33.
$$\begin{array}{r|rrrr} 2 & 4 & 0 & -1 & -18 \\ & & 8 & 16 & 30 \\ \hline & 4 & 8 & 15 & 12 \end{array}$$

$(4x^3-x-18)\div(x-2)=4x^2+8x+15+\frac{12}{x-2}$

35.
$$\begin{array}{r|rrrr} -4 & 2 & 5 & -5 & 20 \\ & & -8 & 12 & -28 \\ \hline & 2 & -3 & 7 & -8 \end{array}$$

$(2x^3+5x^2-5x+20)\div(x+4)=2x^2-3x+7-\frac{8}{x+4}$

37.
$$\begin{array}{r|rrrrr} 2 & 3 & -4 & 8 & -5 & -5 \\ & & 6 & 4 & 24 & 38 \\ \hline & 3 & 2 & 12 & 19 & 33 \end{array}$$

$$\frac{3x^4-4x^3+8x^2-5x-5}{x-2}=3x^3+2x^2+12x+19+\frac{33}{x-2}$$

39.
$$\begin{array}{r|rrrrr} -1 & 3 & 3 & -1 & 3 & 2 \\ & & -3 & 0 & 1 & -4 \\ \hline & 3 & 0 & -1 & 4 & -2 \end{array}$$

$$\frac{3x^4+3x^3-x^2+3x+2}{x+1}=3x^3-x+4-\frac{2}{x+1}$$

41.
$$\begin{array}{r|rrrrr} 3 & 2 & 0 & -1 & 0 & 2 \\ & & 6 & 18 & 51 & 153 \\ \hline & 2 & 6 & 17 & 51 & 155 \end{array}$$

$$\frac{2x^4-x^2+2}{x-3}=2x^3+6x^2+17x+51+\frac{155}{x-3}$$

Objective C Exercises

43. $f(3)=0$

45.
$$\begin{array}{r|rrr} 3 & 2 & -3 & -1 \\ & & 6 & 9 \\ \hline & 2 & 3 & 8 \end{array}$$

$P(3)=8$

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47.
$$\begin{array}{r|cccc} 4 & 1 & -2 & 3 & -1 \\ & & 4 & 8 & 44 \\ \hline & 1 & 2 & 11 & 43 \end{array}$$
$R(4) = 43$

49.
$$\begin{array}{r|cccc} -2 & 2 & -4 & 3 & -1 \\ & & -4 & 16 & -38 \\ \hline & 2 & -8 & 19 & -39 \end{array}$$
$P(-2) = -39$

51.
$$\begin{array}{r|cccc} -3 & 2 & -1 & 0 & 3 \\ & & -6 & 21 & -63 \\ \hline & 2 & -7 & 21 & -60 \end{array}$$
$Z(-3) = -60$

53.
$$\begin{array}{r|ccccc} 2 & 1 & 3 & -2 & 4 & -9 \\ & & 2 & 10 & 16 & 40 \\ \hline & 1 & 5 & 8 & 20 & 31 \end{array}$$
$Q(2) = 31$

55.
$$\begin{array}{r|ccccc} -3 & 2 & -1 & 0 & 2 & -5 \\ & & -6 & 21 & -63 & 183 \\ \hline & 2 & -7 & 21 & -61 & 178 \end{array}$$
$F(-3) = 178$

57.
$$\begin{array}{r|cccc} 5 & 1 & 0 & 0 & -3 \\ & & 5 & 25 & 125 \\ \hline & 1 & 5 & 25 & 122 \end{array}$$
$P(5) = 122$

59.
$$\begin{array}{r|ccccc} -3 & 4 & 0 & -3 & 0 & 5 \\ & & -12 & 36 & -99 & 297 \\ \hline & 4 & -12 & 33 & -99 & 302 \end{array}$$
$R(-3) = 302$

61.
$$\begin{array}{r|cccccc} 2 & 1 & 0 & -4 & -2 & 5 & -2 \\ & & 2 & 4 & 0 & -4 & 2 \\ \hline & 1 & 2 & 0 & -2 & 1 & 0 \end{array}$$
$Q(2) = 0$

63.
$$\begin{array}{r|cccccc} -2 & 2 & 0 & -1 & 0 & 4 & -1 \\ & & -4 & 8 & -14 & 28 & -64 \\ \hline & 2 & -4 & 7 & -14 & 32 & -65 \end{array}$$
$R(-2) = -65$

Applying the Concepts

65. a.
$$\begin{array}{r} a^2 - ab + b^2 \\ a+b \overline{)\, a^3 \qquad\qquad + b^3} \\ \underline{a^3 + a^2 b} \\ -a^2 b \\ \underline{-a^2 b - ab^2} \\ ab^2 + b^3 \\ \underline{ab^2 + b^3} \\ 0 \end{array}$$

$$\frac{a^3 + b^3}{a+b} = a^2 - ab + b^2$$

b.
$$\begin{array}{r} x^4 - x^3 y + x^2 y^2 - xy^3 + y^4 \\ x+y \overline{)\, x^5 \qquad\qquad\qquad + y^5} \\ \underline{x^5 + x^4 y} \\ -x^4 y \\ \underline{-x^4 y - x^3 y^2} \\ x^3 y^2 \\ \underline{x^3 y^2 + x^2 y^3} \\ -x^2 y^3 \\ \underline{-x^2 y^3 - xy^4} \\ xy^4 + y^5 \\ \underline{xy^4 + y^5} \\ 0 \end{array}$$

$$\frac{x^5 + y^5}{x+y} = x^4 - x^3 y + x^2 y^2 - xy^3 + y^4$$

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c.
$$\begin{array}{r} x^5 - x^4y + x^3y^2 - x^2y^3 + xy^4 - y^5 \\ x+y\overline{)\,x^6 \qquad\qquad\qquad\qquad\qquad\qquad -y^6} \\ \underline{x^6 + x^5y} \\ -x^5y \\ \underline{-x^5y - x^4y^2} \\ x^4y^2 \\ \underline{x^4y^2 + x^3y^3} \\ -x^3y^3 \\ \underline{-x^3y^3 - x^2y^4} \\ x^2y^4 \\ \underline{x^2y^4 + xy^5} \\ -xy^5 - y^6 \\ \underline{-xy^5 - y^6} \\ 0 \end{array}$$

$$\frac{x^6 - y^6}{x+y} = x^5 - x^4y + x^3y^2 - x^2y^3 + xy^4 - y^5$$

67. a.
$$\begin{array}{r} \frac{1}{2}x^2 + \frac{3}{4}x - 1 \\ 4x+8\overline{)\,2x^3 + 7x^2 + 2x - 8} \\ \underline{2x^3 + 4x^2} \\ 3x^2 + 2x \\ \underline{3x^2 + 6x} \\ -4x - 8 \\ \underline{-4x - 8} \\ 0 \end{array}$$

$$(2x^3 + 7x^2 + 2x - 8) \div (4x + 8) = \frac{1}{2}x^2 + \frac{3}{4}x - 1$$

b.
$$\begin{array}{r} \frac{2}{3}x^2 + \frac{7}{2}x + \frac{10}{3} \\ 6x-12\overline{)\,4x^3 + 13x^2 - 22x + 24} \\ \underline{4x^3 - 8x^2} \\ 21x^2 - 22x \\ \underline{21x^2 - 42x} \\ 20x + 24 \\ \underline{20x - 40} \\ 64 \end{array}$$

$$(4x^3 + 13x^2 - 22x + 24) \div (6x - 12) = \frac{2}{3}x^2 + \frac{7}{2}x + \frac{10}{3} + \frac{64}{6x-12}$$

c.
$$\begin{array}{r} 2x^2 - x + 1 \\ x^2 - x + 1\overline{)\,2x^4 - 3x^3 + 4x^2 + x - 10} \\ \underline{2x^4 - 2x^3 + 2x^2} \\ -x^3 + 2x^2 + x \\ \underline{-x^3 + x^2 - x} \\ x^2 + 2x - 10 \\ \underline{x^2 - x + 1} \\ 3x - 11 \end{array}$$

$$(2x^4 - 3x^3 + 4x^2 + x - 10) \div (x^2 - x + 1)$$
$$= 2x^2 - x + 1 + \frac{3x + 11}{x^2 - x + 1}$$

d.
$$\begin{array}{r} x^2 + 6x + 17 \\ x^2 - 2x - 3\overline{)\,x^4 + 4x^3 + 2x^2 - x + 5} \\ \underline{x^4 - 2x^3 - 3x^2} \\ 6x^3 + 5x^2 - x \\ \underline{6x^3 - 12x^2 - 18x} \\ 17x^2 + 17x + 5 \\ \underline{17x^2 - 34x - 51} \\ 51x + 56 \end{array}$$

$$(x^4 + 4x^3 + 2x^2 - x + 5) \div (x^2 - 2x - 3)$$
$$= x^2 + 6x + 17 + \frac{51x + 56}{x^2 - 2x - 3}$$

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Chapter Review

1. $(-2a^2b^4)(3ab^2) = -6a^3b^6$

2. $(-3x^2y^3)^2 = (-3)^2x^4y^6 = 9x^4y^6$

3. $\dfrac{(2a^4b^{-3}c^2)^3}{(2a^3b^2c^{-1})^4} = \dfrac{8a^{12}b^{-9}c^6}{16a^{12}b^8c^{-4}} = \dfrac{1}{2}a^{12-12}b^{-9-8}c^{6-(-4)} = \dfrac{1}{2}b^{-17}c^{10} = \dfrac{c^{10}}{2b^{17}}$

4. $2.54 \times 10^{-3} = 0.00254$

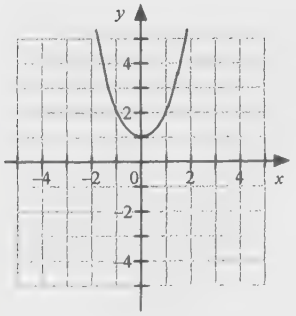
5. $P(-2) = 2(-2)^3 - (-2) + 7$
$P(-2) = -16 + 2 + 7$
$P(-2) = -7$

6. $(5x^2 - 8xy + 2y^2) - (x^2 - 3y^2) = (5x^2 - x^2) - 8xy + (2y^2 + 3y^2) = 4x^2 - 8xy + 5y^2$

7. $(2x^{-1}y^2z^5)(-3x^3yz^{-3})^2 = 2x^{-1}y^2z^5(9x^6y^2z^{-6})$
$= 18x^5y^4z^{-1} = \dfrac{18x^5y^4}{z}$

8. $(2a^{12}b^3)(-9b^2c^6)(3ac) = -54a^{13}b^5c^7$

9. $y = x^2 + 1$



10. **a.** 3
b. 8
c. 5

11.

2	1	−2	3	−5
		2	0	6
	1	0	3	1

$P(2) = 1$

12. $-2x(4x^2 + 7x - 9) = -2x(4x^2) - 2x(7x) - 2x(-9) = -8x^3 - 14x^2 + 18x$

13. $2ab^3(4a^2 - 2ab + 3b^2) = 2ab^3(4a^2) + 2ab^3(-2ab) + 2ab^3(3b^2) = 8a^3b^3 - 4a^2b^4 + 6ab^5$

14.

$$\begin{array}{rrrrrr} & & 3y^2 & + & 4y & - & 7 \\ \times & & & & 2y & + & 3 \\ \hline & & 9y^2 & + & 12y & - & 21 \\ 6y^3 & + & 8y^2 & - & 14y & & \\ \hline 6y^3 & + & 17y^2 & - & 2y & - & 21 \end{array}$$

15. $(5a - 7)(2a + 9) = 10a^2 + 45a - 14a - 63 = 10a^2 + 31a - 63$

16. $0.000000127 = 1.27 \times 10^{-7}$

17. $(5y - 7)^2 = (5y - 7)(5y - 7)$
$= 25y^2 - 35y - 35y + 49$
$= 25y^2 - 70y + 49$

18.

$$\begin{array}{r} 5x + 4 \\ 3x - 2\overline{)\,15x^2 + 2x - 2} \\ \underline{15x^2 - 10x} \\ 12x - 2 \\ \underline{12x - 8} \\ 6 \end{array}$$

$\dfrac{15x^2 + 2x - 2}{3x - 2} = 5x + 4 + \dfrac{6}{3x - 2}$

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19.
$$\begin{array}{r|rrrr} -6 & 4 & 27 & 10 & 2 \\ & & -24 & -18 & 48 \\ \hline & 4 & 3 & -8 & 50 \end{array}$$
$$\frac{4x^3+27x^2+10x+2}{x+6}=4x^2+3x-8+\frac{50}{x+6}$$

20.
$$\begin{array}{r|rrrrr} 4 & 1 & 0 & 0 & 0 & -4 \\ & & 4 & 16 & 64 & 256 \\ \hline & 1 & 4 & 16 & 64 & 252 \end{array}$$
$$\frac{x^4-4}{x-4}=x^3+4x^2+16x+64+\frac{252}{x-4}$$

21. $8.1\times10^9=8{,}100{,}000{,}000$

22. $$\frac{-18a^6b}{27a^3b^4}=\frac{-2\cdot\cancel{3}\cdot\cancel{3}a^6b}{3\cdot\cancel{3}\cdot\cancel{3}a^3b^4}=\frac{-2a^{6-3}b^{1-4}}{3}=\frac{-2a^3b^{-3}}{3}=-\frac{2a^3}{3b^3}$$

23. $(5a+2b)(5a-2b)=25a^2-4b^2$

24.
$$\begin{array}{r|rrrr} -3 & -2 & 2 & 0 & -4 \\ & & 6 & -24 & 72 \\ \hline & -2 & 8 & -24 & 68 \end{array}$$
$P(-3)=68$

25.
$$\begin{array}{rrrrr} (12y^2+17y-4)+(9y^2-13y+3) & & & & \\ 12y^2 & + & 17y & - & 4 \\ +\ 9y^2 & - & 13y & + & 3 \\ \hline 21y^2 & + & 4y & - & 1 \end{array}$$

26.
$$\begin{array}{rrrrrrrr} & & & & 6b^3 & - & 2b^2 & - & 5 \\ & & & & & & 2b^2 & - & 1 \\ \hline & & & - & 6b^3 & + & 2b^2 & + & 5 \\ 12b^5 & - & 4b^4 & & & - & 10b^2 & & \\ \hline 12b^5 & - & 4b^4 & - & 6b^3 & - & 8b^2 & + & 5 \end{array}$$

27. $(a+7)(a-7)=a^2-7a+7a-49=a^2-49$

28. $765{,}000{,}000{,}000=7.65\times10^{11}$

29. Strategy: To find the area, replace the variables L and W in the equation $A=L\cdot W$ by the given values and solve for A.

Solution:
$A=LW$
$A=(2w-1)(w)$
$A=2w^2-w$
The area of the table is $(2w^2-w)$ ft^2.

30. Strategy: To find how far the earth is from the Great Galaxy of Andromeda, use the equation $d=rt$ where $r=6.7\times10^8$ mph and $t=2.2\times10^6$ years. $2.2\times10^6\times24\times365=1.9272\times10^{10}$.

Solution:
$d=rt=(6.7\times10^8)(1.9272\times10^{10})$
$=6.7\times1.9272\times10^{18}=12.91224\times10^{18}$
$=1.291224\times10^{19}$
The distance from Earth to the Great Galaxy of Andromeda is 1.291224×10^{19} mi.

31. Strategy: To find the number of seconds in one week in scientific notation:
- Multiply the number of seconds in a minute (60) by the minutes in an hour (60) by the hours in a day (24) by the days in a week (7).
- Convert that product to scientific notation.

Solution: $60\times60\times24\times7=604{,}800=6.048\times10^5$
The number of seconds in a week is 6.048×10^5.

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32. Strategy Side of a square: $3x-2$
Use the equation for the area of a square.

Solution
$$S^2 = A$$
$$(3x-2)^2 = A$$
$$9x^2-6x-6x+4 = A$$
$$9x^2-12x+4 = A$$
The area is $(9x^2-12x+4)\ \text{in}^2$.

33. Strategy To find the area, replace the variables L and W in the equation $A = LW$ by the given values and solve for A.

Solution
$$A = L \cdot W$$
$$A = (5x+3)(2x-7) = 10x^2-29x-21$$
The area is $(10x^2-29x-21)\ \text{cm}^2$.

Chapter Test

1. $2x(2x^2-3x) = 4x^3-6x^2$

2.
$$\begin{array}{r|cccc} -2 & -1 & 0 & 4 & -8 \\ & & 2 & -4 & 0 \\ \hline & -1 & 2 & 0 & -8 \end{array}$$
$P(-2) = -8$

3. $\dfrac{12x^2}{-3x^8} = -\dfrac{4}{x^6}$

4. $(-2xy^2)(3x^2y^4) = -6x^3y^6$

5.
$$\begin{array}{r} x-1 \\ x+1\overline{)\,x^2+0+1} \\ \underline{x^2+x\ \ } \\ -x+1 \\ \underline{-x-1} \\ 2 \end{array}$$
$(x^2+1)\div(x+1) = x-1+\dfrac{2}{x+1}$

6. $(x-3)(x^2-4x+5) = x^3-4x^2+5x-3x^2+12x-15$
$= x^3-7x^2+17x-15$

7. $(-2a^2b)^3 = -2^3a^6b^3 = -8a^6b^3$

8. $\dfrac{(3x^{-2}y^3)^3}{3x^4y^{-1}} = \dfrac{27x^{-6}y^9}{3x^4y^{-1}} = \dfrac{9y^{10}}{x^{10}}$

9. $(a-2b)(a+5b) = a^2+5ab-2ab-10b^2 = a^2+3ab-10b^2$

10. $P(x) = 3x^2-8x+1$
$P(2) = 3(2)^2-8(2)+1$
$P(2) = -3$

11.
$$\begin{array}{r} x+7 \\ x-1\overline{)\,x^2+6x-7} \\ \underline{x^2-1x\ \ } \\ 7x-7 \\ \underline{7x-7} \\ 0 \end{array}$$
$(x^2+6x-7)\div(x-1) = x+7$

12. $-3y^2(-2y^2+3y-6) = 6y^4-9y^3+18y^2$

13. $(-2x^3+x^2-7)(2x-3) = -4x^4+6x^3+2x^3-3x^2-14x+21 = -4x^4+8x^3-3x^2-14x+21$

14. $(4y-3)(4y+3) = 16y^2+12y-12y-9 = 16y^2-9$

15. $(ab^2)(a^3b^5) = a^4b^7$

16. $\dfrac{2a^{-1}b}{2^{-2}a^{-2}b^{-3}} = 2^3ab^4 = 8ab^4$

17. $\dfrac{(2a^{-4}b^2)^3}{4a^{-2}b^{-1}} = \dfrac{2^3a^{-12}b^6}{4a^{-2}b^{-1}} = \dfrac{2b^7}{a^{10}}$

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18. $(3a^2-2a-7)-(5a^3+2a-10)=-5a^3+3a^2-4a+3$

19. $(2x-5)^2=(2x-5)(2x-5)=4x^2-10x-10x+25$
$=4x^2-20x+25$

20.
$$\begin{array}{r} x^2-5x+10 \\ x+3\overline{)\,x^3-2x^2-5x+7} \\ \underline{x^3+3x^2} \\ -5x^2-5x \\ \underline{-5x^2-15x} \\ 10x+7 \\ \underline{10x+30} \\ -23 \end{array}$$
$x^3-2x^2-5x+7-\dfrac{23}{x+3}$

21. $(2x-7y)(5x-4y)=10x^2-8xy-35xy+28y^2$
$=10x^2-43xy+28y^2$

22. $(3x^3-2x^2-4)+(8x^2-8x+7)=3x^3+6x^2-8x+3$

23. $0.00000000302=3.02\times10^{-9}$

24. $(3.7\times10^{-8})(2.19\times10^{27})=(3.7)(2.19)\times10^{-8+27}$
$=8.103\times10^{19}$
The mass of the moon is 8.103×10^{19} tons.

25. $r=(x-5)$
$A=\pi r^2=\pi(x-5)^2=\pi(x-5)(x-5)$
$=\pi(x^2-5x-5x+25)=\pi x^2-10\pi x+25\pi$
The area of the circle is $(\pi x^2-10\pi x+25\pi)\,\text{m}^2$.

Cumulative Review

1. $-8\geq-3$ False
$-3\geq-3$ True
$3\geq-3$ True
The inequality is true for –3 and 3.

2. The additive inverse of 83 is –83.

3. $8-2[-3-(-1)^2]\div4=$
$8-2[-3+1]^2\div4=$
$8-2[-2]^2\div4=$
$8-2(4)\div4=$
$8-8\div4=$
$8-2=$
6

4. $\dfrac{2(4)-(-2)}{-2-6}=\dfrac{8+2}{-8}=\dfrac{10}{-8}=-\dfrac{5}{4}$

5. $-5\sqrt{300}=-5\sqrt{2^2\cdot3\cdot5^2}$
$=-5\sqrt{2^2\cdot5^2}\sqrt{3}=-5\cdot2\cdot5\sqrt{3}$
$=-50\sqrt{3}$

6. The Inverse Property of Addition.

7. $2x-4[x-2(3-23x)+4]=2x-4[x-6+46x+4]=2x-4(7x+2)=2x-188x+8=-186x+8$

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8. $\frac{2}{3}-y=\frac{5}{6}$

$\frac{2}{3}-y-\frac{2}{3}=\frac{5}{6}-\frac{2}{3}$

$-y=\frac{1}{6}$

$(-1)(-y)=(-1)\frac{1}{6}$

$y=-\frac{1}{6}$

The solution is $-\frac{1}{6}$.

9. $8x-3-x=-6x+3x-8$

$7x-3=3x-14$

$7x-3-3x=3x-14-3x$

$4x-3=-14$

$4x-3+3=-14+3$

$4x=-11$

$\frac{4x}{4}=\frac{-11}{4}$

$x=-\frac{11}{4}$

The solution is $-\frac{11}{4}$.

10. $3-|2-3x|=-2$

$-|2-3x|=-5$

$|2-3x|=5$

$2-3x=5$ $\quad 2-3x=-5$

$-3x=3$ $\quad -3x=-7$

$x=-1$ $\quad x=\frac{7}{3}$

The solutions are -1 and $\frac{7}{3}$.

11. $P(-2)=3(-2)^2-2(-2)+2$

$P(-2)=12+4+2$

$P(-2)=18$

12. No two ordered pairs have the same first coordinate and different second coordinates. The relation is function.

13. $m=\frac{y_2-y_1}{x_2-x_1}=\frac{2-3}{4-(-2)}=\frac{-1}{6}$

The slope is $-\frac{1}{6}$.

14. Use the point-slope formula.

$y-y_1=m(x-x_1)$

$y-2=-\frac{3}{2}[x-(-1)]$

$y-2=-\frac{3}{2}x-\frac{3}{2}$

$y=-\frac{3}{2}x+\frac{1}{2}$

15. First find the slope of the given line.

$3x+2y=4$

$2y=-3x+4$

$y=-\frac{3}{2}+2;\ m=-\frac{3}{2}$

The perpendicular line will have a slope that is the negative reciprocal of $-\frac{3}{2}$.

$m=\frac{2}{3}$

Now use the point-slope formula to find the equation of the line.

$y-y_1=m(x-x_1)$

$y-4=\frac{2}{3}[x-(-2)]$

$y-4=\frac{2}{3}x+\frac{4}{3}$

$y=\frac{2}{3}x+\frac{16}{3}$

The equation of the perpendicular line is $y=\frac{2}{3}x+\frac{16}{3}$.

16. $2x-3y=2$

$x+y=-3$

$D=\begin{vmatrix}2 & -3\\ 1 & 1\end{vmatrix}=5$ $\quad x=\frac{D_x}{D}=-\frac{7}{5}$

$D_x=\begin{vmatrix}2 & -3\\ -3 & 1\end{vmatrix}=-7$ $\quad y=\frac{D_y}{D}=-\frac{8}{5}$

$D_y=\begin{vmatrix}2 & 2\\ 1 & -3\end{vmatrix}=-8$

The solution is $\left(-\frac{7}{5},-\frac{8}{5}\right)$.

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17. (1) $x - y + z = 0$

(2) $2x + y - 3z = -7$

(3) $-x + 2y + 2z = 5$

Add equations (1) and (3) to eliminate x.

$$\begin{array}{rrrrr} x & - & y & + & z = 0 \\ -x & + & 2y & + & 2z = 5 \\ \hline & & y & + & 3z = 5 \end{array}$$

Add –2 times equation (1) and equation (2) to eliminate x.

$$\begin{array}{rrrrr} -2x & + & 2y & - & 2z = 0 \\ 2x & + & y & - & 3z = -7 \\ \hline & & 3y & - & 5z = -7 \end{array}$$

Now solve the system of two equations in two variables.

$y + 3z = 5$

$3y - 5z = -7$

Add –3 times the first of these equations to the second.

$$\begin{array}{rrrrr} -3y & - & 9z & = & -15 \\ 3y & - & 5z & = & -7 \\ \hline & - & 14z & = & -22 \end{array}$$

$$z = \frac{11}{7}$$

Next find y.

$y + 3z = 5$

$$y + 3\left(\frac{11}{7}\right) = 5$$

$$y = \frac{2}{7}$$

Replace y and z in equation (1) and solve for x.

$$x - \frac{2}{7} + \frac{11}{7} = 0$$

$$x = -\frac{9}{7}$$

The solution is $\left(-\frac{9}{7}, \frac{2}{7}, \frac{11}{7}\right)$.

18. $-2x - (-xy) + 7x - 4xy = -2x + 7x + xy - 4xy$

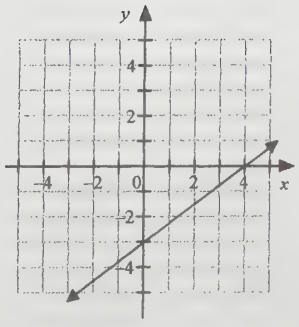
$= 5x - 3xy$

19.

$$\begin{array}{rrrrrr} & & 2x^2 & - & 3x & + 1 \\ \times & & & & 2x & + 3 \\ \hline & & 6x^2 & - & 9x & + 3 \\ 4x^3 & - & 6x^2 & + & 2x & \\ \hline 4x^3 & & & - & 7x & + 3 \end{array}$$

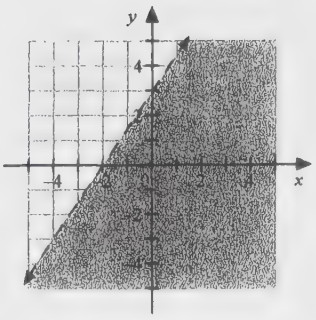
20. $0.00000501 = 5.01 \times 10^{-6}$

21.

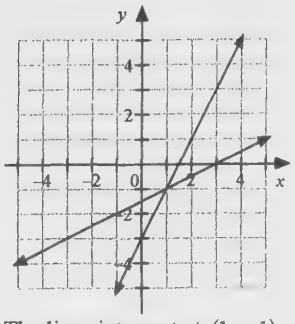


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22.



23.



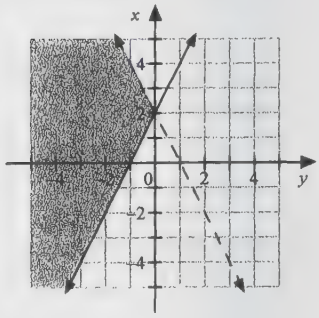
The lines intersect at $(1, -1)$.

24. Solve each inequality

$2x + y < 2$ $\quad -6x + 3y \geq 6$

$y < 2 - 2x$ $\quad 3y \geq 6 + 6x$

$y \geq 2 + 2x$



25. $\left(4a^{-2}b^{3}\right)\left(2a^{3}b^{-1}\right)^{-2} = 4a^{-2}b^{3}\left(2^{-2}a^{-6}b^{2}\right) = 4 \cdot 2^{-2}a^{-2-6}b^{3+2}$

$= 4 \cdot \frac{1}{4}a^{-8}b^{5} = \frac{b^{5}}{a^{8}}$

26. $\frac{\left(5x^{3}y^{-3}z\right)^{-2}}{y^{4}z^{-2}} = \frac{5^{-2}x^{-6}y^{6}z^{-2}}{y^{4}z^{-2}} = 5^{-2}x^{-6}y^{6-4}z^{-2-(-2)}$

$= \frac{1}{25}x^{-6}y^{2} = \frac{y^{2}}{25x^{6}}$

27. Strategy

- Smaller integer: x
 Larger integer: $24 - x$
- The difference between four times the smaller and nine is three less than twice the larger.
 $4x - 9 = 2(24 - x) - 3$

Solution

$4x - 9 = 2(24 - x) - 3$

$4x - 9 = 48 - 2x - 3$

$4x - 9 = 45 - 2x$

$6x - 9 = 45$

$6x = 54$

$x = 9$

$24 - x = 24 - 9 = 15$

The integers are 9 and 15.

28. Strategy

- The number of ounces of pure gold: x

	Amount	Cost	Value
Pure gold	x	360	$360x$
Alloy	80	120	$80(120)$
Mixture	$x + 80$	200	$200(x + 80)$

- The sum of the values before mixing equals the value after mixing.
 $360x + 80(120) = 200(x + 80)$

Solution

$360x + 80(120) = 200(x + 80)$

$360x + 9600 = 200x + 16{,}000$

$160x + 9600 = 16{,}000$

$160x = 6400$

$x = 40$

40 oz. of pure gold must be mixed with the alloy.

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29. Strategy

- Faster cyclist: $1.5x$
 Slower cyclist: x

	Rate	Time	Distance
Faster cyclist	$1.5x$	2	$2(1.5x)$
Slower cyclist	x	2	$2x$

- The sum of the distances is 25 mi.

Solution

$$\begin{aligned} 2(1.5x)+2x &= 25 \\ 3x+2x &= 25 \\ 5x &= 25 \\ x &= 5 \\ 1.5x &= 1.5(5) = 7.5 \end{aligned}$$

The slower cyclist travels at 5 mph, the faster cyclist at 7.5 mph.

30. Strategy

- Amount invested at 10%: x

	Principal	Rate	Interest
Amount at 7.5%	3000	0.075	0.075(3000)
Amount at 10%	x	0.10	$0.10x$
Amount at 9%	$3000+x$	0.09	$0.09(3000+x)$

- The amount of interest earned at 9% equals the total amount of the interest earned at 7.5% and 10%.

Solution

$$\begin{aligned} 0.075(3000)+0.10x &= 0.09(3000+x) \\ 225+0.10x &= 270+0.09x \\ 001x+225 &= 270 \\ 0.01x &= 45 \\ x &= 4500 \end{aligned}$$

The additional investment is \$4500.

31. $m = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{300-100}{6-2} = \dfrac{200}{4} = 50$. The slope represents the average speed of travel in miles per hour.

32. Strategy

- Length: x
 Width: 40% of $x = 0.40x$
- Use the rectangle for the perimeter of a rectangle.

Solution

$$\begin{aligned} 2L+2W &= P \\ 2(x)+2(0.40x) &= 42 \\ 2x+0.80x &= 42 \\ 2.8x &= 42 \\ x &= 15 \\ 0.40x &= 6 \end{aligned}$$

The length is 15 m and the width is 6 m.

33.

$$\begin{aligned} A &= s^2 \\ A &= (2x+3)^2 \\ A &= (2x+3)(2x+3) \\ A &= 4x^2+6x+6x+9 \\ A &= 4x^2+12x+9 \end{aligned}$$

The area is $\left(4x^2+12x+9\right)\text{m}^2$.

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Chapter 7
Factoring

Prep Test

1. $2 \cdot 3 \cdot 5$

2. $-12y + 15$

3. $-(a-b) = -a+b$

4. $2(a-b)-5(a-b)$
$= 2a-2b-5a+5b$
$= -3a+3b$

5. $4x = 0$
$x = 0$

6. $2x+1=0$
$2x=-1$
$x=-\frac{1}{2}$

7. $(x+4)(x-6)$
$= x^2-6x+4x-24$
$= x^2-2x-24$

8. $(2x-5)(3x+2)$
$= 6x^2+4x-15x-10$
$= 6x^2-11x-10$

9. $\frac{x^5}{x^2} = x^3$

10. $\frac{6x^4y^3}{2xy^2} = 3x^2y$

Go Figure

If $x+y$, xy, and $\frac{x}{y}$ all equal the same number, then they also equal each other. So, $xy = \frac{x}{y}$. Solving for y, divide both sides by x, and multiply both sides by y to get $y^2 = 1$. Then y must be either 1 or –1.
Use $x+y=xy$ check our solution,
If $y = 1$, then $x + 1 = x(1)$
Subtracting x from both sides we have
$1 = 0$, which is never true. So, $y = 1$ is not a valid solution.
If $y = -1$, then
$x - 1 = x(-1)$
$x - 1 = -x$ add x to both sides
$2x = 1$ divide both sides by 2
$x = \frac{1}{2}$
So the values are $x = \frac{1}{2}$ and $y = -1$.

Section 7.1

Objective A Exercises

1. $5a+5 = 5(a+1)$

3. $16-8a^2 = 8(2-a^2)$

5. $8x+12 = 4(2x+3)$

7. $30a-6 = 6(5a-1)$

9. $7x^2-3x = x(7x-3)$

11. $3a^2+5a^2 = a^2(3+5a^3)$

13. $14y^2+11y = y(14y+11)$

15. $2x^4-4x = 2x(x^3-2)$

17. $10x^4-12x^2 = 2x^2(5x^2-6)$

19. $8a^8-4a^5 = 4a^5(2a^3-1)$

21. $x^2y^2-xy = xy(xy-1)$

23. $3x^2y^4-6xy = 3xy(xy^3-2)$

25. $x^2y-xy^3 = xy(x-y^2)$

27. $5y^3-20y^2+10y = 5y(y^2-4y+2)$

29. $3y^4-9y^3-6y^2 = 3y^2(y^2-3y-2)$

31. $3y^3-9y^2+24y = 3y(y^2-3y+8)$

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33. $6a^5 - 3a^3 - 2a^2 = a^2(6a^3 - 3a - 2)$

35. $2a^2b - 5a^2b^2 + 7ab^2 = ab(2a - 5ab + 7b)$

37. $4b^5 + 6b^3 - 12b = 2b(2b^4 + 3b^2 - 6)$

39. $8x^2y^2 - 4x^2y + x^2 = x^2(8y^2 - 4y + 1)$

41. The GCF of $2a^{5n}$ and a^{2n} is a^{2n}.
$2a^{5n} + a^{2n} = a^{2n}(2a^{3n} + 1)$

43. The GCF of y^{4n} and y^{2n} is y^{2n}.
$y^{4n} + y^{2n} = y^{2n}(y^{2n} + 1)$

45. The GCF of b^{n+5} and b^5 is b^5.
$b^{n+5} - b^5 = b^5(b^n - 1)$

Objective B Exercises

47. $y(a+z) + 7(a+z) = (a+z)(y+7)$

49. $3r(a-b) + s(a-b) = (a-b)(3r+s)$

51. $t(m-7) + 7(7-m) = t(m-7) - 7(m-7)$
$= (m-7)(t-7)$

53. $2y(4a-b) - (b-4a) = 2y(4a-b) + 1(4a-b)$
$= (4a-b)(2y+1)$

55. $x^2 + 2x + 2xy + 4y = x(x+2) + 2y(x+2)$
$= (x+2)(x+2y)$

57. $p^2 - 2p - 3rp + 6r = p(p-2) - 3r(p-2)$
$= (p-2)(p-3r)$

59. $ab + 6b - 4a - 24 = b(a+6) - 4(a+6) = (a+6)(b-4)$

61. $2z^2 - z + 2yz - y = z(2z-1) + y(2z-1)$
$= (2z-1)(z+y)$

63. $8v^2 - 12vy + 14v - 21y = 4v(2v - 3y) + 7(2v - 3y)$
$= (2v - 3y)(4v + 7)$

65. $2x^2 - 5x - 6xy + 15y = x(2x-5) - 3y(2x-5)$
$= (2x-5)(x-3y)$

67. $3y^2 - 6y - ay + 2a = 3y(y-2) - a(y-2)$
$= (y-2)(3y-a)$

69. $3xy - y^2 - y + 3x = y(3x-y) + 1(-y+3x)$
$= y(3x-y) + 1(3x-y)$
$= (3x-y)(y+1)$

71. $3st + t^2 - 2t - 6s = t(3s+t) - 2(t+3s)$
$= t(3s+t) - 2(3s+t)$
$= (3s+t)(t-2)$

Applying the Concepts

73. **a.** 28 is a perfect number, because 28 has factors 1, 2, 4, 7, and 14, and $1+2+4+7+14 = 28$.

b. 496 is a perfect number, because 496 has factors 1, 2, 4, 8, 16, 31, 62, 124, and 248, and their sum is 496.

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Section 7.2

Objective A Exercises

1.

Factors	*Sum*
+1, +2	3

$x^2 + 3x + 2 = (x + 1)(x + 2)$

3.

Factors	*Sum*
−1, +2	1
+1, −2	−1

$x^2 - x - 2 = (x + 1)(x - 2)$

5.

Factors	*Sum*
−1, +12	11
+1, −12	−11
−2, +6	4
+2, −6	−4
−3, +4	1
+3, −4	−1

$a^2 + a - 12 = (a + 4)(a - 3)$

7.

Factors	*Sum*
−1, −2	−3

$a^2 - 3a + 2 = (a - 1)(a - 2)$

9.

Factors	*Sum*
−1, +2	1
+1, −2	−1

$a^2 + a - 2 = (a + 2)(a - 1)$

11.

Factors	*Sum*
−1, −9	−10
−3, −3	−6

$b^2 - 6b + 9 = (b - 3)(b - 3)$

13.

Factors	*Sum*
−1, +8	7
+1, −8	−7
−2, +4	2
+2, −4	−2

$b^2 + 7b - 8 = (b + 8)(b - 1)$

15.

Factors	*Sum*
−1, +55	54
+1, −55	−54
−5, +11	6
+5, −11	−6

$y^2 + 6y - 55 = (y + 11)(y - 5)$

17.

Factors	*Sum*
−1, −6	−7
−2, −3	−5

$y^2 - 5y + 6 = (y - 2)(y - 3)$

19.

Factors	*Sum*
−1, −45	−46
−3, −15	−18
−5, −9	−14

$z^2 - 14z + 45 = (z - 5)(z - 9)$

21.

Factors	*Sum*
−1, +160	159
+1, −160	−159
−2, +80	78
+2, −80	−78
−4, +40	36
+4, −40	−36
−5, +32	27
+5, −32	−27
−8, +20	12
+8, −20	−12
−10, +16	6
+10, −16	−6

$z^2 - 12z - 160 = (z + 8)(z - 20)$

23.

Factors	*Sum*
+1, +27	28
+3, +9	12

$p^2 + 12p + 27 = (p + 3)(p + 9)$

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25.

Factors	*Sum*
+1, +100	101
+2, +50	52
+4, +25	29
+5, +20	25
+10, +10	20

$x^2 + 20x + 100 = (x + 10)(x + 10)$

27.

Factors	*Sum*
+1, +20	21
+2, +10	12
+4, +5	9

$b^2 + 9b + 20 = (b + 4)(b + 5)$

29.

Factors	*Sum*
−1, +42	41
+1, −42	−41
−2, +21	19
+2, −21	−19
−3, +14	11
+3, −14	−11
−6, +7	1
+6, −7	−1

$x^2 - 11x - 42 = (x + 3)(x - 14)$

31.

Factors	*Sum*
−1, +20	19
+1, −20	−19
−2, +10	8
+2, −10	−8
−4, +5	1
+4, −5	−1

$b^2 - b - 20 = (b + 4)(b - 5)$

33.

Factors	*Sum*
−1, +51	50
+1, −51	−50
−3, +17	14
+3, −17	−14

$y^2 - 14y - 51 = (y + 3)(y - 17)$

35.

Factors	*Sum*
−1, +21	20
+1, −21	−20
−3, +7	4
+3, −7	−4

$p^2 - 4p - 21 = (p + 3)(p - 7)$

37.

Factors	*Sum*
−1, −32	−33
−2, −16	−18
−4, −8	−12

$y^2 - 8y + 32$ is nonfactorable over the integers.

39.

Factors	*Sum*
−1, −75	−76
−3, −25	−28
−5, −15	−20

$x^2 - 20x + 75 = (x - 5)(x - 15)$

41.

Factors	*Sum*
−1, −56	−57
−2, −28	−30
−4, −14	−18
−7, −8	−15

$x^2 - 15x + 56 = (x - 7)(x - 8)$

43.

Factors	*Sum*
−1, +56	55
+1, −56	−55
−2, +28	26
+2, −28	−26
−4, +14	10
+4, −14	−10
−7, +8	1
+7, −8	−1

$x^2 + x - 56 = (x + 8)(x - 7)$

45.

Factors	*Sum*
−1, +72	71
+1, −72	−71
−2, +36	34
+2, −36	−34
−3, +24	21
+3, −24	−21
−4, +18	14
+4, −18	−14
−6, +12	6
+6, −12	−6
−8, +9	1
+8, −9	−1

$a^2 - 21a - 72 = (a + 3)(a - 24)$

47.

Factors	*Sum*
−1, −36	−37
−2, −18	−20
−3, −12	−15
−4, −9	−13
−6, −6	−12

$a^2 - 15a + 36 = (a - 3)(a - 12)$

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49.

Factors	*Sum*
$-1, +136$	135
$+1, -136$	-135
$-2, +68$	66
$+2, -68$	-66
$-4, +34$	30
$+4, -34$	-30
$-8, +17$	9
$+8, -17$	-9

$z^2 - 9z - 136 = (z + 8)(z - 17)$

51.

Factors	*Sum*
$-1, +90$	89
$+1, -90$	-89
$-2, +45$	43
$+2, -45$	-43
$-3, +30$	27
$+3, -30$	-27
$-5, +18$	13
$+5, -18$	-13
$-6, +15$	9
$+6, -15$	-9
$-9, +10$	1
$+9, -10$	-1

$c^2 - c - 90 = (c + 9)(c - 10)$

53.

Factors	*Sum*
$+1, +44$	$+45$
$+2, +22$	$+24$
$+4, +11$	$+15$

$z^2 + 15z + 44 = (z + 4)(z + 11)$

55.

Factors	*Sum*
$+1, +34$	$+35$
$+2, +17$	$+19$

$c^2 + 19c + 34 = (c + 2)(c + 17)$

57.

Factors	*Sum*
$-1, +96$	$+95$
$+1, -96$	-95
$-2, +48$	$+46$
$+2, -48$	-46
$-3, +32$	$+29$
$+3, -32$	-29
$-4, +24$	$+20$
$+4, -24$	-20
$-6, +16$	$+10$
$+6, -16$	-10
$-8, +12$	$+4$
$+8, -12$	-4

$x^2 - 4x - 96 = (x + 8)(x - 12)$

59.

Factors	*Sum*
$-1, -112$	-113
$-2, -56$	-58
$-4, -28$	-32
$-8, -14$	-22

$x^2 - 22x + 112 = (x - 8)(x - 14)$

61.

Factors	*Sum*
$-1, +105$	$+104$
$+1, -105$	-104
$-3, +35$	$+32$
$+3, -35$	-32
$-5, +21$	$+16$
$+5, -21$	-16
$-7, +15$	$+8$
$+7, -15$	-8

$b^2 + 8b - 105 = (b + 15)(b - 7)$

63.

Factors	*Sum*
$-1, +36$	$+35$
$+1, -36$	-35
$-2, +18$	$+16$
$+2, -18$	-16
$-3, +12$	$+9$
$+3, -12$	-9
$-4, +9$	$+5$
$+4, -9$	-5
$-6, +6$	0

$a^2 - 9a - 36 = (a + 3)(a - 12)$

65.

Factors	*Sum*
$-1, -102$	-103
$-2, -51$	-53
$-3, -34$	-37
$-6, -17$	-23

$b^2 - 23b + 102 = (b - 6)(b - 17)$

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67.

Factors	*Sum*
+1, +72	+73
+2, +36	+38
+3, +24	+27
+4, +18	+22
+6, +12	+18
+8, +9	+17

$a^2 + 27a + 72 = (a+3)(a+24)$

69.

Factors	*Sum*
+1, +156	+157
+2, +78	+80
+3, +52	+55
+4, +39	+43
+6, +26	+32
+12, +13	+25

$x^2 + 25x + 156 = (x+12)(x+13)$

71.

Factors	*Sum*
−1, +96	+95
+1, −96	−95
−2, +48	+46
+2, −48	−46
−3, +32	+29
+3, −32	−29
−4, +24	+20
+4, −24	−20
−6, +16	+10
+6, −16	−10
−8, +12	+4
+8, −12	−4

$x^2 - 10x - 96 = (x+6)(x-16)$

Objective B Exercises

73. The GCF is 2.

$2x^2 + 6x + 4 = 2(x^2 + 3x + 2)$

Factor the trinomial $x^2 + 3x + 2$.

Factors	*Sum*
+1, +2	+3

$2x^2 + 6x + 4 = 2(x+1)(x+2)$

75. The GCF is −1.

$-x^2 + 7x + 18 = -(x^2 - 7x - 18)$

Factor the trinomial $x^2 - 7x - 18$.

Factors	*Sum*
−1, +18	+17
+1, −18	−17
−2, +9	+7
+2, −9	−7

$18 + 7x - x^2 = -(x+2)(x-9)$

77. The GCF is a.

$ab^2 + 2ab - 15a = a(b^2 + 2b - 15)$

Factor the trinomial $b^2 + 2b - 15$.

Factors	*Sum*
−1, +15	+14
+1, −15	−14
−3, +5	+2
+3, −5	−2

$ab^2 + 2ab - 15a = a(b+5)(b-3)$

79. The GCF is x.

$xy^2 - 5xy + 6x = x(y^2 - 5y + 6)$

Factor the trinomial $y^2 - 5y + 6$.

Factors	*Sum*
−1, −6	−7
−2, −3	−5

$xy^2 - 5xy + 6x = x(y-2)(y-3)$

81. The GCF is z.

$z^3 - 7z^2 + 12z = z(z^2 - 7z + 12)$

Factor the trinomial $z^2 - 7z + 12$.

Factors	*Sum*
−1, −12	−13
−2, −6	−8
−3, −4	−7

$z^3 - 7z^2 + 12z = z(z-3)(z-4)$

83. The GCF is $-3y$.

$-3y^3 + 15y^2 - 18y = -3y(y^2 - 5y + 6)$

Factor the trinomial $y^2 - 5y + 6$.

Factors	*Sum*
−1, −6	−7
−2, −3	−5

$-3y^3 + 15y^2 - 18y = -3y(y-2)(y-3)$

85. The GCF is 3.

$3x^2 + 3x - 36 = 3(x^2 + x - 12)$

Factor the trinomial $x^2 + x - 12$.

Factors	*Sum*
−1, +12	+11
+1, −12	−11
−2, +6	+4
+2, −6	−4
−3, +4	+1
+3, −4	−1

$3x^2 + 3x - 36 = 3(x+4)(x-3)$

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87. The GCF is 5.

$5z^2 - 15z - 140 = 5(z^2 - 3z - 28)$

Factor the trinomial $z^2 - 3z - 28$.

Factors	*Sum*
−1, +28	+27
+1, −28	−27
−2, +14	+12
+2, −14	−12
−4, +7	+3
+4, −7	−3

$5z^2 - 15z - 140 = 5(z + 4)(z - 7)$

89. The GCF is $2a$.

$2a^3 + 8a^2 - 64a = 2a(a^2 + 4a - 32)$

Factor the trinomial $a^2 + 4a - 32$.

Factors	*Sum*
−1, +32	+31
+1, −32	−31
−2, +16	+14
+2, −16	−14
−4, +8	+4
+4, −8	−4

$2a^3 + 8a^2 - 64a = 2a(a + 8)(a - 4)$

91. There is no common factor.

Factor the trinomial $x^2 - 5xy + 6y^2$.

Factors	*Sum*
−1, −6	−7
−2, −3	−5

$x^2 - 5xy + 6y^2 = (x - 2y)(x - 3y)$

93. There is no common factor.

Factor the trinomial $a^2 - 9ab + 20b^2$.

Factors	*Sum*
−1, −20	−21
−2, −10	−12
−4, −5	−9

$a^2 - 9ab + 20b^2 = (a - 4b)(a - 5b)$

95. There is no common factor.

Factor the trinomial $x^2 - 3xy - 28y^2$.

Factors	*Sum*
−1, +28	+27
+1, −28	−27
−2, +14	+12
+2, −14	−12
−4, +7	+3
+4, −7	−3

$x^2 - 3xy - 28y^2 = (x + 4y)(x - 7y)$

97. There is no common factor.

Factor the trinomial $y^2 - 15yz - 41z^2$.

Factors	*Sum*
−1, +41	+40
+1, −41	−40

$y^2 - 15yz - 41z^2$ is nonfactorable over the integers.

99. The GCF is z^2.

$z^4 - 12z^3 + 35z^2 = z^2(z^2 - 12z + 35)$

Factor the trinomial $z^2 - 12z + 35$.

Factors	*Sum*
−1, −35	−36
−5, −7	−12

$z^4 - 12z^3 + 35z^2 = z^2(z - 5)(z - 7)$

101. The GCF is b^2.

$b^4 - 22b^3 + 120b^2 = b^2(b^2 - 22b + 120)$

Factor the trinomial $b^2 - 22b + 120$.

Factors	*Sum*
−1, −120	−121
−2, −60	−62
−3, −40	−43
−4, −30	−34
−5, −24	−29
−6, −20	−26
−8, −15	−23
−10, −12	−22

$b^4 - 22b^3 + 120b^2 = b^2(b - 10)(b - 12)$

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103. The GCF is $2y^2$.

$2y^4 - 26y^3 - 96y^2 = 2y^2(y^2 - 13y - 48)$

Factor the trinomial $y^2 - 13y - 48$.

Factors	*Sum*
$-1, +48$	$+47$
$+1, -48$	-47
$-2, +24$	$+22$
$+2, -24$	-22
$-3, +16$	$+13$
$+3, -16$	-13
$-4, +12$	$+8$
$+4, -12$	-8
$-6, +8$	$+2$
$+6, -8$	-2

$2y^4 - 26y^3 - 96y^2 = 2y^2(y + 3)(y - 16)$

105. The GCF is $-x^2$.

$-x^4 - 7x^3 + 8x^2 = -x^2(x^2 + 7x - 8)$

Factor the trinomial $x^2 + 7x - 8$.

Factors	*Sum*
$-1, +8$	$+7$
$+1, -8$	-7
$-2, +4$	$+2$
$+2, -4$	-2

$-x^4 - 7x^3 + 8x^2 = -x^2(x + 8)(x - 1)$

107. The GCF is $4y$.

$4x^2y + 20xy - 56y = 4y(x^2 + 5x - 14)$

Factor the trinomial $x^2 + 5x - 14$.

Factors	*Sum*
$-1, +14$	$+13$
$+1, -14$	-13
$-2, +7$	$+5$
$+2, -7$	-5

$4x^2y + 20xy - 56y = 4y(x + 7)(x - 2)$

109. The GCF is c.

$c^3 + 18c^2 - 40c = c(c^2 + 18c - 40)$

Factor the trinomial $c^2 + 18c - 40$.

Factors	*Sum*
$-1, +40$	$+39$
$+1, -40$	-39
$-2, +20$	$+18$
$+2, -20$	-18
$-4, +10$	$+6$
$+4, -10$	-6
$-5, +8$	$+3$
$+5, -8$	-3

$c^3 + 18c^2 - 40c = c(c + 20)(c - 2)$

111. The GCF is $-4x$.

$-4x^3 - 4x^2 + 24x = -4x(x^2 + x - 6)$

Factor the trinomial $x^2 + x - 6$.

Factors	*Sum*
$-1, +6$	$+5$
$+1, -6$	-5
$-2, +3$	$+1$
$+2, -3$	-1

$-4x^3 - 4x^2 + 24x = -4x(x + 3)(x - 2)$

113. There is no common factor.

Factor the trinomial $y^2 - 7xy - 8x^2$.

Factors	*Sum*
$-1, +8$	$+7$
$+1, -8$	-7
$-2, +4$	$+2$
$+2, -4$	-2

$y^2 - 7xy - 8x^2 = (y + x)(y - 8x)$

115. There is no common factor.

Factor the trinomial $y^2 + 4yz - 21z^2$.

Factors	*Sum*
$-1, +21$	$+20$
$+1, -21$	-20
$-3, +7$	$+4$
$+3, -7$	-4

$y^2 + 4yz - 21z^2 = (y + 7z)(y - 3z)$

117. There is no common factor.

Factor the trinomial $y^2 - 16yz + 15z^2$.

Factors	*Sum*
$-1, -15$	-16
$-3, -5$	-8

$y^2 - 16yz + 15z^2 = (y - z)(y - 15z)$

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119. The GCF is $4y$.

$4x^2y - 68xy - 72y = 4y(x^2 - 17x - 18)$

Factor the trinomial $x^2 - 17x - 18$.

Factors	*Sum*
$-1, +18$	$+17$
$+1, -18$	-17
$-2, +9$	$+7$
$+2, -9$	-7
$-3, +6$	$+3$
$+3, -6$	-3

$4x^2y - 68xy - 72y = 4y(x + 1)(x - 18)$

121. The GCF is $4x$.

$4x^3 + 12x^2 - 160x$

$= 4x(x^2 + 3x - 40)$

Factor the trinomial $x^2 + 3x - 40$.

Factors	*Sum*
$-1, +40$	$+39$
$+1, -40$	-39
$-2, +20$	$+18$
$+2, -20$	-18
$-4, +10$	$+6$
$+4, -10$	-6
$-5, +8$	$+3$
$+5, -8$	-3

$4x^3 + 12x^2 - 160x$

$= 4x(x + 8)(x - 5)$

123. The GCF is $5z$.

$5z^3 - 50z^2 - 120z$

$= 5z(z^2 - 10z - 24)$

Factor the trinomial $z^2 - 10z - 24$.

Factors	*Sum*
$-1, +24$	$+23$
$+1, -24$	-23
$-2, +12$	$+10$
$+2, -12$	-10
$-3, +8$	$+5$
$+3, -8$	-5
$-4, +6$	$+2$
$+4, -6$	-2

$5z^3 - 50z^2 - 120z$

$= 5z(z + 2)(z - 12)$

125. The GCF is $5x$.

$5x^3 + 30x^2 + 40x = 5x(x^2 + 6x + 8)$

Factor the trinomial $x^2 + 6x + 8$.

Factors	*Sum*
$+1, +8$	$+9$
$+2, +4$	$+6$

$5x^3 + 30x^2 + 40x = 5x(x + 2)(x + 4)$

127. The GCF is 4.

$4p^2 - 28p - 480 = 4(p^2 - 7p - 120)$

Factor the trinomial $p^2 - 7p - 120$.

Factors	*Sum*
$-1, +120$	$+119$
$+1, -120$	-119
$-2, +60$	$+58$
$+2, -60$	-58
$-3, +40$	$+37$
$+3, -40$	-37
$-4, +30$	$+26$
$+4, -30$	-26
$-5, +24$	$+19$
$+5, -24$	-19
$-6, +20$	$+14$
$+6, -20$	-14
$-8, -15$	$+7$
$+8, -15$	-7
$-10, +12$	$+2$
$+10, -12$	-2

$4p^2 - 28p - 480 = 4(p + 8)(p - 15)$

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129. The GCF is p^2.

$p^4 + p^3 - 56p^2 = p^2(p^2 + p - 56)$

Factor the trinomial $p^2 + p - 56$.

Factors	*Sum*
$-1, +56$	$+55$
$+1, -56$	-55
$-2, +28$	$+26$
$+2, -28$	-26
$-4, +14$	$+10$
$+4, -14$	-10
$-7, +8$	$+1$
$+7, -8$	-1

$p^4 + p^3 - 56p^2 = p^2(p + 8)(p - 7)$

131. There is no common factor.

Factor the trinomial $a^2 - 10ab + 25b^2$.

Factors	*Sum*
$-1, -25$	-26
$-5, -5$	-10

$a^2 - 10ab + 25b^2 = (a - 5b)(a - 5b)$

133. There is no common factor.

Factor the trinomial $x^2 + 4xy - 60y^2$.

Factors	*Sum*
$-1, +60$	$+59$
$+1, -60$	-59
$-2, +30$	$+28$
$+2, -30$	-28
$-3, +20$	$+17$
$+3, -20$	-17
$-4, +15$	$+11$
$+4, -15$	-11
$-5, +12$	$+7$
$+5, -12$	-7
$-6, +10$	$+4$
$+6, -10$	-4

$x^2 + 4xy - 60y^2 = (x + 10y)(x - 6y)$

135. The GCF is $6x$.

$6x^3 - 6x^2 - 120x = 6x(x^2 - x - 20)$

Factor the trinomial $x^2 - x - 20$.

Factors	*Sum*
$-1, +20$	$+19$
$+1, -20$	-19
$-2, +10$	$+8$
$+2, -10$	-8
$-4, +5$	$+1$
$+4, -5$	-1

$6x^3 - 6x^2 - 120x = 6x(x + 4)(x - 5)$

Applying the Concepts

137. The GCF is y.

$x^2y - 54y - 3xy = y(x^2 - 54 - 3x) = y(x^2 - 3x - 54)$

Factor the trinomial $x^2 - 3x - 54$.

Factors	*Sum*
$-1, +54$	$+53$
$-2, +27$	$+25$
$-3, +18$	$+15$
$-6, +9$	$+3$
$+6, -9$	-3

$x^2y - 54y - 3xy = y(x + 6)(x - 9)$

139. $x^2 + kx + 35$. The factors of 35 must sum to k.

Factors	*Sum*
$-1, -35$	-36
$+1, +35$	$+36$
$-5, -7$	-12
$+5, +7$	$+12$

k can be -36, $+36$, -12, or $+12$.

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141. $x^2 + kx + 21$. The factors of 21 must sum to k.

Factors	*Sum*
$-1, -21$	-22
$+1, +21$	$+22$
$-3, -7$	-10
$+3, +7$	$+10$

k can be 22, –22, 10, or –10.

143. $z^2 + 7z + k,\ k > 0$

Find two positive integers that sum to 7. Their product is k.

Integers	*Product*
$+1, +6$	6
$+2, +5$	10
$+3, +4$	12

k can be 6, 10, or 12.

145. $c^2 - 7c + k,\ k > 0$

Find two negative integers that sum to –7. Their product is k.

Integers	*Product*
$-1, -6$	6
$-2, -5$	10
$-3, -4$	12

k can be 6, 10, or 12.

147. $y^2 + 5y + k,\ k > 0$

Find two positive integers that sum to 5. Their product is k.

Integers	*Product*
$+2, +3$	6
$+1, +4$	4

k can be 4 or 6.

Section 7.3

Objective A Exercises

1. Positive Positive

Factors of 2: 1, 2 Factors of 1: +1, +1

Trial Factors	*Middle Term*
$(1x + 1)(2x + 1)$	$x + 2x = 3x$

$2x^2 + 3x + 1 = (x + 1)(2x + 1)$

3. Positive Positive

Factors of 2: 1, 2 Factors of 3: +1, +3

Trial Factors	*Middle Term*
$(1y + 1)(2y + 3)$	$3y + 2y = 5y$
$(1y + 3)(2y + 1)$	$y + 6y = 7y$

$2y^2 + 7y + 3 = (y + 3)(2y + 1)$

5. Positive Negative

Factors of 2: 1, 2 Factors of 1: −1, −1

Trial Factors	*Middle Term*
$(1a - 1)(2a - 1)$	$-a - 2a = -3a$

$2a^2 - 3a + 1 = (a - 1)(2a - 1)$

7. Positive Negative

Factors of 2: 1, 2 Factors of 5: −1, −5

Trial Factors	*Middle Term*
$(1b - 1)(2b - 5)$	$-5b - 2b = -7b$
$(1b - 5)(2b - 1)$	$-b - 10b = -11b$

$2b^2 - 11b + 5 = (b - 5)(2b - 1)$

9. Positive

Factors of 2: 1, 2 Factors of −1: −1, +1

Trial Factors	*Middle Term*
$(1x + 1)(2x + 1)$	$x - 2x = -x$
$(1x + 1)(2x - 1)$	$-x + 2x = x$

$2x^2 + x - 1 = (x + 1)(2x - 1)$

11. Positive

Factors of 2: 1, 2 Factors of −3: −1, +3; +1, −3

Trial Factors	*Middle Term*
$(1x - 1)(2x + 3)$	$3x - 2x = x$
$(1x + 3)(2x - 1)$	$-x + 6x = 5x$
$(1x + 1)(2x - 3)$	$-3x + 2x = -x$
$(1x - 3)(2x + 1)$	$x - 6x = -5x$

$2x^2 - 5x - 3 = (x - 3)(2x + 1)$

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13. Positive
Factors of 2: 1, 2 Factors of −10: −1, +10
+1, −10
−2, +5
+2, −5

Trial Factors	*Middle Term*
$(1t-1)(2t+10)$	Common factor
$(1t+10)(2t-1)$	$-t+20t=19t$
$(1t+1)(2t-10)$	Common factor
$(1t-10)(2t+1)$	$t-20t=-19t$
$(1t-2)(2t+5)$	$5t-4t=t$
$(1t+5)(2t-2)$	Common factor
$(1t+2)(2t-5)$	$-5t+4t=-t$
$(1t-5)(2t+2)$	Common factor

$2t^2-t-10=(t+2)(2t-5)$

15. Positive Negative
Factors of 3: 1, 3 Factors of 5: −1, −5

Trial Factors	*Middle Term*
$(1p-1)(3p-5)$	$-5p-3p=-8p$
$(1p-5)(3p-1)$	$-p-15p=-16p$

$3p^2-16p+5=(p-5)(3p-1)$

17. Positive Negative
Factors of 12: 1, 12 Factors of 1: −1, −1
2, 6
3, 4

Trial Factors	*Middle Term*
$(1y-1)(12y-1)$	$-y-12y=-13y$
$(2y-1)(6y-1)$	$-2y-6y=-8y$
$(3y-1)(4y-1)$	$-3y-4y=-7y$

$12y^2-7y+1=(3y-1)(4y-1)$

19. Positive Negative
Factors of 6: 1, 6 Factors of 3: −1, −3
2, 3

Trial Factors	*Middle Term*
$(1z-1)(6z-3)$	Common factor
$(1z-3)(6z-1)$	$-z-18z=-19z$
$(2z-1)(3z-3)$	Common factor
$(2z-3)(3z-1)$	$-2z-9z=-11z$

$6z^2-7z+3$ is nonfactorable over the integers.

21. Positive Negative
Factors of 6: 1, 6 Factors of 4: −1, −4
2, 3 −2, −2

Trial Factors	*Middle Term*
$(1t-1)(6t-4)$	Common factor
$(1t-4)(6t-1)$	$-t-24t=-25t$
$(1t-2)(6t-2)$	Common factor
$(2t-1)(3t-4)$	$-8t-3t=-11t$
$(2t-4)(3t-1)$	$-2t-12t=-14t$
$(2t-2)(3t-2)$	Common factor

$6t^2-11t+4=(2t-1)(3t-4)$

23. Positive Positive
Factors of 8: 1, 8 Factors of 4: +1, +4
2, 4 +2, +2

Trial Factors	*Middle Term*
$(1x+1)(8x+4)$	Common factor
$(1x+4)(8x+1)$	$x+32x=33x$
$(1x+2)(8x+2)$	Common factor
$(2x+1)(8x+4)$	Common factor
$(2x+4)(8x+1)$	Common factor
$(2x+2)(8x+2)$	Common factor

$8x^2+33x+4=(x+4)(8x+1)$

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25. Positive
Factors of 5: 1, 5 Factors of -7: $-1, +7$
$+1, -7$

Trial Factors	*Middle Term*
$(1x-1)(5x+7)$	$7x-5x=2x$
$(1x+7)(5x-1)$	$-x+35x=34x$
$(1x+1)(5x-7)$	$-7x+5x=-2x$
$(1x-7)(5x+1)$	$x-35x=-34x$

$5x^2-62x-7$ is nonfactorable over the integers.

27. Positive Positive
Factors of 12: 1, 12 Factors of 5: $+1, +5$
2, 6
3, 4

Trial Factors	*Middle Term*
$(1y+1)(12y+5)$	$5y+12y=17y$
$(1y+5)(12y+1)$	$y+60y=61y$
$(2y+1)(6y+5)$	$10y+6y=16y$
$(2y+5)(6y+1)$	$2y+30y=32y$
$(3y+1)(4y+5)$	$15y+4y=19y$
$(3y+5)(4y+1)$	$3y+20y=23y$

$12y^2+19y+5=(3y+1)(4y+5)$

29. Positive
Factors of 7: 1, 7 Factors of -14: $-1, +14$
$+1, -14$
$-2, +7$
$+2, -7$

Trial Factors	*Middle Term*
$(1a-1)(7a+14)$	Common factor
$(1a+14)(7a-1)$	$-a+98a=97a$
$(1a+1)(7a-14)$	Common factor
$(1a-14)(7a+1)$	$a-98a=-97a$
$(1a-2)(7a+7)$	Common factor
$(1a+7)(7a-2)$	$-2a+49a=47a$
$(1a+2)(7a-7)$	Common factor
$(1a-7)(7a+2)$	$2a-49a=-47a$

$7a^2+47a-14=(a+7)(7a-2)$

31. Positive Negative
Factors of 3: 1, 3 Factors of 16: $-1, -16$
$-2, -8$
$-4, -4$

Trial Factors	*Middle Term*
$(1b-1)(3b-16)$	$-16b-3b=-19b$
$(1b-16)(3b-1)$	$-b-48b=-49b$
$(1b-2)(3b-8)$	$-8b-6b=-14b$
$(1b-8)(3b-2)$	$-2b-24b=-26b$
$(1b-4)(3b-4)$	$-4b-12b=-16b$

$3b^2-16b+16=(b-4)(3b-4)$

33. Positive
Factors of 2: 1, 2 Factors of -14: $-1, +14$
$+1, -14$
$-2, +7$
$+2, -7$

Trial Factors	*Middle Term*
$(1z-1)(2z+14)$	Common factor
$(1z+14)(2z-1)$	$-z+28z=27z$
$(1z+1)(2z-14)$	Common factor
$(1z-14)(2z+1)$	$z-28z=-27z$
$(1z-2)(2z+7)$	$7z-4z=3z$
$(1z+7)(2z-2)$	Common factor
$(1z+2)(2z-7)$	$-7z+4z=-3z$
$(1z-7)(2z+2)$	Common factor

$2z^2-27z-14=(z-14)(2z+1)$

35. Positive
Factors of 3: 1, 3 Factors of -16: $-1, +16$
$+1, -16$
$-2, +8$
$+2, -8$
$-4, +4$

Trial Factors	*Middle Term*
$(1p-1)(3p+16)$	$16p-3p=13p$
$(1p+16)(3p-1)$	$-p+48p=47p$
$(1p+1)(3p-16)$	$-16p+3p=-13p$
$(1p-16)(3p+1)$	$p-48p=-47p$
$(1p-2)(3p+8)$	$8p-6p=2p$
$(1p+8)(3p-2)$	$-2p+24p=22p$
$(1p+2)(3p-8)$	$-8p+6p=-2p$
$(1p-8)(3p+2)$	$2p-24p=-22p$
$(1p-4)(3p+4)$	$4p-12p=-8p$
$(1p+4)(3p-4)$	$-4p+12p=8p$

$3p^2+22p-16=(p+8)(3p-2)$

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37. The GCF is 2.

$4x^2+6x+2=2(2x^2+3x+1)$

Factor the trinomial.

Positive	Positive
Factors of 2: 1, 2	Factors of 1: +1, +1

Trial Factors	*Middle Term*
$(1x+1)(2x+1)$	$x+2x=3x$

$4x^2+6x+2=2(x+1)(2x+1)$

39. The GCF is 5.

$15y^2-50y+35=5(3y^2-10y+7)$

Factor the trinomial.

Positive	Negative
Factors of 3: 1, 3	Factors of 7: −1, −7

Trial Factors	*Middle Term*
$(1y-1)(3y-7)$	$-7y-3y=-10y$
$(1y-7)(3y-1)$	$-y-21y=-22y$

$15y^2-50y+35=5(y-1)(3y-7)$

41. The GCF is x.

$2x^3-11x^2+5x=x(2x^2-11x+5)$

Factor the trinomial $2x^2-11x+5$.

Positive	Negative
Factors of 2: 1, 2	Factors of 5: −1, −5

Trial Factors	*Middle Term*
$(1x-1)(2x-5)$	$-5x-2x=-7x$
$(1x-5)(2x-1)$	$-x-10x=-11x$

$2x^3-11x^2+5x=x(x-5)(2x-1)$

43. The GCF is b.

$3a^2b-16ab+16b=b(3a^2-16a+16)$

Factor the trinomial $3a^2-16a+16$.

Positive	Negative
Factors of 3: 1, 3	Factors of 16: −1, −16 −2, −8 −4, −4

Trial Factors	*Middle Term*
$(1a-1)(3a-16)$	$-16a-3a=-19a$
$(1a-16)(3a-1)$	$-a-48a=-49a$
$(1a-2)(3a-8)$	$-8a-6a=-14a$
$(1a-8)(3a-2)$	$-2a-24a=-26a$
$(1a-4)(3a-4)$	$-4a-12a=-16a$

$3a^2b-16ab+16b=b(a-4)(3a-4)$

45. There is no common factor.

Factor the trinomial.

Positive	Positive
Factors of 3: 1, 3	Factors of 10: +1, +10 +2, +5

Trial Factors	*Middle Term*
$(1z+1)(3z+10)$	$10z+3z=13z$
$(1z+10)(3z+1)$	$z+30z=31z$
$(1z+2)(3z+5)$	$5z+6z=11z$
$(1z+5)(3z+2)$	$2z+8z=10z$

$3z^2+95z+10$ is nonfactorable over the integers.

47. The GCF is $3x$.

$36x-3x^2-3x^3=3x(12-x-x^2)$

Factor the trinomial.

Positive	
Factors of 12: 1, 12 2, 6 3, 4	Factors of −1: −1, +1

Trial Factors	*Middle Term*
$(1-1x)(12+1x)$	$x-12x=-11x$
$(1+1x)(12-1x)$	$-x+12x=11x$
$(2-1x)(6+1x)$	$2x-6x=-4x$
$(2+1x)(6-1x)$	$-2x+6x=4x$
$(3-1x)(4+1x)$	$3x-4x=-x$
$(3+1x)(4-1x)$	$-3x+4x=x$

$36x-3x^2-3x^3=-3x(x-3)(x+4)$

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49. The GCF is 4.

$80y^2 - 36y + 4 = 4(20y^2 - 9y + 1)$

Factor the trinomial.

Positive		Negative	
Factors of 20:	1, 20	Factors of 1:	−1, −1
	2, 10		
	4, 5		

Trial Factors	*Middle Term*
$(1y - 1)(20y - 1)$	$-y - 20y = -21y$
$(2y - 1)(10y - 1)$	$-2y - 10y = -12y$
$(4y - 1)(5y - 1)$	$-4y - 5y = -9y$

$80y^2 - 36y + 4 = 4(4y - 1)(5y - 1)$

51. The GCF is z.

$8z^3 + 4z^2 + 3z = z(8z^2 + 14z + 3)$

Factor the trinomial.

Positive		Positive	
Factors of 8:	1, 8	Factors of 3:	+1, +3
	2, 4		

Trial Factors	*Middle Term*
$(1z + 1)(8z + 3)$	$3z + 8z = 11z$
$(1z + 3)(8z + 1)$	$z + 24z = 25z$
$(2z + 1)(4z + 3)$	$6z + 4z = 10z$
$(2z + 3)(4z + 1)$	$2z + 12z = 14z$

$8z^3 + 14z^2 + 3z = z(2z + 3)(4z + 1)$

53. The GCF is y. $6x^2y - 11xy - 10y = y(6x^2 - 11x - 10)$

Factor the trinomial.

Positive			
Factors of 6:	1, 6	Factors of −10:	−1, +10
	2, 3		+1, −10
			−2, +5
			+2, −5

Trial Factors	*Middle Term*
$(1x - 1)(6x + 10)$	Common factor
$(1x + 10)(6x - 1)$	$-x + 60x = 59x$
$(1x + 1)(6x - 10)$	Common factor
$(1x - 10)(6x + 1)$	$x - 60x = -59x$
$(1x - 2)(6x + 5)$	$5x - 12x = -7x$
$(1x + 5)(6x - 2)$	Common factor
$(1x + 2)(6x - 5)$	$-5x + 12x = 7x$
$(1x - 5)(6x + 2)$	Common factor
$(2x - 1)(3x + 10)$	$20x - 3x = 17x$
$(2x + 10)(3x - 1)$	Common factor
$(2x + 1)(3x - 10)$	$-20x + 3x = -17x$
$(2x - 10)(3x + 1)$	Common factor
$(2x - 2)(3x + 5)$	Common factor
$(2x + 5)(3x - 2)$	$-4x + 15x = 11x$
$(2x + 2)(3x - 5)$	Common factor
$(2x - 5)(3x + 2)$	$4x - 15x = -11x$

$6x^2y - 11xy - 10y = y(2x - 5)(3x + 2)$

55. The GCF is 5. $10t^2 - 5t - 50 = 5(2t^2 - t - 10)$

Factor the trinomial.

Positive			
Factors of 2:	1, 2	Factors of −10:	−1, +10
			+1, −10
			−2, +5
			+2, −5

Trial Factors	*Middle Term*
$(1t - 1)(2t + 10)$	Common factor
$(1t + 10)(2t - 1)$	$-t + 20t = 19t$
$(1t + 1)(2t - 10)$	Common factor
$(1t - 10)(2t + 1)$	$t - 20t = -19t$
$(1t - 2)(2t + 5)$	$5t - 4t = t$
$(1t + 5)(2t - 2)$	Common factor
$(1t + 2)(2t - 5)$	$-5t + 4t = -t$
$(1t - 5)(2t + 2)$	Common factor

$10t^2 - 5t - 50 = 5(t + 2)(2t - 5)$

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57. The GCF is p. $3p^3 - 16p^2 + 5p = p(3p^2 - 16p + 5)$

Factor the trinomial.

Positive	Negative
Factors of 3: 1, 3	Factors of 5: $-1, -5$

Trial Factors	*Middle Term*
$(1p - 1)(3p - 5)$	$-5p - 3p = -8p$
$(1p - 5)(3p - 1)$	$-p - 15p = -16p$

$3p^3 - 16p^2 + 5p = p(p - 5)(3p - 1)$

59. The GCF is 2. $26z^2 + 98z - 24 = 2(13z^2 + 49z - 12)$

Factor the trinomial.

Positive

Factors of 13: 1, 13	Factors of -12: $-1, +12$
	$+1, -12$
	$-2, +6$
	$+2, -6$
	$-3, +4$
	$+3, -4$

Trial Factors	*Middle Term*
$(1z - 1)(13z + 12)$	$2z - 13z = -z$
$(1z + 12)(13z - 1)$	$-z + 156z = 155z$
$(1z + 1)(13z - 12)$	$-12z + 13z = z$
$(1z - 12)(13z + 1)$	$z - 156z = -155z$
$(1z - 2)(13z + 6)$	$6z - 26z = -20z$
$(1z + 6)(13z - 2)$	$-2z + 78z = 76z$
$(1z + 2)(13z - 6)$	$-6z + 26z = 20z$
$(1z - 6)(13z + 2)$	$2z - 78z = -76z$
$(1z - 3)(13z + 4)$	$4z - 39z = -35z$
$(1z + 4)(13z - 3)$	$-3z + 52z = 49z$
$(1z + 3)(13z - 4)$	$-4z + 39z = 35z$
$(1z - 4)(13z + 3)$	$3z - 52z = -49z$

$26z^2 + 98z - 24 = 2(z + 4)(13z - 3)$

59. The GCF is 2. $26z^2 + 98z - 24 = 2(13z^2 + 49z - 12)$

Factor the trinomial.

Positive

Factors of 13: 1, 13	Factors of -12: $-1, +12$
	$+1, -12$
	$-2, +6$
	$+2, -6$
	$-3, +4$
	$+3, -4$

Trial Factors	*Middle Term*
$(1z - 1)(13z + 12)$	$2z - 13z = -z$
$(1z + 12)(13z - 1)$	$-z + 156z = 155z$
$(1z + 1)(13z - 12)$	$-12z + 13z = z$
$(1z - 12)(13z + 1)$	$z - 156z = -155z$
$(1z - 2)(13z + 6)$	$6z - 26z = -20z$
$(1z + 6)(13z - 2)$	$-2z + 78z = 76z$
$(1z + 2)(13z - 6)$	$-6z + 26z = 20z$
$(1z - 6)(13z + 2)$	$2z - 78z = -76z$
$(1z - 3)(13z + 4)$	$4z - 39z = -35z$
$(1z + 4)(13z - 3)$	$-3z + 52z = 49z$
$(1z + 3)(13z - 4)$	$-4z + 39z = 35z$
$(1z - 4)(13z + 3)$	$3z - 52z = -49z$

$26z^2 + 98z - 24 = 2(z + 4)(13z - 3)$

61. The GCF is $2y$.

$10y^3 - 44y^2 + 16y = 2y(5y^2 - 22y + 8)$

Factor the trinomial.

Positive	Negative
Factors of 5: 1, 5	Factors of 8: $-1, -8$
	$-2, -4$

Trial Factors	*Middle Term*
$(1y - 1)(5y - 8)$	$-8y - 5y = -13y$
$(1y - 8)(5y - 1)$	$-y - 40y = -41y$
$(1y - 2)(5y - 4)$	$-4y - 10y = -14y$
$(1y - 4)(5y - 2)$	$-2y - 20y = -22y$

$10y^3 - 44y^2 + 16y = 2y(y - 4)(5y - 2)$

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63. The GCF is yz.

$4yz^3 + 5yz^2 - 6yz = yz(4z^2 + 5z - 6)$

Factor the trinomial.

Positive		Factors of -6:	
Factors of 4:	1, 4		$-1, +6$
	2, 2		$+1, -6$
			$-2, +3$
			$+2, -3$

Trial Factors	*Middle Term*
$(1z - 1)(4z + 6)$	Common factor
$(1z + 6)(4z - 1)$	$-z + 24z = 23z$
$(1z + 1)(4z - 6)$	Common factor
$(1z - 6)(4z + 1)$	$z - 24z = -23z$
$(1z - 2)(4z + 3)$	$3z - 8z = -5z$
$(1z + 3)(4z - 2)$	Common factor
$(1z + 2)(4z - 3)$	$-3z + 8z = 5z$
$(1z - 3)(4z + 2)$	Common factor
$(2z - 1)(2z + 6)$	Common factor
$(2z + 1)(2z - 6)$	Common factor
$(2z - 2)(2z + 3)$	Common factor
$(2z + 2)(2z - 3)$	Common factor

$4yz^3 + 5yz^2 - 6yz = yz(z + 2)(4z - 3)$

65. The GCF is $3a$. $42a^3 + 45a^2 - 27a = 3a(14a^2 + 15a - 9)$

Factor the trinomial.

Positive		Factors of -9:	
Factors of 14:	1, 14		$-1, +9$
	2, 7		$+1, -9$
			$-3, +3$

Trial Factors	*Middle Term*
$(1a - 1)(14a + 9)$	$9a - 14a = -5a$
$(1a + 9)(14a - 1)$	$-a + 126a = 125a$
$(1a + 1)(14a - 9)$	$-9a + 14a = 5a$
$(1a - 9)(14a + 1)$	$a - 126a = -125a$
$(1a - 3)(14a + 3)$	$-3a + 42a = 39a$
$(1a + 3)(14a - 3)$	$3a - 42a = -39a$
$(2a - 1)(7a + 9)$	$18a - 7a = 11a$
$(2a + 9)(7a - 1)$	$-2a + 63a = 61a$
$(2a + 1)(7a - 9)$	$-18a + 7a = -11a$
$(2a - 9)(7a + 1)$	$2a - 63a = -61a$
$(2a - 3)(7a + 3)$	$6a - 21a = -15a$
$(2a + 3)(7a - 3)$	$-6a + 21a = 15a$

$42a^3 + 45a^2 - 27a = 3a(2a + 3)(7a - 3)$

67. The GCF is y.

$9x^2y - 30xy^2 + 25y^2 = y(9x^2 - 30xy + 25y^2)$

Factor the trinomial.

Positive		Negative	
Factors of 9:	1, 9	Factors of 25:	$-1, -25$
	3, 3		$-5, -5$

Trial Factors	*Middle Term*
$(1x - 1y)(9x - 25y)$	$-25xy - 9xy = -34xy$
$(1x - 25y)(9x - 1y)$	$-xy - 225xy = -226xy$
$(1x - 5y)(9x - 5y)$	$-5xy - 45xy = -50xy$
$(3x - 1y)(3x - 25y)$	$-75xy - 3xy = -78xy$
$(3x - 5y)(3x - 5y)$	$-15xy - 15xy = -30xy$

$9x^2y - 30xy^2 + 25y^3 = y(3x - 5y)(3x - 5y)$

69. The GCF is xy.

$9x^3y - 24x^2y^2 + 16xy^3 = xy(9x^2 - 24xy + 16y^2)$

Factor the trinomial.

Positive		Negative	
Factors of 9:	1, 9	Factors of 16:	$-1, -16$
	3, 3		$-2, -8$
			$-4, -4$

Trial Factors	*Middle Term*
$(1x - 1y)(9x - 16y)$	$-16xy - 9xy = -25xy$
$(1x - 16y)(9x - 1y)$	$-xy - 144xy = -145xy$
$(1x - 2y)(9x - 8y)$	$-8xy - 18xy = -26xy$
$(1x - 8y)(9x - 2y)$	$-2xy - 72xy = -74xy$
$(1x - 4y)(9x - 4y)$	$-4xy - 36xy = -40xy$
$(3x - 1y)(3x - 16y)$	$-48xy - 3xy = -51xy$
$(3x - 2y)(3x - 8y)$	$-24xy - 6xy = -30xy$
$(3x - 4y)(3x - 4y)$	$-12xy - 12xy = -24xy$

$9x^3y - 24x^2y^2 + 16xy^3 = xy(3x - 4y)(3x - 4y)$

Objective B Exercises

71. $6x^2 - 17x + 12$

$6 \cdot 12 = 72$

Factors of 72 whose sum is -17: -9 and -8

$6x^2 - 17x + 12 = 6x^2 - 9x - 8x + 12 = 3x(2x - 3) - 4(2x - 3) = (2x - 3)(3x - 4)$

73. $5b^2 + 33b - 14$

$5(-14) = -70$

Factors of -70 whose sum is 33: 35 and -2

$5b^2 + 33b - 14 = 5b^2 + 35b - 2b - 14 = 5b(b + 7) - 2(b + 7) = (b + 7)(5b - 2)$

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75. $6a^2+7a-24$
$6(-24)=-144$

Factors of -144 whose sum is 7: 16 and -9
$6a^2+7a-24=6a^2+16a-9a-24=2a(3a+8)-3(3a+8)=(3a+8)(2a-3)$

77. $4z^2+11z+6$
$4\cdot 6=24$

Factors of 24 whose sum is 11: 8 and 3
$4z^2+11z+6=4z^2+8z+3z+6=4z(z+2)+3(z+2)=(z+2)(4z+3)$

79. $22p^2+51p-10$
$22(-10)=-220$

Factors of -220 whose sum is 51: 55 and -4
$22p^2+51p-10=22p^2+55p-4p-10=11p(2p+5)-2(2p+5)$
$=(2p+5)(11p-2)$

81. $8y^2+17y+9$
$8\cdot 9=72$

Factors of 72 whose sum is 17: 9 and 8
$8y^2+17y+9=8y^2+8y+9y+9=8y(y+1)+9(y+1)=(y+1)(8y+9)$

83. $18t^2-9t-5$
$18(-5)=-90$

Factors of -90 whose sum is -9: -15 and 6
$18t^2-9t-5=18t^2-15t+6t-5=3t(6t-5)+1(6t-5)=(6t-5)(3t+1)$

85. $6b^2+71b-12$
$6(-12)=-72$

Factors of -72 whose sum is 71: 72 and -1
$6b^2+71b-12=6b^2+72b-b-12=6b(b+12)-1(b+12)=(b+12)(6b-1)$

87. $9x^2+12x+4$
$9\cdot 4=36$

Factors of 36 whose sum is 12: 6 and 6
$9x^2+12x+4=9x^2+6x+6x+4=3x(3x+2)+2(3x+2)=(3x+2)(3x+2)$

89. $6b^2-13b+6$
$6\cdot 6=36$

Factors of 36 whose sum is -13: -9 and -4
$6b^2-13b+6=6b^2-9b-4b+6=3b(2b-3)-2(2b-3)=(2b-3)(3b-2)$

91. $33b^2+34b-35$
$33(-35)=-1155$

Factors of -1155 whose sum is 34: 55 and -21
$33b^2+34b-35=33b^2+55b-21b-35=11b(3b+5)-7(3b+5)=(3b+5)(11b-7)$

93. $18y^2-39y+20$
$18\cdot 20=360$

Factors of 360 whose sum is -39: -24 and -15
$18y^2-39y+20=18y^2-24y-15y+20=6y(3y-4)-5(3y-4)=(3y-4)(6y-5)$

95. $15a^2+26a-21$
$15(-21)=-315$

Factors of -315 whose sum is 26: 35 and -9
$15a^2+26a-21=15a^2+35a-9a-21=5a(3a+7)-3(3a+7)=(3a+7)(5a-3)$

97. $8y^2-26y+15$
$8\cdot 15=120$

Factors of 120 whose sum is -26: -20 and -6
$8y^2-26y+15=8y^2-20y-6y+15=4y(2y-5)-3(2y-5)=(2y-5)(4y-3)$

99. $8z^2+2z-15$
$8(-15)=-120$

Factors of -120 whose sum is 2: 12 and -10
$8z^2+2z-15=8z^2+12z-10z-15=4z(2z+3)-5(2z+3)=(2z+3)(4z-5)$

101. $15x^2-82x+24$
$15\cdot 24=360$

Factors of 360 whose sum is -82: none
Nonfactorable over the integers.

103. $10z^2-29z+10$
$10\cdot 10=100$

Factors of 100 whose sum is -29: -25 and -4
$10z^2-29z+10=10z^2-25z-4z+10=5z(2z-5)-2(2z-5)=(2z-5)(5z-2)$

105. $36z^2+72z+35$
$36\cdot 35=1260$

Factors of 1260 whose sum is 72: 30 and 42
$36z^2+72z+35=36z^2+30z+42z+35=6z(6z+5)+7(6z+5)=(6z+5)(6z+7)$

107. $3x^2+xy-2y^2$
$3(-2)=-6$

Factors of -6 whose sum is 1: 3 and -2
$3x^2+xy-2y^2=3x^2+3xy-2xy-2y^2=3x(x+y)-2y(x+y)=(x+y)(3x-2y)$

109. $3a^2+5ab-2b^2$
$3(-2)=-6$

Factors of -6 whose sum is 5: 6 and -1
$3a^2+5ab-2b^2=3a^2+6ab-ab-2b^2=3a(a+2b)-b(a+2b)=(a+2b)(3a-b)$

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111. $4y^2 - 11yz + 6z^2$
$4 \cdot 6 = 24$

Factors of 24 whose sum is –11: –8 and –3
$4y^2 - 11yz + 6z^2 = 4y^2 - 8yz - 3yz + 6z^2 = 4y(y - 2z) - 3z(y - 2z) = (y - 2z)(4y - 3z)$

113. $28 + 3z - z^2$
$28(-1) = -28$

Factors of –28 whose sum is 3: 7 and –4
$28 + 3z - z^2 = -z^2 + 3z + 28 = -z^2 + 7z - 4z + 28 = -z(z + 4) + 7(z + 4) = -(z - 7)(z + 4)$

115. $8 - 7x - x^2$
$8(-1) = -8$

Factors of –8 whose sum is –7: –8 and 1
$8 - 7x - x^2 = -x^2 - 7x + 8 = -x^2 - 8x + x + 8 = -x(x - 1) - 8(x - 1) = -(x - 1)(x + 8)$

117. $9x^2 + 33x - 60$
Common factor 3:
$3(3x^2 + 11x - 20)$
$3(-20) = -60$

Factors of –60 whose sum is 11: 15 and –4
$3(3x^2 + 11x - 20) = 3(3x^2 + 15x - 4x - 20) = 3[3x(x + 5) - 4(x + 5)] = 3(x + 5)(3x - 4)$

119. $24x^2 - 52x + 24$
Common factor 4:
$4(6x^2 - 13x + 6)$
$6 \cdot 6 = 36$

Factors of 36 whose sum is –13: –9 and –4
$4(6x^2 - 13x + 6) = 4(6x^2 - 9x - 4x + 6) = 4[3x(2x - 3) - 2(2x - 3)] = 4(2x - 3)(3x - 2)$

121. $35a^4 + 9a^3 - 2a^2$
Common factor a^2:
$a^2(35a^2 + 9a - 2)$
$35(-2) = -70$

Factors of –70 whose sum is 9: 14 and –5
$a^2(35a^2 + 9a - 2) = a^2(35a^2 + 14a - 5a - 2) = a^2[7a(5a + 2) - 1(5a + 2)] = a^2(5a + 2)(7a - 1)$

123. $15b^2 - 115b + 70$
Common factor 5:
$5(3b^2 - 23b + 14)$
$3 \cdot 14 = 42$

Factors of 42 whose sum is –23: –21 and –2
$5(3b^2 - 23b + 14) = 5(3b^2 - 21b - 2b + 14) = 5[3b(b - 7) - 2(b - 7)] = 5(b - 7)(3b - 2)$

125. $3x^2 - 26xy + 35y^2$
$3 \cdot 35 = 105$

Factors of 105 whose sum is –26: –21 and –5
$3x^2 - 25xy + 36y^2 = 3x^2 - 21xy - 5xy + 35y^2 = 3x(x - 7y) - 5y(x - 7y) = (x - 7y)(3x - 5y)$

127. $216y^2 - 3y - 3$
Common factor 3:
$3(72y^2 - y - 1)$
$72(-1) = -72$

Factors of –72 whose sum is –1: –9 and 8
$3(72y^2 - y - 1) = 3(72y^2 - 9y + 8y - 1) = 3[9y(8y - 1) + 1(8y - 1)] = 3(8y - 1)(9y + 1)$

129. $21 - 20x - x^2$
$21(-1) = -21$

Factors of –21 whose sum is –20: –21 and 1
$21 - 20x - x^2 = -x^2 - 20x + 21 = -x^2 + x - 21x + 21 = -x(x + 21) + (x + 21) = -(x - 1)(x + 21)$

131. $15a^2 + 11ab - 14b^2$
$15(-14) = -210$

Factors of –210 whose sum is 1: 21 and –10
$15a^2 + 11ab - 14b^2 = 15a^2 + 21ab - 10ab - 14b^2 = 3a(5a + 7b) - 2b(5a + 7b) = (5a + 7b)(3a - 2b)$

133. $33z - 8z^2 - z^3$
Common factor z:
$z(33 - 8z - z^2)$
$33(-1) = -33$

Factors of –33 whose sum is –8: –11 and 3
$z(33 - 8z - z^2) = -z(z^2 + 8z - 33) = -z[z^2 + 11z - 32 - 33] = -z[z(z - 3) + 11(z - 3)]$
$= -z(z + 11)(z - 3)$

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Applying the Concepts

135. $3(a+2)^2-(a+2)-4$ Let $x=a+2$

$=3x^2-x-4$

$=(3x-44)(x+1)$

$=[3(a+2)-4][a+2+1]$

$=(3a+6-4)(a+3)$

$=(3a+2)(a+3)$

137. $2x^2+kx+3$

$2\times 3=6$

Factors of 6 *Sum*

$+1, +6$ $+7$

$-1, -6$ -7

$+2, +3$ $+5$

$-2, -3$ -5

k can be 7, –7, 5, or –5.

139. $3x^2+kx+2$

$3\times 2=6$

Factors of 6 *Sum*

$+1, +6$ $+7$

$-1, -6$ -7

$+2, +3$ $+5$

$-2, -3$ -5

k can be 7, –7, 5, or –5.

Section 7.4

Objective A Exercises

1. $4=(2)^2;\ 25x^6=(5x^3)^2;\ 100x^4y^4=(10x^2y^2)^2$

3. $4z^4$

5. $9a^2b^3$

7. $x^2-16=x^2-4^2=(x+4)(x-4)$

9. $4x^2-1=(2x)^2-1^2=(2x+1)(2x-1)$

11. $16x^2-121=(4x)^2-11^2=(4x+11)(4x-11)$

13. $1-9a^2=1^2-(3a)^2=(1+3a)(1-3a)$

15. $x^2y^2-100=(xy)^2-10^2=(xy+10)(xy-10)$

17. x^2+4 is nonfactorable over the integers.

19. $25-a^2b^2=5^2-(ab)^2=(5+ab)(5-ab)$

21. $a^{2n}-1=(a^n)^2-1^2=(a^n+1)(a^n-1)$

23. $x^2-12x+36=(x-6)^2$

25. $b^2-2b+1=(b-1)^2$

27. $16x^2-40x+25=(4x-5)^2$

29. $4a^2+4a-1$ is nonfactorable over the integers.

31. $b^2+7b+14$ is nonfactorable over the integers.

33. $x^2+6xy+9y^2=(x+3y)^2$

35. $25a^2-40ab+16b^2=(5a-4b)^2$

37. $x^{2n}+6x^n+9=(x^n+3)^2$

39. $(x-4)^2-9=[(x-4)-3][(x-4)+3]$

$=(x-4-3)(x-4+3)=(x-7)(x-1)$

41. $(x-y)^2-(a+b)^2=[(x-y)-(a+b)][(x-y)+(a+b)]=(x-y-a-b)(x-y+a+b)$

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Objective B Exercises

43. $8=(2)^3;\ x^9=(x^3)^3;\ 27c^{15}d^{18}=(3c^5d^6)^3$

45. $2x^3$

47. $4a^2b^6$

49. $x^3-27=x^3-3^3=(x-3)(x^2+3x+9)$

51. $8x^3-1=(2x)^3-1^3=(2x-1)(4x^2+2x+1)$

53. $x^3-y^3=(x-y)(x^2+xy+y^2)$

55. $m^3+n^3=(m+n)(m^2-mn+n^2)$

57. $64x^3+1=(4x)^3+1^3=(4x+1)(16x^2-4x+1)$

59. $27x^3-8y^3=(3x)^3-(2y)^3=(3x-2y)(9x^2+6xy+4y^2)$

61. $x^3y^3+64=(xy)^3+4^3=(xy+4)(x^2y^2-4xy+16)$

63. $16x^3-y^3$ is nonfactorable over the integers.

65. $8x^3-9y^3$ is nonfactorable over the integers.

67.
$$\begin{aligned}(a-b)^3-b^3&=[(a-b)-b]\left[(a-b)^2+b(a-b)+b^2\right]\\&=(a-2b)(a^2-2ab+b^2+ab-b^2+b^2)\\&=(a-2b)(a^2-ab+b^2)\end{aligned}$$

69.
$$\begin{aligned}x^{6n}+y^{3n}&=(x^{2n})^3+(y^n)^3\\&=(x^{2n}+y^n)(x^{4n}-x^{2n}y^n+y^{2n})\end{aligned}$$

71. $x^{3n}+8=(x^n)^3+2^3=(x^n+2)(x^{2n}-2x^n+4)$

Objective C Exercises

73. Let $u=xy$.
$$\begin{aligned}x^2y^2-8xy+15&=u^2-8u+15=(u-5)(u-3)\\&=(xy-5)(xy-3)\end{aligned}$$

75. Let $u=xy$.
$$\begin{aligned}x^2y^2-17xy+60&=u^2-17u+60=(u-12)(u-5)\\&=(xy-12)(xy-5)\end{aligned}$$

77. Let $u=x^2$.
$$\begin{aligned}x^4-9x^2+18&=u^2-9u+18=(u-3)(u-6)\\&=(x^2-3)(x^2-6)\end{aligned}$$

79. Let $u=b^2$.
$$\begin{aligned}b^4-13b^2-90&=u^2-13u-90=(u+5)(u-18)\\&=(b^2+5)(b^2-18)\end{aligned}$$

81. Let $u=x^2y^2$.
$$\begin{aligned}x^4y^4-8x^2y^2+12&=u^2-8u+12=(u-2)(u-6)\\&=(x^2y^2-2)(x^2y^2-6)\end{aligned}$$

83. Let $u=x^n$.
$$\begin{aligned}x^{2n}+3x^n+2&=u^2+3u+2=(u+1)(u+2)\\&=(x^n+1)(x^n+2)\end{aligned}$$

85. Let $u=xy$.
$$\begin{aligned}3x^2y^2-14xy+15&=3u^2-14u+15=(3u-5)(u-3)\\&=(3xy-5)(xy-3)\end{aligned}$$

87. Let $u=ab$.
$$\begin{aligned}6a^2b^2-23ab+21&=6u^2-23u+21=(2u-3)(3u-7)\\&=(2ab-3)(3ab-7)\end{aligned}$$

89. Let $u=x^2$.
$$\begin{aligned}2x^4-13x^2-15&=2u^2-13u-15=(u+1)(2u-15)\\&=(x^2+1)(2x^2-15)\end{aligned}$$

91. Let $u=x^n$.
$$\begin{aligned}2x^{2n}-7x^n+3&=2u^2-7u+3=(2u-1)(u-3)\\&=(2x^n-1)(x^n-3)\end{aligned}$$

93. Let $u=a^n$.
$$\begin{aligned}6a^{2n}+19a^n+10&=6u^2+19u+10=(2u+5)(3u+2)\\&=(2a^n+5)(3a^n+2)\end{aligned}$$

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Objective D Exercises

95. $12x^2 - 36x + 27 = 3(4x^2 - 12x + 9) = 3(2x - 3)^2$

97. $27a^4 - a = a(27a^3 - 1) = a(3a - 1)(9a^2 + 3a + 1)$

99. $20x^2 - 5 = 5(4x^2 - 1) = 5(2x + 1)(2x - 1)$

101. $y^5 + 6y^4 - 55y^3 = y^3(y^2 + 6y - 55) = y^3(y + 11)(y - 5)$

103. $16x^4 - 81 = (4x^2 + 9)(4x^2 - 9) = (4x^2 + 9)(2x + 3)(2x - 3)$

105. $16a - 2a^4 = 2a(8 - a^3) = 2a(2 - a)(4 + 2a + a^2)$

107. $a^3b^6 - b^3 = b^3(a^3b^3 - 1) = b^3(ab - 1)(a^2b^2 + ab + 1)$

109. $8x^4 - 40x^3 + 50x^2 = 2x^2(4x^2 - 20x + 25) = 2x^2(2x - 5)^2$

111. $x^4 - y^4 = (x^2 + y^2)(x^2 - y^2) = (x^2 + y^2)(x + y)(x - y)$

113. $x^6 + y^6 = (x^2)^3 + (y^2)^3 = (x^2 + y^2)(x^4 - x^2y^2 + y^4)$

115. $a^4 - 25a^2 - 144$ is nonfactorable over the integers.

117. $16a^4 - 2a = 2a(8a^3 - 1) = 2a(2a - 1)(4a^2 + 2a + 1)$

119. $24a^2b^2 - 14ab^3 - 90b^4 = 2b^2(12a^2 - 7ab - 45b^2)$
$= 2b^2(3a + 5b)(4a - 9b)$

121. $x^3 - 2x^2 - 4x + 8 = x^2(x - 2) - 4(x - 2) = (x - 2)(x^2 - 4)$
$= (x - 2)(x + 2)(x - 2) = (x - 2)^2(x + 2)$

123. $4x^4 - x^2 - 4x^2y^2 + y^2 = x^2(4x^2 - 1) - y^2(4x^2 - 1)$
$= (4x^2 - 1)(x^2 - y^2)$
$= (2x + 1)(2x - 1)(x + y)(x - y)$

125. $x^{2n+1} + 2x^{n+1} + x = x(x^{2n} + 2x^n + 1) = x(x^n + 1)^2$

127. $3b^{n+2} + 4b^{n+1} - 4b^n = b^n(3b^2 + 4b - 4) = b^n(b + 2)(3b - 2)$

Applying the Concepts

129. $16x^2 + 24x + 9 = (4x + 3)(4x + 3)$

The dimensions of the square are $(4x + 3)$ m by $(4x + 3)$ m.

Yes, it is possible for x to equal zero.

If $x = 0$, then $4x + 3 = 4(0) + 3 = 3$, and the dimensions are 3 m by 3 m.

The length of a side must be a positive number.

$4x + 3 > 0$
$4x > -3$
$x > -\frac{3}{4}$

The possible values of x are $x > -\frac{3}{4}$.

Section 7.5

Objective A Exercises

3. $(y - 3)(y - 5) = 0$
$y - 3 = 0 \quad y - 5 = 0$
$y = 3 \quad y = 5$
The solutions are 3 and 5.

5. $(z + 8)(z - 9) = 0$
$z + 8 = 0 \quad z - 9 = 0$
$z = -8 \quad z = 9$
The solutions are –8 and 9.

7. $x(x + 2) = 0$
$x = 0 \quad x + 2 = 0$
$x = -2$
The solutions are –2 and 0.

9. $a(a + 12) = 0$
$a = 0 \quad a + 12 = 0$
$a = -12$
The solutions are 0 and –12.

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11. $t(4t-7)=0$

$t=0 \quad 4t-7=0$

$4t=7$

$t=\frac{7}{4}$

The solutions are $\frac{7}{4}$ and 0.

13. $4b(2b+5)=0$

$4b=0 \quad 2b+5=0$

$b=0 \quad 2b=-5$

$b=-\frac{5}{2}$

The solutions are $-\left(\frac{5}{2}\right)$ and 0.

15. $(b-8)(b+3)=0$

$b-8=0 \quad b+3=0$

$b=8 \quad b=-3$

The solutions are –3 and 8.

17. $x^2-121=0$

$(x+11)(x-11)=0$

$x+11=0 \quad x-11=0$

$x=-11 \quad x=11$

The solutions are –11 and 11.

19. $16x^2-1=0$

$(4x-1)(4x+1)=0$

$4x-1=0 \quad 4x+1=0$

$4x=1 \quad 4x=-1$

$x=\frac{1}{4} \quad x=-\frac{1}{4}$

The solutions are $-\left(\frac{1}{4}\right)$ and $\frac{1}{4}$.

21. $16x^2-49=0$

$(4x+7)(4x-7)=0$

$4x+7=0 \quad 4x-7=0$

$4x=-7 \quad 4x=7$

$x=-\frac{7}{4} \quad x=\frac{7}{4}$

The solutions are $-\left(\frac{7}{4}\right)$ and $\frac{7}{4}$.

23. $x^2-8x+15=0$

$(x-3)(x-5)=0$

$x-3=0 \quad x-5=0$

$x=3 \quad x=5$

The solutions are 3 and 5.

25. $z^2+z-72=0$

$(z+9)(z-8)=0$

$z+9=0 \quad z-8=0$

$z=-9 \quad z=8$

The solutions are –9 and 8.

27. $3a^2+14a+8=0$

$(3a+2)(a+4)=0$

$3a+2=0 \quad a+4=0$

$3a=-2 \quad a=-4$

$a=-\frac{2}{3}$

The solutions are $-\left(\frac{2}{3}\right)$ and –4.

29. $6y^2-19y+15=0$

$(3y-5)(2y-3)=0$

$3y-5=0 \quad 2y-3=0$

$3y=5 \quad 2y=3$

$y=\frac{5}{3} \quad y=\frac{3}{2}$

The solutions are $\frac{5}{3}$ and $\frac{3}{2}$.

31. $a^2-5a=0$

$a(a-5)=0$

$a=0 \quad a-5=0$

$a=5$

The solutions are 0 and 5.

33. $2a^2-8a=0$

$2a(a-4)=0$

$2a=0 \quad a-4=0$

$a=0 \quad a=4$

The solutions are 0 and 4.

35. $a^2-5a=24$

$a^2-5a-24=0$

$(a+3)(a-8)=0$

$a+3=0 \quad a-8=0$

$a=-3 \quad a=8$

The solutions are –3 and 8.

37. $y^2-7y=8$

$y^2-7y-8=0$

$(y+1)(y-8)=0$

$y+1=0 \quad y-8=0$

$y=-1 \quad y=8$

The solutions are –1 and 8.

39. $3t^2+t=10$

$3t^2+t-10=0$

$(3t-5)(t+2)=0$

$3t-5=0 \quad t+2=0$

$3t=5 \quad t=-2$

$t=\frac{5}{3}$

The solutions are $\frac{5}{3}$ and –2.

41. $5t^2-16t=-12$

$5t^2-16t+12=0$

$(5t-6)(t-2)=0$

$5t-6=0 \quad t-2=0$

$5t=6 \quad t=2$

$t=\frac{6}{5}$

The solutions are $\frac{6}{5}$ and 2.

43. $x(x-11)=12$

$x^2-11x=12$

$x^2-11x-12=0$

$(x+1)(x-12)=0$

$x+1=0 \quad x-12=0$

$x=-1 \quad x=12$

The solutions are –1 and 12.

45. $y(y+8)=-15$

$y^2+8y=-15$

$y^2+8y+15=0$

$(y+3)(y+5)=0$

$y+3=0 \quad y+5=0$

$y=-3 \quad y=-5$

The solutions are –3 and –5.

47. $p(p-1)=20$

$p^2-p=20$

$p^2-p-20=0$

$(p+4)(p-5)=0$

$p+4=0 \quad p-5=0$

$p=-4 \quad p=5$

The solutions are –4 and 5.

49. $y(y-8)=-15$

$y^2-8y=-15$

$y^2-8y+15=0$

$(y-3)(y-5)=0$

$y-3=0 \quad y-5=0$

$y=3 \quad y=5$

The solutions are 3 and 5.

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51. $p(p-14)=15$
$p^2-14p=15$
$p^2-14p-15=0$
$(p+1)(p-15)=0$
$p+1=0 \quad p-15=0$
$p=-1 \quad p=15$
The solutions are –1 and 15.

53. $(x+4)(x-1)=14$
$x^2+3x-4=14$
$x^2+3x-18=0$
$(x+6)(x-3)=0$
$x+6=0 \quad x-3=0$
$x=-6 \quad x=3$
The solutions are –6 and 3.

55. $(z-8)(z+4)=-35$
$z^2-4z-32=-35$
$z^2-4z+3=0$
$(z-1)(z-3)=0$
$z-1=0 \quad z-3=0$
$z=1 \quad z=3$
The solutions are 1 and 3.

57. $(a+3)(a+4)=72$
$a^2+7a+12=72$
$a^2+7a-60=0$
$(a+12)(a-5)=0$
$a+12=0 \quad a-5=0$
$a=-12 \quad a=5$
The solutions are –12 and 5.

59. $(2x+5)(x+1)=-1$
$2x^2+7x+5=-1$
$2x^2+7x+6=0$
$(2x+3)(x+2)=0$
$2x+3=0 \quad x+2=0$
$2x=-3 \quad x=-2$
$x=-\frac{3}{2}$
The solutions are $-\left(\frac{3}{2}\right)$ and –2.

Objective B Exercises

61. Strategy
The negative number: x
The square of a negative number is fifteen more than twice the negative number.

Solution
$x^2=2x+15$
$x^2-2x-15=0$
$(x-5)(x+3)=0$
$x-5=0 \quad x+3=0$
$x=5 \quad x=-3$
Because 5 is not a negative number, it is not a solution. The number is –3.

63. Strategy
The two numbers: x, y
The sum of the two numbers is eight. The sum of the squares of the numbers is thirty-four.

Solution
$x+y=8$
$y=-x+8$
$x^2+(-x+8)^2=34$
$2x^2-16x+64=34$
$2x^2-16x+30=0$
$(2x-6)(x-5)=0$
$2x-6=0 \quad x-5=0$
$x=3 \quad x=5$
The numbers are 3 and 5.

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67. Strategy

The length of the rectangle: x
The width of the rectangle: y
The length of a rectangle is 8 cm more than three times the width. The area of the rectangle is 380 cm^2.

Solution

$$x = 3y + 8$$
$$xy = 380$$
$$(3y+8)y = 380$$
$$3y^2 + 8y = 380$$
$$3y^2 + 8y - 372 = 0$$
$$(3y+38)(y-10) = 0$$
$$3y + 38 = 0 \qquad y - 10 = 0$$
$$y = -\frac{38}{3} \qquad y = 10$$

Because $-\frac{38}{3}$ is not positive, it can not be a solution. Therefore the width is 10 cm and the length is 38 cm.

65. Strategy

The two numbers: x, y
The sum of the squares of two consecutive positive even integers is one hundred sixty-four.

Solution

$$x^2 + y^2 = 164$$
$$x + 2 = y$$
$$x^2 + (x+2)^2 = 164$$
$$2x^2 + 4x + 4 = 164$$
$$2x^2 + 4x - 160 = 0$$
$$(2x+20)(x-8) = 0$$
$$2x + 20 = 0 \qquad x - 8 = 0$$
$$x = -10 \qquad x = 8$$

Because –10 is not positive, it can not be a solution. Therefore the numbers are 8 and 10.

69.

$$S = \frac{n^2 + n}{2}$$
$$171 = \frac{n^2 + n}{2}$$
$$342 = n^2 + n$$
$$n^2 + n - 342 = 0$$
$$(n+19)(n-18) = 0 \quad (n \geq 1)$$
$$n = 18$$

Eighteen consecutive numbers beginning with 1 give a sum of 171.

71.

$$N = \frac{t^2 - t}{2}$$
$$45 = \frac{t^2 - t}{2}$$
$$90 = t^2 - t$$
$$t^2 - t - 90 = 0$$
$$(t+9)(t-10) = 0 \quad (t \geq 1)$$
$$t = 10$$

There are 10 teams in the league.

73.

$$S = vt + 16t^2$$
$$320 = 16t + 16t^2$$
$$16t^2 + 16t - 320 = 0$$
$$16(t^2 + t - 20) = 0$$
$$(t+5)(t-4) = 0 \quad (t \geq 0)$$
$$t = 4$$

In 4 s the object will hit the ground.

75.

$$h = vt - 16t^2$$
$$64 = 64t - 16t^2$$
$$16t^2 - 64t + 64 = 0$$
$$16(t^2 - 4t + 4) = 0$$
$$(t-2)(t-2) = 0$$
$$t = 2$$

In 2 s the ball will be 64 ft above the ground.

77. Strategy

The height of the triangle: h
The length of the base of the triangle: b
The height of a triangle is 4 m more than twice the length of the base. The area of the triangle is 35 cm^2.

Solution

$$h = 2b + 4$$
$$0.5bh = 35$$
$$0.5b(2b+4) = 35$$
$$b^2 + 2b = 35$$
$$b^2 + 2b - 35 = 0$$
$$(b+7)(b-5) = 0$$
$$b + 7 = 0 \qquad b - 5 = 0$$
$$b = -7 \qquad b = 5$$

Because –7 is not positive, it can not be a solution. Therefore the base is 5 m and the height is 14 m.

79. Strategy

The width of the border: w
The garden measures 8 ft by 10 ft. A uniform border increases the area to 143 ft^2.

Solution

$$(2w+8)(2w+10) = 143$$
$$4w^2 + 36w + 80 = 143$$
$$4w^2 + 36w - 63 = 0$$
$$(2w+21)(2w-3) = 0$$
$$2w + 21 = 0 \qquad 2w - 3 = 0$$
$$w = -11.5 \qquad w = 1.5$$

Because –11.5 is not positive, it can not be a solution. Therefore the width is 1.5 ft.

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81. Strategy

- Radius of the original circle: r
 Radius of the larger circle: $3+r$
- To find the radius of the original circle, subtract the area of the small circle from the area of the larger circle and set equal to 100.

Solution

$$A_2 - A_1 = 100$$
$$\pi(3+r)^2 - \pi r^2 = 100$$
$$\pi(9+6r+r^2) - \pi r^2 = 100$$
$$9\pi + 6\pi r + \pi r^2 - \pi r^2 = 100$$
$$9\pi + 6\pi r = 100$$
$$6\pi r = 100 - 9\pi$$
$$r = \frac{100-9\pi}{6\pi}$$
$$r = 3.805 \approx 3.81$$

The radius of the original circle is 3.81 in.

Applying the Concepts

83. Strategy

Width of cardboard: W
Length of cardboard: $W+10$
Volume of box: 192 in.^{3}
Height of box: 2 in.
Length of box: $W+10-4 = W+6$
Width of box: $W-4$
The equation for the volume of a box is $V = L \cdot W \cdot H$. Substitute in the equation and solve for W.

Solution

$$V = L \cdot W \cdot H$$
$$192 = (W+6)(W-4)(2)$$
$$192 = (W^2 + 2W - 24)2$$
$$192 = 2W^2 + 4W - 48$$
$$0 = 2W^2 + 4W - 240$$
$$0 = 2(W^2 + 2W - 120)$$
$$0 = 2(W+12)(W-10)$$
$$W+12 = 0 \qquad W-10 = 0$$
$$W = -12 \qquad W = 10$$

Since the width cannot be a negative number, -12 is not a solution.

$$W = 10$$
$$W + 10 = 10 + 10 = 20$$

The length of the piece of cardboard is 20 in. The width is 10 in.

85.
$$(x+3)(2x-1) = (3-x)(5-3x)$$
$$2x^2 + 5x - 3 = 15 - 14x + 3x^2$$
$$-x^2 + 19x - 18 = 0$$
$$x^2 - 19x + 18 = 0$$
$$(x-1)(x-18) = 0$$
$$x-1 = 0 \quad x-18 = 0$$
$$x = 1 \qquad x = 18$$

The solution is 1 or 18.

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Chapter Review

1. $5x^3+10x^2+35x$
The GCF is $5x$: $5x\left(x^2+2x+7\right)$

2. $12a^2b+3ab^2$
The GCF is $3ab$: $3ab(4a+b)$

3. $14y^9-49y^6+7y^3$
The GCF is $7y^3$:
$7y^3\left(2y^6-7y^3+1\right)$

4. $4x(x-3)-5(3-x)=4x(x-3)+5(x-3)$
$=(x-3)(4x+5)$

5. $10x^2+25x+4xy+10y=5x(2x+5)+2y(2x+5)$
$=(2x+5)(5x+2y)$

6. $21ax-35bx-10by+6ay=7x(3a-5b)+2y(-5b+3a)$
$=7x(3a-5b)+2y(3a-5b)$
$=(3a-5b)(7x+2y)$

7. $b^2-13b+30$
Factors of 30 whose sum is –13: –10 and –3
$b^2-10b-3b+30=b(b-10)-3(b-10)=(b-10)(b-3)$

8. $c^2+8c+12$
Factors of 12 whose sum is 8: 6 and 2
$c^2+6c+2x+12=c(c+6)+2(c+6)=(c+6)(c+2)$

9. $y^2+5y-36$
Factors of –36 whose sum is 5: 9 and –4
$y^2+9y-4y-36=y(y+9)-4(y+9)=(y+9)(y-4)$

10. $3a^2-15a-42$
The GCF is 3: $3\left(a^2-5a-14\right)$
Factors of –14 whose sum is –5: –7 and 2
$3\left(a^2-7a+2a-14\right)=3[a(a-7)+2(a-7)]=3(a-7)(a+2)$

11. $4x^3-20x^2-24x$
The GCF is $4x$: $4x\left(x^2-5x-6\right)$
Factors of –6 whose sum is –5: –6 and 1
$4x\left(x^2-5x-6\right)=4x\left(x^2-6x+x-6\right)$
$=4x[x(x-6)+1(x-6)]=4x(x-6)(x+1)$

12. $n^4-2n^3-3n^2$
The GCF is n^2: $n^2\left(n^2-2n-3\right)$
Factors of –3 whose sum is –2: –3 and 1
$n^2\left(n^2-3n+n-3\right)=n^2[n(n-3)+1(n-3)]$
$=n^2(n-3)(n+1)$

13. $6x^2-29x+28$

Factors of 6	*Negative Factors of* 28
1 and 6	–1 and –28
2 and 3	–2 and –14
	–4 and –7

Trial Factors	*Middle Term*
$(x-1)(6x-28)$	$-28x-6x=-34x$
$(x-4)(6x-7)$	$-7x-24x=-31x$
$(2x-1)(3x-28)$	$-56x-6x=-62x$
$(2x-4)(3x-7)$	$-14x-12x=-26x$
$(2x-7)(3x-4)$	$-8x-21x=-29x$

$6x^2-29x+28=(2x-7)(3x-4)$

14. $12y^2+16y-3$

Factors of 12	*Factors of* –3
1 and 12	1 and –3
2 and 6	–1 and 3
3 and 4	

Trial Factors	*Middle Term*
$(3y+1)(4y-3)$	$-9y+4y=-5y$
$(3y-1)(4y+3)$	$9y-4y=5y$
$(2y+1)(6y-3)$	$-6y+6y=0$
$(6y-1)(2y+3)$	$18y-2y=16y$

$12y^2+16y-3=(6y-1)(2y+3)$

15. $2x^2-5x+6$

Factors of 2	*Negative Factors of* 6
1 and 2	–1 and –6
	–2 and –3

Trial Factors	*Middle Term*
$(x-1)(2x-6)$	$-6x-2x=-8x$
$(x-2)(2x-3)$	$-3x-4x=-7x$
$(2x-1)(x-6)$	$-12x-x=-13x$
$(2x-2)(x-3)$	$-6x-2x=-8x$

$2x^2-5x+6$ is nonfactorable over the integers.

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16. $3x^2 - 17x + 10$
$3 \cdot 10 = 30$
Factors of 30 whose sum is –17: –15 and –2
$3x^2 - 15x - 2x + 10 = 3x(x-5) - 2(x-5) = (x-5)(3x-2)$
$3x^2 - 17x + 10 = (x-5)(3x-2)$

17. $2a^2 - 19a - 60$
$2(-60) = -120$
Factors of –120 whose sum is –19: –24 and 5
$2a^2 - 24a + 5a - 60 = 2a(a-12) + 5(a-12) = (a-12)(2a+5)$
$2a^2 - 19a - 60 = (a-12)(2a+5)$

18. $18a^2 - 3a - 10$
$18(-10) = -180$
Factors of –180 whose sum is –3: –15 and 12
$18a^2 - 15a + 12a - 10 = 3a(6a-5) + 2(6a-5)$
$= (6a-5)(3a+2)$
$18a^2 - 3a - 10 = (6a-5)(3a+2)$

19. Let $u = xy$.
$x^2y^2 - 9 = u^2 - 9 = (u+3)(u-3) = (xy+3)(xy-3)$

20. $4x^2 + 12xy + 9y^2 = (2x+3y)^2$

21. Let $u = x^n$.
$x^{2n} - 12x^n + 36 = u^2 - 12u + 36 = (u-6)^2 = (x^n - 6)^2$

22. $64a^3 - 27b^3 = (4a)^3 - (3b)^3$
$= (4a - 3b)(16a^2 + 12ab + 9b^2)$

23. Let $u = x^2$.
$15x^4 + x^2 - 6 = 15u^2 + u - 6 = (3u+2)(5u-3)$
$= (3x^2+2)(5x^2-3)$

24. Let $u = x^2y^2$.
$21x^4y^4 + 23x^2y^2 + 6 = 21u^2 + 23u + 6 = (7u+3)(3u+2)$
$= (7x^2y^2+3)(3x^2y^2+2)$

25. $3a^6 - 15a^4 - 18a^2 = 3a^2(a^4 - 5a^2 - 6) = 3a^2(a^2-6)(a^2+1)$

26. $4x^2 + 27x = 7$
$4x^2 + 27x - 7 = 0$
$(4x-1)(x+7) = 0$
$4x - 1 = 0 \quad x + 7 = 0$
$4x = 1 \qquad x = -7$
$x = \frac{1}{4}$
The solutions are $\frac{1}{4}$ and –7.

27. $(x+1)(x-5) = 16$
$x^2 - 4x - 5 = 16$
$x^2 - 4x - 21 = 0$
$(x-7)(x+3) = 0$
$x - 7 = 0 \quad x + 3 = 0$
$x = 7 \qquad x = -3$
The solutions are 7 and –3.

28. Strategy
- Width: x
 Length: $2x - 20$
- Use the equation for the area of a rectangle: $A = LW$

Solution
$LW = A$
$x(2x - 20) = 6000$
$2x^2 - 20x - 6000 = 0$
$2(x^2 - 10x - 3000) = 0$
$2(x-60)(x+50) = 0$
$x - 60 = 0 \quad x + 50 = 0$
$x = 60 \qquad x = -50$
Because the width of a rectangle cannot be a negative number, –50 is not a solution. $2x - 20 = 120 - 20 = 100$
The width is 60 yd and the length is 100 yd.

29. Strategy
- Width of the picture frame: x
 New width: $12 + 2x$
 New length: $15 + 2x$
- Use the equation for the area of a rectangle.

Solution
$LW = A$
$(12 + 2x)(15 + 2x) = 270$
$180 + 24x + 30x + 4x^2 = 270$
$4x^2 + 54x - 90 = 0$
$2(2x^2 + 27x - 45) = 0$
$2(2x-3)(x+15) = 0$
$2x - 3 = 0 \quad x + 15 = 0$
$2x = 3 \qquad x = -15$
$x = 1.5$
Because the width of a picture frame cannot be a negative number, –15 is not a solution. The width of the frame is 1.5 in.

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30. Strategy
- Side of original square: x
 Side of new square: $x+4$
- Use the equation for the area of a square.

Solution

$$s^2 = A$$
$$(x+4)^2 = 576$$
$$x^2+8x+16 = 576$$
$$x^2+8x-560 = 0$$
$$(x+28)(x-20) = 0 \quad (x>0)$$
$$x = 20$$

The side of the original square is 20 ft.

Chapter Test

1. $ab+6a-3b-18 = a(b+6)-3(b+6) = (b+6)(a-3)$

2. $2y^4-14y^3-16y^2 = 2y^2(y^2-7y-8) = 2y^2(y+1)(y-8)$

3. $8x^2+20x-48$

$8(-48) = -384$

Factors of -384 whose sum is 20: 32 and -12

$$8x^2+20x-48 = 8x^2-12x+32x-48$$
$$= 4x(2x-3)+16(2x-3)$$
$$= (4x+16)(2x-3) = 4(x+4)(2x-3)$$

4. There is no common factor.

Factor the trinomial.

Positive

Factors of 6	*Positive Factors of* 8
1 and 6	1 and 8
2 and 3	2 and 4

Trial Factors	*Middle Term*
$(x+1)(6x+8)$	$8x+6x = 14x$
$(x+8)(6x+1)$	$x+48x = 49x$
$(2x+1)(3x+8)$	$16x+3x = 19x$
$(2x+8)(3x+1)$	$2x+24x = 26x$

$6x^2+19x+8 = (2x+1)(3x+8)$

5.

Factors	*Sum*
$-1, -48$	-49
$-2, -24$	-26
$-3, -16$	-19
$-4, -12$	-16
$-6, -8$	-14

$a^2-19a+48 = (a-3)(a-16)$

6. $6x^3-8x^2+10x = 2x(3x^2-4x+5)$

7.

Factors	*Sum*
$-1, +15$	$+14$
$+1, -15$	-14
$-3, +5$	$+2$
$+3, -5$	-2

$x^2+2x-15 = (x+5)(x-3)$

8. $4x^2-1 = 0$

$(2x-1)(2x+1) = 0$

$2x-1 = 0 \quad 2x+1 = 0$

$x = \frac{1}{2} \qquad x = -\frac{1}{2}$

The solutions are $\left(\frac{1}{2}, -\frac{1}{2}\right)$.

9. $5x^2-45x-15 = 5(x^2-9x-3)$

10. $p^2+12p+36 = (p+6)^2$

11. $x(x-8) = -15$

$x^2-8x = -15$

$x^2-8x+15 = 0$

$(x-3)(x-5) = 0$

$x-3 = 0 \quad x-5 = 0$

$x = 3 \qquad x = 5$

The solutions are 3 and 5.

12. $3x^2+12xy+12y^2 = 3(x^2+4xy+4y^2)$

$3(x+2y)(x+2y) = 3(x+2y)^2$

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13. $27x^3 - 8 = (3x)^3 - 2^3 = (3x - 2)(9x^2 + 6x + 4)$

14. $6x^2y^2 + 9xy^2 + 3y^2$

Factors of 2 whose sum is 3: 2, 1

Common Factor $3y^2$:

$3y^2(2x^2 + 3x + 1)$

$2 \cdot 2 = 2$

$$\begin{aligned} 3y^2(2x^2 + 3x + 1) &= 3y^2(2x^2 + 2x + x + 1) \\ &= 3y^2(2x(x + 1) + (x + 1)) \\ &= 3y^2(2x + 1)(x + 1) \end{aligned}$$

15. Let $u = a^2$

$$\begin{aligned} 6a^4 - 13a^2 - 5 &= 6u^2 - 13u - 5 \\ &= (2u - 5)(3u + 1) \\ &= (2a^2 - 5)(3a^2 + 1) \end{aligned}$$

16. $a(x - 2) + b(x - 2) = (x - 2)(a + b)$

17. $x(p + 1) - (p + 1) = (p + 1)(x - 1)$

18. $3a^2 - 75 = 3(a^2 - 25)$

$= 3(a + 5)(a - 5)$

19. There is no common factor.

Factor the trinomial.

Factors of 2	*Factors of −5*
1 and 2	−1 and 5
	1 and −5

Trial Factors	*Middle Term*
$(x - 1)(2x + 5)$	$5x - 2x = 3x$
$(x + 1)(2x - 5)$	$-5x + 2x = -3x$

$2x^2 + 4x - 5$ is nonfactorable over the integers.

20.

Factors	*Sum*
−1, +36	+35
+1, −36	−35
−2, +18	+16
+2, −18	−16
−3, +12	+9
+3, −12	−9
−4, +9	+5
+4, −9	−5
−6, +6	0

$x^2 - 9x - 36 = (x + 3)(x - 12)$

21. $4a^2 - 12ab + 9b^2 = (2a - 3b)^2$

22. $4x^2 - 49y^2 = (2x + 7y)(2x - 7y)$

23. $(2a - 3)(a + 7) = 0$

$2a - 3 = 0 \quad a + 7 = 0$

$a = \frac{3}{2} \qquad a = -7$

The solutions are $\frac{3}{2}$ and 7.

24. Strategy

The length of the rectangle: x

The width of the rectangle: y

The length of a rectangle is 3 cm more than twice its width. The area of the rectangle is 90 cm^2.

Solution

$$\begin{aligned} x + 2y &= 3 \\ xy &= 90 \\ (2y + 3)y &= 90 \\ 2y^2 + 3y &= 90 \\ 2y^2 + 3y - 90 &= 0 \\ (2y + 15)(y - 6) &= 0 \end{aligned}$$

$2y + 15 = 0 \qquad y - 6 = 0$

$y = -7.5 \qquad y = 6$

Because −7.5 is not positive, it can not be a solution. Therefore the width is 6 cm and the length is 15 cm.

25. Strategy

The two numbers: x, y

The sum of the two numbers is ten.

The sum of the squares of the two numbers is fifty-eight.

Solution

$$\begin{aligned} x + y &= 10 \\ y &= -x + 10 \\ x^2 + y^2 &= 58 \\ x^2 + (-x + 10)^2 &= 58 \\ 2x^2 - 20x + 100 &= 58 \\ 2x^2 - 20x + 42 &= 0 \\ (2x - 6)(x - 7) &= 0 \end{aligned}$$

$2x - 6 = 0 \quad x - 7 = 0$

$x = 3 \qquad x = 7$

The numbers are 3 and 7.

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Cumulative Review

1. $-2-(-3)-5-(-11)=-2+3-5+11=1-5+11$
$=-4+11=7$

2. $(3-7)^2\div(-2)-3(-4)=(-4)^2\div(-2)-3(-4)$
$=16\div(-2)-(-12)=-8+12=4$

3. $-2a^2\div(2b)-c=-2(-4)^2\div[2(2)]-(-1)=-2(16)\div4+1$
$=-32\div4+1=-8+1=-7$

4. $-\frac{3}{4}\left(-20x^2\right)=15x^2$

5. $-2[4x-2(3-2x)-8x]=-2[4x-6+4x-8x]=-2[-6]$
$=12$

6. $-\frac{5}{7}x=-\frac{10}{21}$

$-\frac{7}{5}\left(-\frac{5}{7}x\right)=-\frac{7}{5}\left(-\frac{10}{21}\right)$

$x=\frac{2}{3}$

The solution is $\frac{2}{3}$.

7. $3x-2=12-5x$
$3x+5x=12+2$
$8x=14$
$x=\frac{14}{8}=\frac{7}{4}$

The solution is $\frac{7}{4}$.

8. $-2+4[3x-2(4-x)-3]=4x+2$
$-2+4[3x-8+2x-3]=4x+2$
$-2+4[5x-11]=4x+2$
$-2+20x-44=4x+2$
$20x-46=4x+2$
$16x=48$
$x=3$

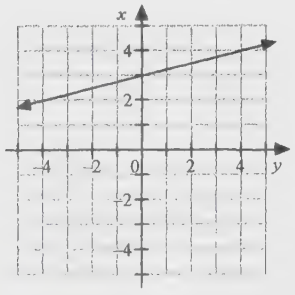
The solution is 3.

9. $P\cdot B=A$
$120\%\cdot B=54$
$1.2B=54$
$B=\frac{54}{1.2}$
$B=45$

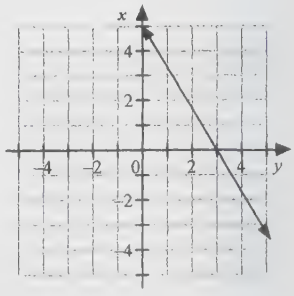
The number is 45.

10. $f(x)=-x^2+3x-1$
$f(2)=-(2)^2+3(2)-1$
$f(2)=-4+6-1$
$f(2)=1$

11.



12.



13. $m=\frac{2}{3}$, $(x_1, y_1)=(-3, 4)$
$y-y_1=m(x-x_1)$
$y-4=\frac{2}{3}[x-(-3)]$
$y-4=\frac{2}{3}(x+3)$
$y-4=\frac{2}{3}x+2$
$y=\frac{2}{3}x+6$

The equation of the line is $y=\frac{2}{3}x+6$.

14. (1) $8x-y=2$
(2) $y=5x+1$
Substitute in equation (1).
$8x-(5x+1)=2$
$8x-5x-1=2$
$3x=3$
$x=1$
Substitute in equation (2).
$y=5x+1$
$y=5+1$
$y=6$
The solution is (1, 6).

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15. (1) $5x + 2y = -9$

(2) $12x - 7y = 2$

Eliminate y and add the equations.

$$\begin{aligned} 7(5x + 2y) &= -9(7) & 35x + 14y &= -63 \\ 2(12x - 7y) &= 2(2) & 24x - 14y &= 4 \\ & & 59x &= -59 \\ & & x &= -1 \end{aligned}$$

Replace x in equation (1).

$$\begin{aligned} 5x + 2y &= -9 \\ -5 + 2y &= -9 \\ 2y &= -4 \\ y &= -2 \end{aligned}$$

The solution is $(-1, -2)$.

16. $\left(-3a^3b^2\right)^2 = (-3)^2a^6b^4 = 9a^6b^4$

17.

$$\begin{array}{r} x^2 - 5x + 4 \\ \hline x + 2 \\ \hline 2x^2 - 10x + 8 \\ x^3 - 5x^2 + 4x \\ \hline x^3 - 3x^2 - 6x + 8 \end{array}$$

18.

$$\begin{array}{r} 4x \quad +8 \\ 2x - 3\overline{)8x^2 + 4x - 3} \\ 8x^2 - 12x \\ \hline 16x - 3 \\ 16x - 24 \\ \hline 21 \end{array}$$

$$\left(8x^2 + 4x - 3\right) \div (2x - 3) = 4x + 8 + \frac{21}{2x - 3}$$

19. $\left(x^{-4}y^3\right)^2 = x^{-8}y^6 = \dfrac{y^6}{x^8}$

20. $3a - 3b - ax + bx = 3(a - b) - x(a - b) = (a - b)(3 - x)$

21. $15xy^2 - 20xy^4$

The GCF is $5xy^2$: $5xy^2\left(3 - 4y^2\right)$

22. $x^2 - 5xy - 14y^2$

Factors of -14 whose sum is -5: -7 and 2

$$\begin{aligned} x^2 - 7xy + 2xy - 14y^2 &= x(x - 7y) + 2y(x - 7y) \\ &= (x - 7y)(x + 2y) \end{aligned}$$

23. $3x^2 + 19x - 14 = 0$

$(3x - 2)(x + 7) = 0$

$3x - 2 = 0 \quad x + 7 = 0$

$3x = 2 \qquad x = -7$

$x = \dfrac{2}{3}$

The solutions are $\dfrac{2}{3}$ and -7.

24. $6x^2 + 60 = 39x$

$6x^2 - 39x + 60 = 0$

$3\left(2x^2 - 13x + 20\right) = 0$

$3(2x - 5)(x - 4) = 0$

$2x - 5 = 0 \quad x - 4 = 0$

$2x = 5 \qquad x = 4$

$x = \dfrac{5}{2}$

The solutions are $\dfrac{5}{2}$ and 4.

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25. Strategy To find the measure of the third angle, write and solve an equation using the fact that the sum of the angles of a triangle is 180° and that the measure of a right angle is 90°.

Solution
$$A + B + C = 180°$$
$$A + 31° + 90° = 180°$$
$$A + 121° = 180°$$
$$A = 59°$$
The measure of the third angle is 59°.

26. Strategy To find the width of the garden, solve the formula for the perimeter of a rectangle for W.

Solution
$$P = 2L + 2W$$
$$86 = 2(28) + 2W$$
$$86 = 56 + 2W$$
$$30 = 2W$$
$$15 = W$$
The width of the garden is 15 ft.

27. Strategy
- Shorter piece: x
 Longer piece: $10 - x$
- Four times the length of the shorter piece is 2 ft less than three times the length of the longer piece.

Solution
$$4x = 3(10 - x) - 2$$
$$4x = 30 - 3x - 2$$
$$4x = 28 - 3x$$
$$7x = 28$$
$$x = 4$$
$$10 - x = 10 - 4 = 6$$
The shorter piece is 4 ft long and the longer piece is 6 ft long.

28. Strategy
- Amount of additional money: x

	Principal	Rate	Interest
Amount at 8%	4000	0.08	0.08(4000)
Additional amount	x	0.11	$0.11x$

- The total annual interest is $1035.

Solution
$$0.08(4000) + 0.11x = 1035$$
$$320 + 0.11x = 1035$$
$$0.11x = 715$$
$$x = 6500$$
An additional $6500 must be invested.

29. Strategy
- Time driving to resort: x

	Rate	Time	Distance
To resort	42	x	$42x$
From resort	56	$7 - x$	$56(7 - x)$

- The distances are the same.

Solution
$$42x = 56(7 - x)$$
$$42x = 392 - 56x$$
$$98x = 392$$
$$x = 4$$
$$42x = 42(4) = 168$$
The distance to the resort is 168 mi.

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30. Strategy

- Height: x
 Base: $3x$
- Use the equation for the area of a triangle.

Solution

$$\frac{1}{2}bh = A$$

$$\frac{1}{2}(3x)(x) = 24$$

$$\frac{3}{2}x^2 = 24$$

$$x^2 = 24 \cdot \frac{2}{3}$$

$$x^2 = 16$$

$$x = 4$$

$3x = 3(4) = 12$

The base of the triangle is 12 in.

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Chapter 8
Rational Expressions

Prep Test

1. 50

2. $-\frac{3}{8}\cdot\frac{4}{9}=-\frac{1}{6}$

3. $-\frac{4}{5}\div\frac{8}{15}=-\frac{4}{5}\cdot\frac{15}{8}=-\frac{3}{2}$

4. $-\frac{5}{6}+\frac{7}{8}=-\frac{20}{24}+\frac{21}{24}=\frac{1}{24}$

5. $-\frac{3}{8}-\left(-\frac{7}{12}\right)=-\frac{9}{24}+\frac{14}{24}=\frac{5}{24}$

6. $\frac{2(2)-3}{(2)^2-2+1}=\frac{4-3}{4-1}=\frac{1}{3}$

7.
$$\begin{aligned}4(2x+1)&=3(x-2)\\8x+4&=3x-6\\5x&=-10\\x&=-2\end{aligned}$$

8.
$$\begin{aligned}10\left(\frac{t}{2}+\frac{t}{5}\right)&=10(1)\\5t+2t&=10\\7t&=10\\t&=\frac{10}{7}\end{aligned}$$

9. Strategy

Rate of the second plane: r

Rate of the first plane: $r-20$

	Rate	Time	Distance
2nd plane	r	2.	$2.r$
1st plane	$r-20$	2.	$2.(r-20)$

The total distance traveled by the two planes is 480 mi.

$2r+2(r-20)=480$

Solution

$2r+2(r-20)=480$

$2r+2r-40=480$

$4r=520$

$r=130$

$r-20=130-20=110$

The rate of the first plane is 110 mph.

The rate of the second plane is 130 mph.

Go Figure

If 6 machines can fill 12 boxes in 7 minutes, then dividing by 6, each machine can fill 2 boxes in 7 minutes, or each machine can fill $\frac{2}{7}$ of a box per minute. To find the number of boxes that can be filled by 14 machines, multiply the output of each machine, $\frac{2}{7}$, by 14, or $\frac{2}{7}\cdot 14=4$. That means that fourteen machines can fill 4 boxes per minute. Multiply by 12 minutes, $4\cdot 12=48$. Fourteen machines can fill 48 boxes of cereal in 12 minutes.

Section 8.1

Objective A Exercises

3. $\frac{9x^3}{12x^4}=\frac{\overset{1}{\cancel{3}}\cdot 3x^3}{2\cdot 2\cdot\underset{1}{\cancel{3}}x^4}=\frac{3}{4x}$

5. $\frac{(x+3)^2}{(x+3)^3}=(x+3)^{2-3}=(x+3)^{-1}=\frac{1}{x+3}$

7. $\frac{3n-4}{4-3n}=\frac{\overset{1}{-(4-3n)}}{\underset{1}{4-3n}}=-1$

9. $\frac{6y(y+2)}{9y^2(y+2)}=\frac{2\cdot\overset{1}{\cancel{3}}y\overset{1}{\cancel{(y+2)}}}{\underset{1}{\cancel{3}}\cdot 3y^2\underset{1}{\cancel{(y+2)}}}=\frac{2}{3y}$

11. $\frac{6x(x-5)}{8x^2(5-x)}=\frac{\overset{1}{2}\cdot 3x\overset{-1}{(x-5)}}{\underset{1}{2}\cdot 2\cdot 2x^2\underset{-1}{(5-x)}}=-\frac{3}{4x}$

13. $\frac{a^2+4a}{ab+4b}=\frac{a\overset{1}{(a+4)}}{b\underset{1}{(a+4)}}=\frac{a}{b}$

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15. $\dfrac{4-6x}{3x^2-2x}=\dfrac{2(2-3x)}{x(3x-2)}=-\dfrac{2}{x}$

17. $\dfrac{y^2-3y+2}{y^2-4y+3}=\dfrac{(y-1)(y-2)}{(y-1)(y-3)}=\dfrac{y-2}{y-3}$

19. $\dfrac{x^2+3x-10}{x^2+2x-8}=\dfrac{(x+5)(x-2)}{(x+4)(x-2)}=\dfrac{x+5}{x+4}$

21. $\dfrac{x^2+x-12}{x^2-6x+9}=\dfrac{(x+4)(x-3)}{(x-3)(x-3)}=\dfrac{x+4}{x-3}$

23. $\dfrac{x^2-3x-10}{25-x^2}=\dfrac{(x+2)(x-5)}{(5+x)(5-x)}=-\dfrac{x+2}{x+5}$

25. $\dfrac{2x^3+2x^2-4x}{x^3+2x^2-3x}=\dfrac{2x(x^2+x-2)}{x(x^2+2x-3)}=\dfrac{2x(x+2)(x-1)}{x(x+3)(x-1)}=\dfrac{2(x+2)}{x+3}$

27. $\dfrac{6x^2-7x+2}{6x^2+5x-6}=\dfrac{(2x-1)(3x-2)}{(2x+3)(3x-2)}=\dfrac{2x-1}{2x+3}$

29. $\dfrac{x^2+3x-28}{24-2x-x^2}=\dfrac{(x+7)(x-4)}{(6+x)(4-x)}=-\dfrac{x+7}{x+6}$

Objective B Exercises

31. $\dfrac{14a^2b^3}{15x^5y^2}\cdot\dfrac{25x^3y}{16ab}=\dfrac{2\cdot7\cdot a^2b^3\cdot5x^3y}{3\cdot8x^5y^2\cdot2\cdot2\cdot2ab}=\dfrac{35ab^2}{24x^2y}$

33. $\dfrac{18a^4b^2}{25x^2y^3}\cdot\dfrac{50x^5y^6}{27a^6b^2}=\dfrac{2\cdot3\cdot3a^4b^2\cdot2\cdot5x^5y^6}{5\cdot5x^2y^3\cdot3\cdot3\cdot3a^6b^2}=\dfrac{4x^3y^3}{3a^2}$

35. $\dfrac{8x-12}{14x+7}\cdot\dfrac{42x+21}{32x-48}=\dfrac{4(2x-3)}{7(2x+1)}\cdot\dfrac{21(2x+1)}{16(2x-3)}=\dfrac{2\cdot2(2x-3)\cdot3\cdot7(2x+1)}{7(2x+1)\cdot2\cdot2\cdot2\cdot2(2x-3)}=\dfrac{3}{4}$

37. $\dfrac{4a^2x-3a^2}{2by+5b}\cdot\dfrac{2b^3y+5b^3}{4ax-3a}=\dfrac{a^2(4x-3)}{b(2y+5)}\cdot\dfrac{b^3(2y+5)}{a(4x-3)}=\dfrac{a^2(4x-3)b^3(2y+5)}{b(2y+5)a(4x-3)}=ab^2$

39. $\dfrac{x^2+x-2}{xy^2}\cdot\dfrac{x^3y}{x^2+5x+6}=\dfrac{(x+2)(x-1)}{xy^2}\cdot\dfrac{x^3y}{(x+2)(x+3)}=\dfrac{(x+2)(x-1)x^3y}{xy^2(x+2)(x+3)}=\dfrac{x^2(x-1)}{y(x+3)}$

41. $\dfrac{x^5y^3}{x^2+13x+30}\cdot\dfrac{x^2+2x-3}{x^7y^2}=\dfrac{x^5y^3}{(x+3)(x+10)}\cdot\dfrac{(x+3)(x-1)}{x^7y^2}=\dfrac{x^5y^3(x+3)(x-1)}{(x+3)(x+10)x^7y^2}=\dfrac{y(x-1)}{x^2(x+10)}$

43. $\dfrac{3a^3+4a^2}{5ab-3b}\cdot\dfrac{3b^3-5ab^3}{3a^2+4a}=\dfrac{a^2(3a+4)}{b(5a-3)}\cdot\dfrac{b^3(3-5a)}{a(3a+4)}=\dfrac{a^2(3a+4)b^3(3-5a)}{b(5a-3)a(3a+4)}=-ab^2$

45. $\dfrac{x^2-8x+7}{x^2+3x-4}\cdot\dfrac{x^2+3x-10}{x^2-9x+14}=\dfrac{(x-1)(x-7)}{(x+4)(x-1)}\cdot\dfrac{(x+5)(x-2)}{(x-2)(x-7)}=\dfrac{(x-1)(x-7)(x+5)(x-2)}{(x+4)(x-1)(x-2)(x-7)}=\dfrac{x+5}{x+4}$

47. $\dfrac{y^2+y-20}{y^2+2y-15}\cdot\dfrac{y^2+4y-21}{y^2+3y-28}=\dfrac{(y+5)(y-4)}{(y+5)(y-3)}\cdot\dfrac{(y+7)(y-3)}{(y+7)(y-4)}=\dfrac{(y+5)(y-4)(y+7)(y-3)}{(y+5)(y-3)(y+7)(y-4)}=1$

49. $\dfrac{25-n^2}{n^2-2n-35}\cdot\dfrac{n^2-8n-20}{n^2-3n-10}=\dfrac{(5+n)(5-n)}{(n+5)(n-7)}\cdot\dfrac{(n+2)(n-10)}{(n+2)(n-5)}=\dfrac{(5+n)(5-n)(n+2)(n-10)}{(n+5)(n-7)(n+2)(n-5)}=-\dfrac{n-10}{n-7}$

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51. $\dfrac{8x^3+4x^2}{x^2-3x+2}\cdot\dfrac{x^2-4}{16x^2+8x}=\dfrac{4x^2(2x+1)}{(x-1)(x-2)}\cdot\dfrac{(x+2)(x-2)}{8x(2x+1)}$

$=\dfrac{2\cdot 2\,x^2\,(2x+1)(x+2)\,(x-2)}{(x-1)(x-2)\cdot 2\cdot 2\cdot 2x\,(2x+1)}$

$=\dfrac{x(x+2)}{2(x-1)}$

53. $\dfrac{x^2-11x+28}{x^2-13x+42}\cdot\dfrac{x^2+7x+10}{20-x-x^2}=\dfrac{(x-4)(x-7)}{(x-6)(x-7)}\cdot\dfrac{(x+2)(x+5)}{(5+x)(4-x)}=\dfrac{(x-4)(x-7)(x+2)(x+5)}{(x-6)(x-7)(5+x)(4-x)}=-\dfrac{x+2}{x-6}$

55. $\dfrac{x^2-4x-32}{x^2-8x-48}\cdot\dfrac{3x^2+17x+10}{3x^2-22x-16}=\dfrac{(x+4)(x-8)}{(x+4)(x-12)}\cdot\dfrac{(3x+2)(x+5)}{(3x+2)(x-8)}=\dfrac{(x+4)(x-8)(3x+2)(x+5)}{(x+4)(x-12)(3x+2)(x-8)}=\dfrac{x+5}{x-12}$

Objective C Exercises

57. $\dfrac{9x^3y^4}{16a^4b^2}\div\dfrac{45x^4y^2}{14a^7b}=\dfrac{9x^3y^4}{16a^4b^2}\cdot\dfrac{14a^7b}{45x^4y^2}$

$=\dfrac{3\cdot 3x^3y^4\cdot 2\cdot 7a^7b}{2\cdot 2\cdot 2\cdot 2a^4b^2\cdot 3\cdot 3\cdot 5x^4y^2}=\dfrac{7a^3y^2}{40bx}$

59. $\dfrac{28x+14}{45x-30}\div\dfrac{14x+7}{30x-20}=\dfrac{28x+14}{45x-30}\cdot\dfrac{30x-20}{14x+7}$

$=\dfrac{14(2x+1)}{15(3x-2)}\cdot\dfrac{10(3x-2)}{7(2x+1)}$

$=\dfrac{2\cdot 7(2x+1)\cdot 2\cdot 5(3x-2)}{3\cdot 5(3x-2)\cdot 7(2x+1)}=\dfrac{4}{3}$

61. $\dfrac{5a^2y+3a^2}{2x^3+5x^2}\div\dfrac{10ay+6a}{6x^3+15x^2}=\dfrac{5a^2y+3a^2}{2x^3+5x^2}\cdot\dfrac{6x^3+15x^2}{10ay+6a}$

$=\dfrac{a^2(5y+3)}{x^2(2x+5)}\cdot\dfrac{3x^2(2x+5)}{2a(5y+3)}=\dfrac{3a}{2}$

63. $\dfrac{x^3y^2}{x^2-3x-10}\div\dfrac{xy^4}{x^2-x-20}=\dfrac{x^3y^2}{x^2-3x-10}\cdot\dfrac{x^2-x-20}{xy^4}$

$=\dfrac{x^3y^2}{(x+2)(x-5)}\cdot\dfrac{(x+4)(x-5)}{xy^4}$

$=\dfrac{x^2(x+4)}{y^2(x+2)}$

65. $\dfrac{x^2y^5}{x^2-11x+30}\div\dfrac{xy^6}{x^2-7x+10}=\dfrac{x^2y^5}{x^2-11x+30}\cdot\dfrac{x^2-7x+10}{xy^6}$

$=\dfrac{x^2y^5}{(x-5)(x-6)}\cdot\dfrac{(x-2)(x-5)}{xy^6}$

$=\dfrac{x(x-2)}{y(x-6)}$

67. $\dfrac{3x^2y-9xy}{a^2b}\div\dfrac{3x^2-x^3}{ab^2}=\dfrac{3x^2y-9xy}{a^2b}\cdot\dfrac{ab^2}{3x^2-x^3}$

$=\dfrac{3xy(x-3)}{a^2b}\cdot\dfrac{ab^2}{x^2(3-x)}=-\dfrac{3by}{ax}$

69. $\dfrac{x^2+3x-40}{x^2+2x-35}\div\dfrac{x^2+3x-18}{x^2+2x-48}=\dfrac{x^2+3x-40}{x^2+2x-35}\cdot\dfrac{x^2+3x-18}{x^2+2x-48}=\dfrac{(x+8)(x-5)}{(x+7)(x-5)}\cdot\dfrac{(x+6)(x-3)}{(x+8)(x-6)}=\dfrac{(x+6)(x-3)}{(x+7)(x-6)}$

71. $\dfrac{y^2-y-56}{y^2+8y+7}\div\dfrac{y^2-13y+40}{y^2-4y-5}=\dfrac{y^2-y-56}{y^2+8y+7}\cdot\dfrac{y^2-4y-5}{y^2-13y+40}=\dfrac{(y+7)(y-8)}{(y+1)(y+7)}\cdot\dfrac{(y+1)(y-5)}{(y-5)(y-8)}=1$

73. $\dfrac{x^2-x-2}{x^2-7x+10}\div\dfrac{x^2-3x-4}{40-3x-x^2}=\dfrac{x^2-x-2}{x^2-7x+10}\cdot\dfrac{40-3x-x^2}{x^2-3x-4}=\dfrac{(x+1)(x-2)}{(x-2)(x-5)}\cdot\dfrac{(8+x)(5-x)}{(x+1)(x-4)}=-\dfrac{x+8}{x-4}$

75. $\dfrac{6n^2+13n+6}{4n^2-9}\div\dfrac{6n^2+n-2}{4n^2-1}=\dfrac{6n^2+13n+6}{4n^2-9}\cdot\dfrac{4n^2-1}{6n^2+n-2}=\dfrac{(2n+3)(3n+2)}{(2n+3)(2n-3)}\cdot\dfrac{(2n+1)(2n-1)}{(3n+2)(2n-1)}=\dfrac{2n+1}{2n-3}$

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Applying the Concepts

79.
$$\frac{n}{n+3}\div\frac{?}{}=\frac{n}{n-2}$$
$$\frac{n}{n+3}=\frac{n}{n-2}\cdot ?$$
$$?=\frac{n}{n+3}\div\frac{n}{n-2}$$
$$=\frac{\cancel{n}}{n+3}\cdot\frac{n-2}{\cancel{n}}$$
$$=\frac{n-2}{n+3}$$

81.
$$\left(\frac{y}{3}\right)\div\left(\frac{y}{2}\cdot\frac{y}{4}\right)=\frac{y}{3}\div\frac{y^2}{8}=\frac{y}{3}\cdot\frac{8}{y^2}=\frac{\cancel{y}^{1}}{3}\cdot\frac{2\cdot2\cdot2}{\cancel{y^2}_{\,y}}=\frac{8}{3y}$$

83.
$$\frac{x-2}{x+5}\div\frac{x-3}{x+5}\cdot\frac{x-3}{x-2}=\frac{\cancel{x-2}^{1}}{\cancel{x+5}_{1}}\cdot\frac{\cancel{x+5}^{1}}{\cancel{x-3}_{1}}\cdot\frac{\cancel{x-3}^{1}}{\cancel{x-2}_{1}}=1$$

Section 8.2

Objective A Exercises

1.
$8x^3y=2\cdot2\cdot2\cdot x\cdot x\cdot x\cdot y$
$12xy^2=2\cdot2\cdot3\cdot x\cdot y\cdot y$
$\text{LCM}=2\cdot2\cdot2\cdot3\cdot x\cdot x\cdot x\cdot y\cdot y=24x^3y^2$

3.
$10x^4y^2=2\cdot5\cdot x\cdot x\cdot x\cdot x\cdot y\cdot y$
$15x^3y=3\cdot5\cdot x\cdot x\cdot x\cdot y$
$\text{LCM}=2\cdot3\cdot5\cdot x\cdot x\cdot x\cdot x\cdot y\cdot y=30x^4y^2$

5.
$8x^2=2\cdot2\cdot2\cdot x\cdot x$
$4x^2+8x=4x(x+2)=2\cdot2\cdot x(x+2)$
$\text{LCM}=2\cdot2\cdot2\cdot x\cdot x(x+2)=8x^2(x+2)$

7.
$2x^2y=2\cdot x\cdot x\cdot y$
$3x^2+12x=3x(x+4)$
$\text{LCM}=2\cdot3\cdot x\cdot x\cdot y(x+4)=6x^2y(x+4)$

9.
$9x(x+2)=3\cdot3\cdot x(x+2)$
$12(x+2)^2=2\cdot2\cdot3(x+2)(x+2)$
$\text{LCM}=2\cdot2\cdot3\cdot3\cdot x(x+2)(x+2)=36x(x+2)^2$

11.
$3x+3=3(x+1)$
$2x^2+4x+2=2\left(x^2+2x+1\right)=2(x+1)(x+1)$
$\text{LCM}=2\cdot3(x+1)(x+1)=6(x+1)^2$

13.
$(x-1)(x+2)$
$(x-1)(x+3)$
$\text{LCM}=(x-1)(x+2)(x+3)$

15.
$(2x+3)^2=(2x+3)(2x+3)$
$(2x+3)(x-5)$
$\text{LCM}=(2x+3)(2x+3)(x-5)$
$=(2x+3)^2(x-5)$

17.
$(x-1)$
$(x-2)$
$(x-1)(x-2)$
$\text{LCM}=(x-1)(x-2)$

19.
$x^2-x-6=(x+2)(x-3)$
$x^2+x-12=(x-3)(x+4)$
$\text{LCM}=(x+2)(x-3)(x+4)$

21.
$x^2+5x+4=(x+1)(x+4)$
$x^2-3x-28=(x+4)(x-7)$
$\text{LCM}=(x+1)(x+4)(x-7)$

23.
$x^2-2x-24=(x+4)(x-6)$
$x^2-36x=(x+6)(x-6)$
$\text{LCM}=(x+4)(x-6)(x+6)$

25.
$x^2-7x-30=(x+3)(x-10)$
$x^2-5x-24=(x+3)(x-8)$
$\text{LCM}=(x+3)(x-10)(x-8)$

27.
$3x^2-11x+6=(x-3)(3x-2)$
$3x^2+4x-4=(x+2)(3x-2)$
$\text{LCM}=(x-3)(3x-2)(x+2)$

29.
$6+x-x^2=-\left(x^2-x-6\right)$
$=-(x+2)(x-3)$
$x+2$
$x-3$
$\text{LCM}=(x+2)(x-3)$

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31. $5+4x-x^2=(1+x)(5-x)$

$x-5-(x^2-4x-5)=-(x+1)(x-5)$

$x+1$

LCM $=(x+1)(x-5)$

33. $x^2-5x+6=(x-2)(x-3)$

$1-x=-(x-1)$

$x-6$

LCM $=(x-1)(x-2)(x-3)(x-6)$

Objective B Exercises

35. The LCM is ab^2.

$\frac{5}{ab^2}=\frac{5}{ab^2}$

$\frac{6}{ab}=\frac{6}{ab}\cdot\frac{b}{b}=\frac{6b}{ab^2}$

37. The LCM is $18x^2y$.

$\frac{5y}{6x^2}=\frac{5y}{6x^2}\cdot\frac{3y}{3y}=\frac{15y^2}{18x^2y}$

$\frac{7}{9xy}=\frac{7}{9xy}\cdot\frac{2x}{2x}=\frac{14x}{18x^2y}$

39. The LCM is $y^2(y+5)$.

$\frac{a}{y^2}=\frac{a}{y^2}\cdot\frac{y+5}{y+5}=\frac{ay+5a}{y^2(y+5)}$

$\frac{6}{y(y+5)}=\frac{6}{y(y+5)}\cdot\frac{y}{y}=\frac{6y}{y^2(y+5)}$

41. The LCM is $y(y+7)^2$.

$\frac{a^2}{y(y+7)}=\frac{a^2}{y(y+7)}\cdot\frac{y+7}{y+7}$

$=\frac{a^2y+7a^2}{y(y+7)^2}$

$\frac{a}{(y+7)^2}=\frac{a}{(y+7)^2}\cdot\frac{y}{y}=\frac{ay}{y(y+7)^2}$

43. $\frac{b^2}{4-y}=\frac{b^2}{-(y-4)}=-\frac{b^2}{y-4}$

The LCM is $y(y-4)$.

$\frac{b}{y(y-4)}=\frac{b}{y(y-4)}$

$\frac{b^2}{4-y}=-\frac{b^2}{y-4}\cdot\frac{y}{y}=-\frac{b^2y}{y(y-4)}$

45. $\frac{3}{7-y}=\frac{3}{-(y-7)}=-\frac{3}{y-7}$

The LCM is $(y-7)^2$.

$\frac{3}{7-y}=-\frac{3}{y-7}\cdot\frac{y-7}{y-7}=-\frac{3y-21}{(y-7)^2}$

$\frac{2}{(y-7)^2}=\frac{2}{(y-7)^2}$

47. The LCM is $y^2(y-3)$.

$\frac{2}{y-3}=\frac{2}{y-3}\cdot\frac{y^2}{y^2}=\frac{2y^2}{y^2(y-3)}$

$\frac{3}{y^3-3y^2}=\frac{3}{y^2(y-3)}$

49. The LCM is $(2x-1)(x+4)$.

$\frac{x^2}{2x-1}=\frac{x^2}{2x-1}\cdot\frac{x+4}{x+4}=\frac{x^3+4x^2}{(2x-1)(x+4)}$

$\frac{x+1}{x+4}=\frac{x+1}{x+4}\cdot\frac{2x-1}{2x-1}=\frac{2x^2+x-1}{(2x-1)(x+4)}$

51. The LCM is $(x+5)(x-5)$.

$\frac{3x}{x-5}=\frac{3x}{x-5}\cdot\frac{x+5}{x+5}=\frac{3x^2+15x}{(x+5)(x-5)}$

$\frac{4}{x^2-25}=\frac{4}{(x+5)(x-5)}$

53. The LCM is $(3x-2)(x+2)$.

$\frac{x-3}{3x^2+4x-4}=\frac{x-3}{(3x-2)(x+2)}$

$\frac{2}{x+2}=\frac{2}{x+2}\cdot\frac{3x-2}{3x-2}=\frac{6x-4}{(3x-2)(x+2)}$

55. The LCM is $(x-3)(x+5)(x+1)$.

$\frac{x-1}{x^2+2x-15}=\frac{x-1}{(x-3)(x+5)}\cdot\frac{x+1}{x+1}=\frac{x^2-1}{(x-3)(x+5)(x+1)}$

$\frac{x}{x^2+6x+5}=\frac{x}{(x+5)(x+1)}\cdot\frac{x-3}{x-3}=\frac{x^2-3x}{(x-3)(x+5)(x+1)}$

Objective C Exercises

57. $\frac{3}{y^2}+\frac{8}{y^2}=\frac{3+8}{y^2}=\frac{11}{y^2}$

59. $\frac{3}{x+4}-\frac{10}{x+4}=\frac{3-10}{x+4}=-\frac{7}{x+4}$

61. $\frac{3x}{2x+3}+\frac{5x}{2x+3}=\frac{3x+5x}{2x+3}=\frac{8x}{2x+3}$

63. $\frac{2x+1}{x-3}+\frac{3x+6}{x-3}=\frac{2x+1+(3x+6)}{x-3}=\frac{5x+7}{x-3}$

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65. $\dfrac{5x-1}{x+9}+\dfrac{3x+4}{x+9}=\dfrac{5x-1-(3x+4)}{x+9}=\dfrac{4x-1-3x-4}{x+9}$
$=\dfrac{2x-5}{x+9}$

67. $\dfrac{x-7}{2x+7}-\dfrac{4x-3}{2x+7}=\dfrac{x-7-(4x-3)}{2x+7}=\dfrac{x-7-4x+3}{2x+7}$
$=\dfrac{-3x-4}{2x+7}$

69. $\dfrac{x}{x^2+2x-15}-\dfrac{3}{x^2+2x-15}=\dfrac{x-3}{x^2+2x-15}$
$=\dfrac{\cancel{(x-3)}^{1}}{(x+5)\cancel{(x-3)}_{1}}=\dfrac{1}{x+5}$

71. $\dfrac{2x+3}{x^2-x-30}-\dfrac{x-2}{x^2-x-30}=\dfrac{2x+3-(x-2)}{x^2-x-30}$
$=\dfrac{2x+3-x+2}{(x+5)(x-6)}$
$=\dfrac{\cancel{(x+5)}^{1}}{\cancel{(x+5)}_{1}(x-6)}=\dfrac{1}{x-6}$

73. $\dfrac{4y+7}{2y^2+7y-4}-\dfrac{y-5}{2y^2+7y-4}=\dfrac{4y+7-(y-5)}{2y^2+7y-4}$
$=\dfrac{4y+7-y+5}{(y+4)(2y-1)}$
$=\dfrac{3y+12}{(y+4)(2y-1)}$
$=\dfrac{3\cancel{(y+4)}^{1}}{\cancel{(y+4)}_{1}(2y-1)}=\dfrac{3}{2y-1}$

75. $\dfrac{2x^2+3x}{x^2-9x+20}+\dfrac{2x^2-3}{x^2-9x+20}-\dfrac{4x^2+2x+1}{x^2-9x+20}=\dfrac{2x^2+3x+(2x^2-3)-(4x^2+2x+1)}{x^2-9x+20}=\dfrac{2x^2+3x+2x^2-3-4x^2-2x-1}{x^2-9x+20}$
$=\dfrac{\cancel{(x-4)}^{1}}{\cancel{(x-4)}_{1}(x-5)}=\dfrac{1}{x-5}$

Objective D Exercises

79. The LCM of the denominators is ab.
$\dfrac{7}{a}=\dfrac{7}{a}\cdot\dfrac{b}{b}=\dfrac{7b}{ab}$
$\dfrac{5}{b}=\dfrac{5}{b}\cdot\dfrac{a}{a}=\dfrac{5a}{ab}$
$\dfrac{7}{a}+\dfrac{5}{b}=\dfrac{7b}{ab}+\dfrac{5a}{ab}=\dfrac{7b+5a}{ab}$

81. The LCM of the denominators is $12a$.
$\dfrac{5}{3a}=\dfrac{5}{3a}\cdot\dfrac{4}{4}=\dfrac{20}{12a}$
$\dfrac{3}{4a}=\dfrac{3}{4a}\cdot\dfrac{3}{3}=\dfrac{9}{12a}$
$\dfrac{5}{3a}-\dfrac{3}{4a}=\dfrac{20}{12a}-\dfrac{9}{12a}=\dfrac{20-9}{12a}=\dfrac{11}{12a}$

83. The LCM of the denominators is $12y$.
$\dfrac{7}{4y}=\dfrac{7}{4y}\cdot\dfrac{3}{3}=\dfrac{21}{12y}$
$\dfrac{11}{6y}=\dfrac{11}{6y}\cdot\dfrac{2}{2}=\dfrac{22}{12y}$
$\dfrac{8}{3y}=\dfrac{8}{3y}\cdot\dfrac{4}{4}=\dfrac{32}{12y}$
$\dfrac{7}{4y}+\dfrac{11}{6y}-\dfrac{8}{3y}=\dfrac{21}{12y}+\dfrac{22}{12y}-\dfrac{32}{12y}$
$=\dfrac{21+22-32}{12y}=\dfrac{11}{12y}$

85. The LCM of the denominators is $20y^2$.
$\dfrac{6}{y^2}=\dfrac{6}{y^2}\cdot\dfrac{20}{20}=\dfrac{120}{20y^2}$
$\dfrac{3}{4y}=\dfrac{3}{4y}\cdot\dfrac{5y}{5y}=\dfrac{15y}{20y^2}$
$\dfrac{2}{5y}=\dfrac{2}{5y}\cdot\dfrac{4y}{4y}=\dfrac{8y}{20y^2}$
$\dfrac{6}{y^2}+\dfrac{3}{4y}-\dfrac{2}{5y}=\dfrac{120}{20y^2}+\dfrac{15y}{20y^2}-\dfrac{8y}{20y^2}$
$=\dfrac{120+15y-8y}{20y^2}$
$=\dfrac{120+7y}{20y^2}$

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87. The LCM of the denominators is $12ab$.

$$\frac{5}{2a}=\frac{5}{2a}\cdot\frac{6b}{6b}=\frac{30b}{12ab}$$
$$\frac{7}{3b}=\frac{7}{3b}\cdot\frac{4a}{4a}=\frac{28a}{12ab}$$
$$\frac{2}{b}=\frac{2}{b}\cdot\frac{12a}{12a}=\frac{24a}{12ab}$$
$$\frac{3}{4a}=\frac{3}{4a}\cdot\frac{3b}{3b}=\frac{9b}{12ab}$$
$$\frac{5}{2a}+\frac{7}{3b}-\frac{2}{b}-\frac{3}{4a}=\frac{30b}{12ab}+\frac{28a}{12ab}-\frac{24a}{12ab}-\frac{9b}{12ab}$$
$$=\frac{30b+28a-24a-9b}{12ab}=\frac{21b+4a}{12ab}$$

89. The LCM of the denominators is $12x$.

$$\frac{4x-3}{6x}=\frac{4x-3}{6x}\cdot\frac{2}{2}=\frac{8x-6}{12x}$$
$$\frac{2x+3}{4x}=\frac{2x+3}{4x}\cdot\frac{3}{3}=\frac{6x+9}{12x}$$
$$\frac{4x-3}{6x}+\frac{2x+3}{4x}=\frac{8x-6}{12x}+\frac{6x+9}{12x}=\frac{8x-6+(6x+9)}{12x}$$
$$=\frac{14x+3}{12x}$$

91. The LCM of the denominators is $6x$.

$$\frac{2x-3}{2x}=\frac{2x-3}{2x}\cdot\frac{3}{3}=\frac{6x-9}{6x}$$
$$\frac{x+3}{3x}=\frac{x+3}{3x}\cdot\frac{2}{2}=\frac{2x+6}{6x}$$
$$\frac{2x-3}{2x}+\frac{x+3}{3x}=\frac{6x-9}{6x}+\frac{2x+6}{6x}=\frac{6x-9+(2x+6)}{6x}$$
$$=\frac{8x-3}{6x}$$

93. The LCM of the denominators is $36y$.

$$\frac{3y-12}{12y}=\frac{3y-12}{12y}\cdot\frac{3}{3}=\frac{9y-6}{36y}$$
$$\frac{y-3}{18y}=\frac{y-3}{18y}\cdot\frac{2}{2}=\frac{2y-6}{36y}$$
$$\frac{3y-2}{12y}-\frac{y-3}{18y}=\frac{9y-6}{36y}-\frac{2y-6}{36y}=\frac{9y-6-(2y-6)}{36y}$$
$$=\frac{9y-6-2y+6}{36y}=\frac{7y}{36y}=\frac{7}{36}$$

95. The LCM of the denominators is $3x^2$.

$$\frac{x-2}{3x^2}=\frac{x-2}{3x^2}$$
$$\frac{x+4}{x}=\frac{x+4}{x}\cdot\frac{3x}{3x}=\frac{3x^2+12x}{3x^2}$$
$$\frac{x-2}{3x^2}-\frac{x+4}{x}=\frac{x-2}{3x^2}-\frac{3x^2+12x}{3x^2}=\frac{x-2-(3x^2+12x)}{3x^2}$$
$$=\frac{x-2-3x^2-12x}{3x^2}=\frac{-3x^2-11x-2}{3x^2}$$

97. The LCM of the denominators is $6x^2$.

$$\frac{x+5}{3x^2}=\frac{x+5}{3x^2}\cdot\frac{2}{2}=\frac{2x+10}{6x^2}$$
$$\frac{2x+1}{2x}=\frac{2x+1}{2x}\cdot\frac{3x}{3x}=\frac{6x^2+3x}{6x^2}$$
$$\frac{x+5}{3x^2}+\frac{2x+1}{2x}=\frac{2x+10}{6x^2}+\frac{6x^2+3x}{6x^2}$$
$$=\frac{2x+10+(6x^2+3x)}{6x^2}=\frac{6x^2+5x+10}{6x^2}$$

99. The LCM of the denominators is x.

$$2x=\frac{2x}{1}\cdot\frac{x}{x}=\frac{2x^2}{x}$$
$$\frac{1}{x}=\frac{1}{x}$$
$$2x+\frac{1}{x}=\frac{2x^2}{x}+\frac{1}{x}=\frac{2x^2+1}{x}$$

101. The LCM of the denominators is $x+1$.

$$3=\frac{3}{1}\cdot\frac{x+1}{x+1}=\frac{3x+3}{x+1}$$
$$\frac{x-1}{x+1}=\frac{x-1}{x+1}$$
$$3+\frac{x-1}{x+1}=\frac{3x+3}{x+1}+\frac{x-1}{x+1}=\frac{3x+3+(x-1)}{x+1}=\frac{4x+2}{x+1}$$

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103. The LCM of the denominators is x^2y.

$$\frac{x+2}{xy}=\frac{x+2}{xy}\cdot\frac{x}{x}=\frac{x^2+2x}{x^2y}$$

$$\frac{3x-2}{x^2y}-\frac{3x-2}{x^2y}$$

$$\frac{x+2}{xy}-\frac{3x-2}{x^2y}=\frac{x^2+2x}{x^2y}-\frac{3x-2}{x^2y}=\frac{x^2+2x-(3x-2)}{x^2y}$$

$$=\frac{x^2+2x-3x+2}{x^2y}=\frac{x^2-x+2}{x^2y}$$

105. The LCM of the denominators is $12x^2y^2$.

$$\frac{4x-3}{3x^2y}=\frac{4x-3}{3x^2y}\cdot\frac{4y}{4y}=\frac{16xy-12y}{12x^2y^2}$$

$$\frac{2x+1}{4xy^2}=\frac{2x+1}{4xy^2}\cdot\frac{3x}{3x}=\frac{6x^2+3x}{12x^2y^2}$$

$$\frac{4x-3}{3x^2y}+\frac{2x+1}{4xy^2}=\frac{16xy-12y}{12x^2y^2}+\frac{6x^2+3x}{12x^2y^2}$$

$$=\frac{16xy-12y+\left(6x^2+3x\right)}{12x^2y^2}$$

$$=\frac{16xy-12y+6x^2+3x}{12x^2y^2}$$

107. The LCM of the denominators is $24x^2y$.

$$\frac{x-2}{8x^2}=\frac{x-2}{8x^2}\cdot\frac{3y}{3y}=\frac{3xy-6y}{24x^2y}$$

$$\frac{x+7}{12xy}=\frac{x+7}{12xy}\cdot\frac{2x}{2x}=\frac{2x^2+14x}{24x^2y}$$

$$\frac{x-2}{8x^2}-\frac{x+7}{12xy}=\frac{3xy-6y}{24x^2y}-\frac{2x^2+14x}{24x^2y}$$

$$=\frac{3xy-6y-\left(2x^2+14x\right)}{24x^2y}$$

$$=\frac{3xy-6y-2x^2-14x}{24x^2y}$$

109. The LCM s is $(x-2)(x+3)$.

$$\frac{4}{x-2}=\frac{4}{x-2}\cdot\frac{x+3}{x+3}=\frac{4x+12}{(x-2)(x+3)}$$

$$\frac{5}{x+3}=\frac{5}{x+3}\cdot\frac{x-2}{x-2}=\frac{5x-10}{(x-2)(x+3)}$$

$$\frac{4}{x-2}+\frac{5}{x+3}=\frac{4x+12}{(x-2)(x+3)}+\frac{5x-10}{(x-2)(x+3)}$$

$$=\frac{4x+12+(5x-10)}{(x-2)(x+3)}=\frac{9x+2}{(x-2)(x+3)}$$

111. The LCM s is $(x-7)(x+3)$.

$$\frac{6}{x-7}=\frac{6}{x-7}\cdot\frac{x+3}{x+3}=\frac{6x+18}{(x-7)(x+3)}$$

$$\frac{4}{x+3}=\frac{4}{x+3}\cdot\frac{x-7}{x-7}=\frac{4x-28}{(x-7)(x+3)}$$

$$\frac{6}{x-7}-\frac{4}{x+3}=\frac{6x+18}{(x-7)(x+3)}-\frac{4x-28}{(x-7)(x+3)}$$

$$=\frac{6x+18-(4x-28)}{(x-7)(x+3)}=\frac{6x+18-4x+28}{(x-7)(x+3)}$$

$$=\frac{2x+46}{(x-7)(x+3)}=\frac{2(x+23)}{(x-7)(x+3)}$$

113. The LCM s is $(x+1)(x-3)$.

$$\frac{2x}{x+1}=\frac{2x}{x+1}\cdot\frac{x-3}{x-3}=\frac{2x^2-6x}{(x+1)(x-3)}$$

$$\frac{1}{x-3}=\frac{1}{x-3}\cdot\frac{x+1}{x+1}=\frac{x+1}{(x+1)(x-3)}$$

$$\frac{2x}{x+1}+\frac{1}{x-3}=\frac{2x^2-6x}{(x+1)(x-3)}+\frac{x+1}{(x+1)(x-3)}$$

$$=\frac{2x^2-6x+(x+1)}{(x+1)(x-3)}=\frac{2x^2-5x+1}{(x+1)(x-3)}$$

115. The LCM s is $(2x-1)(x-6)$.

$$\frac{4x}{2x-1}=\frac{4x}{2x-1}\cdot\frac{x-6}{x-6}=\frac{4x^2-24x}{(2x-1)(x-6)}$$

$$\frac{5}{x-6}=\frac{5}{x-6}\cdot\frac{2x-1}{2x-1}=\frac{10x-5}{(2x-1)(x-6)}$$

$$\frac{4x}{2x-1}-\frac{5}{x-6}=\frac{4x^2-24x}{(2x-1)(x-6)}-\frac{10x-5}{(2x-1)(x-6)}$$

$$=\frac{4x^2-24x-(10x-5)}{(2x-1)(x-6)}$$

$$=\frac{4x^2-24x-10x+5}{(2x-1)(x-6)}=\frac{4x^2-34x+5}{(2x-1)(x-6)}$$

117. The LCM is $a-7$.

$$\frac{2a}{a-7}=\frac{2a}{a-7}$$

$$\frac{5}{7-a}=\frac{5}{-(a-7)}\cdot\frac{-1}{-1}=\frac{-5}{a-7}$$

$$\frac{2a}{a-7}+\frac{5}{7-a}=\frac{2a}{a-7}+\frac{-5}{a-7}=\frac{2a+(-5)}{a-7}=\frac{2a-5}{a-7}$$

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119. The LCM is $(x+3)(x-3)$.

$$\frac{x}{x^2-9}=\frac{x}{(x+3)(x-3)}$$

$$\frac{3}{x-3}=\frac{3}{x-3}\cdot\frac{x+3}{x+3}=\frac{3x+9}{(x+3)(x-3)}$$

$$\frac{x}{x^2-9}+\frac{3}{x-3}=\frac{x}{(x+3)(x-3)}+\frac{3x+9}{(x+3)(x-3)}$$

$$=\frac{x+(3x+9)}{(x+3)(x-3)}=\frac{4x+9}{(x+3)(x-3)}$$

121. The LCM is $(x+2)(x-3)$.

$$\frac{2x}{x^2-x-6}=\frac{2x}{(x+2)(x-3)};$$

$$\frac{3}{x+2}=\frac{3}{x+2}\cdot\frac{x-3}{x-3}=\frac{3x-9}{(x+2)(x-3)}$$

$$\frac{2x}{x^2-x-6}-\frac{3}{x+2}=\frac{2x}{(x+2)(x-3)}-\frac{3x-9}{(x+2)(x-3)}$$

$$=\frac{2x-(3x-9)}{(x+2)(x-3)}=\frac{2x-3x+9}{(x+2)(x-3)}$$

$$=\frac{-x+9}{(x+2)(x-3)}$$

123. The LCM is $(y+2)^2$.

$$1=\frac{1}{1}\cdot\frac{(y+2)^2}{(y+2)^2}=\frac{(y+2)^2}{(y+2)^2};\ \frac{(y-2)^2}{(y+2)^2}=\frac{(y-2)^2}{(y+2)^2}$$

$$1-\frac{(y-2)^2}{(y+2)^2}=\frac{(y+2)^2}{(y+2)^2}-\frac{(y-2)^2}{(y+2)^2}=\frac{\left(y^2+4y+4\right)-\left(y^2-4y+4\right)}{(y+2)^2}=\frac{y^2+4y+4-y^2+4y-4}{(y+2)^2}=\frac{8y}{(y+2)^2}$$

125. The LCM is $x-y$.

$$\frac{y}{x-y}=\frac{y}{x-y};\ 2=\frac{2}{1}\cdot\frac{x-y}{x-y}=\frac{2x-2y}{x-y};$$

$$\frac{x}{y-x}=\frac{x}{y-x}\cdot\frac{-1}{-1}=\frac{-x}{x-y}$$

$$\frac{y}{x-y}+2-\frac{x}{y-x}=\frac{y}{x-y}+\frac{2x-2y}{x-y}-\frac{-x}{x-y}$$

$$=\frac{y+(2x-2y)+x}{x-y}=\frac{3x-y}{x-y}$$

127. The LCM is $(a-3)(a-4)$.

$$\frac{2a+3}{a^2-7a+12}=\frac{2a+3}{(a-3)(a-4)};$$

$$\frac{2}{a-3}=\frac{2}{a-3}\cdot\frac{a-4}{a-4}=\frac{2a-8}{(a-3)(a-4)}$$

$$\frac{2a+3}{a^2-7a+12}-\frac{2}{a-3}=\frac{2a+3}{(a-3)(a-4)}-\frac{2a-8}{(a-3)(a-4)}$$

$$=\frac{2a+3-(2a-8)}{(a-3)(a-4)}=\frac{2a+3-2a+8}{(a-3)(a-4)}$$

$$=\frac{11}{(a-3)(a-4)}$$

129. The LCM is $(x+2)(x-5)$.

$$\frac{x+3}{x^2-3x-10}=\frac{x+3}{(x+2)(x-5)};\ \frac{2}{5-x}=\frac{2}{-(x-5)}\cdot\frac{x+2}{x+2}=\frac{-2(x+2)}{(x+2)(x-5)}=\frac{-2x-4}{(x+2)(x-5)}$$

$$\frac{x+3}{x^2-3x-10}+\frac{2}{5-x}=\frac{x+3}{(x+2)(x-5)}+\frac{-2x-4}{(x+2)(x-5)}=\frac{x+3+(-2x-4)}{(x+2)(x-5)}=\frac{-x-1}{(x+2)(x-5)}$$

131. The LCM is $(x-4)(x+5)$.

$$\frac{x}{x-4}=\frac{x}{x-4}\cdot\frac{x+5}{x+5}=\frac{x^2+5x}{(x-4)(x+5)};\ \frac{5}{x+5}=\frac{5}{x+5}\cdot\frac{x-4}{x-4}=\frac{5x-20}{(x-4)(x+5)};\ \frac{11x-8}{x^2+x-20}=\frac{11x-8}{(x-4)(x+5)}$$

$$\frac{x}{x-4}+\frac{5}{x+5}-\frac{11x-8}{x^2+x-20}=\frac{x^2+5x}{(x-4)(x+5)}+\frac{5x-20}{(x-4)(x+5)}-\frac{11x-8}{(x-4)(x+5)}=\frac{x^2+5x+(5x-20)-(11x-8)}{(x-4)(x+5)}$$

$$=\frac{x^2+5x+5x-20-11x+8}{(x-4)(x+5)}=\frac{x^2-x-12}{(x-4)(x+5)}=\frac{(x+3)\cancel{(x-4)}^{1}}{\cancel{(x-4)}_{1}(x+5)}=\frac{x+3}{x+5}$$

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133. The LCM is $(x-8)(x+4)$.

$$\frac{4x+1}{x-8}=\frac{4x+1}{x-8}\cdot\frac{x+4}{x+4}=\frac{4x^2+17x+4}{(x-8)(x+4)};\ \frac{3x+2}{x+4}=\frac{3x+2}{x+4}\cdot\frac{x-8}{x-8}=\frac{3x^2-22x-16}{(x-8)(x+4)};\ \frac{49x+4}{x^2-4x-32}=\frac{49x+4}{(x-8)(x+4)}$$

$$\frac{4x+1}{x-8}-\frac{3x+2}{x+4}-\frac{49x+4}{x^2-4x-32}=\frac{4x^2+17x+4}{(x-8)(x+4)}-\frac{3x^2-22x-16}{(x-8)(x+4)}-\frac{49x+4}{(x-8)(x+4)}$$

$$=\frac{4x^2+17x+4-(3x^2-22x-16)-(49x+4)}{(x-8)(x+4)}=\frac{4x^2+17x+4-3x^2+22x+16-49x-4}{(x-8)(x+4)}$$

$$=\frac{x^2-10x+16}{(x-8)(x+4)}=\frac{(x-2)\cancel{(x-8)}^{1}}{\underset{1}{\cancel{(x-8)}}(x+4)}=\frac{x-2}{x+4}$$

135. The LCM is $(x+5)(x-2)$.

$$\frac{3x+5}{x+5}=\frac{3x+5}{x+5}\cdot\frac{x-2}{x-2}=\frac{3x^2-x-10}{(x+5)(x-2)};\ \frac{x+1}{2-x}=\frac{x+1}{-(x-2)}=\frac{-(x+1)}{x-2}\cdot\frac{x+5}{x+5}=\frac{-(x^2+6x+5)}{(x+5)(x-2)}=\frac{-x^2-6x-5}{(x-5)(x-2)};$$

$$\frac{4x^2-3x-1}{x^2+3x-10}=\frac{4x^2-3x-1}{(x+5)(x-2)}$$

$$\frac{3x+5}{x+5}-\frac{x+1}{2-x}-\frac{4x^2-3x-1}{x^2+3x-10}=\frac{3x^2-x-10}{(x+5)(x-2)}-\frac{-x^2-6x-5}{(x+5)(x-2)}-\frac{4x^2-3x-1}{(x+5)(x-2)}$$

$$=\frac{3x^2-x-10-(-x^2-6x-5)-(4x^2-3x-1)}{(x+5)(x-2)}$$

$$=\frac{3x^2-x-10+x^2+6x+5-4x^2+3x+1}{(x+5)(x-2)}=\frac{8x-4}{(x+5)(x-2)}=\frac{4(2x-1)}{(x+5)(x-2)}$$

Applying the Concepts

137. **a.** $\dfrac{3x+6y}{xy}=\dfrac{3x}{xy}+\dfrac{6y}{xy}=\dfrac{3}{y}+\dfrac{6}{x}$

b. $\dfrac{4a^2+3ab}{a^2b^2}=\dfrac{4a^2}{a^2b^2}+\dfrac{3ab}{a^2b^2}=\dfrac{4}{b^2}+\dfrac{3}{ab}$

c. $\dfrac{3m^2n+2mn^2}{12m^3n^2}=\dfrac{3m^2n}{12m^3n^2}+\dfrac{2mn^2}{12m^3n^2}=\dfrac{1}{4mn}+\dfrac{1}{6m^2}$

139. **a.** $\dfrac{12{,}000\cdot 1.60}{x}=\dfrac{19{,}200}{x}$ dollars

b. $\dfrac{19{,}200}{x}-\dfrac{19{,}200}{x+5}$

$=\dfrac{19{,}200x+96{,}000-19{,}200x}{x(x+5)}$

$=\dfrac{96{,}000}{x(x+5)}$ dollars

c. Let $x=25$,

$\dfrac{96{,}000}{25(30)}=\$128$

Section 8.3

Objective A Exercises

1. The LCM of x and x^2 is x^2.

$$\frac{1+(3/x)}{1-(9/x^2)}=\frac{1+(3/x)}{1-(9/x^2)}\cdot\frac{x^2}{x^2}=\frac{1\cdot x^2+(3/x)\cdot x^2}{1\cdot x^2-(9/x^2)\cdot x^2}$$

$$=\frac{x^2+3x}{x^2-9}=\frac{x\cancel{(x+3)}^{1}}{\underset{1}{\cancel{(x+3)}}(x-3)}=\frac{x}{x-3}$$

3. The LCM is $x+4$.

$$\frac{2-[8/(x+4)]}{3-[12/(x+4)]}=\frac{2-[8/(x+4)]}{3-[12/(x+4)]}\cdot\frac{x+4}{x+4}$$

$$=\frac{2(x+4)-[8/(x+4)](x+4)}{3(x+4)-[12/(x+4)](x+4)}$$

$$=\frac{2(x+4)-8}{3(x+4)-12}=\frac{2x+8-8}{3x+12-12}=\frac{2x}{3x}=\frac{2}{3}$$

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5. The LCM is $y-2$.

$$\frac{1+[5/(y-2)]}{1-[2/(y-2)]}=\frac{1+[5/(y-2)]}{1-[2/(y-2)]}\cdot\frac{y-2}{y-2}$$
$$=\frac{1(y-2)+[5/(y-2)](y-2)}{1(y-2)-[2/(y-2)](y-2)}$$
$$=\frac{(y-2)+5}{(y-2)-2}=\frac{y-2+5}{y-2-2}=\frac{y+3}{y-4}$$

7. The LCM is $x+7$.

$$\frac{4-[2/(x+7)]}{5+[1/(x+7)]}=\frac{4-[2/(x+7)]}{5+[1/(x+7)]}\cdot\frac{x+7}{x+7}$$
$$=\frac{4(x+7)-[2/(x+7)](x+7)}{5(x+7)+[1/(x+7)](x+7)}$$
$$=\frac{4(x+7)-2}{5(x+7)+1}=\frac{4x+28-2}{5x+35+1}=\frac{4x+26}{5x+36}$$
$$=\frac{2(2x+13)}{5x+36}$$

9. The LCM of x and x^2 is x^2.

$$\frac{1-(1/x)-(6/x^2)}{1-(9/x^2)}=\frac{1-(1/x)-(6/x^2)}{1-(9/x^2)}\cdot\frac{x^2}{x^2}$$
$$=\frac{1\cdot x^2-(1/x)\cdot x^2-(6/x^2)\cdot x^2}{1\cdot x^2-(9/x^2)\cdot x^2}$$
$$=\frac{x^2-x-6}{x^2-9}=\frac{(x+2)\cancel{(x-3)}}{(x+3)\cancel{(x-3)}_{1}}=\frac{x+2}{x+3}$$

11. The LCM of x and x^2 is x^2.

$$\frac{1-(5/x)-(6/x^2)}{1+(6/x)+(5/x^2)}=\frac{1-(5/x)-(6/x^2)}{1+(6/x)+(5/x^2)}\cdot\frac{x^2}{x^2}$$
$$=\frac{1\cdot x^2-(5/x)\cdot x^2-(6/x^2)\cdot x^2}{1\cdot x^2+(6/x)\cdot x^2+(5/x^2)\cdot x^2}$$
$$=\frac{x^2-5x-6}{x^2+6x+5}=\frac{\cancel{(x+1)}^{1}(x-6)}{\cancel{(x+1)}_{1}(x+5)}=\frac{x-6}{x+5}$$

13. The LCM of x and x^2 is x^2.

$$\frac{1-(6/x)+(8/x^2)}{(4/x^2)+(3/x)-1}=\frac{1-(6/x)+(8/x^2)}{(4/x^2)+(3/x)-1}\cdot\frac{x^2}{x^2}$$
$$=\frac{1\cdot x^2-(6/x)\cdot x^2+(8/x^2)\cdot x^2}{(4/x^2)\cdot x^2+(3/x)\cdot x^2-1\cdot x^2}$$
$$=\frac{x^2-6x+8}{4+3x-x^2}=\frac{(x-2)\cancel{(x-4)}}{(1+x)\cancel{(4-x)}_{1}}=-\frac{x-2}{x+1}$$

15. The LCM is $x+3$.

$$\frac{x-[4/(x+3)]}{1+[1/(x+3)]}=\frac{x-[4/(x+3)]}{1+[1/(x+3)]}\cdot\frac{x+3}{x+3}$$
$$=\frac{x(x+3)-[4/(x+3)](x+3)}{1(x+3)+[1/(x+3)](x+3)}$$
$$=\frac{x^2+3x-4}{x+3+1}=\frac{\cancel{(x+4)}^{1}(x-1)}{\cancel{(x+4)}_{1}}=x-1$$

17. The LCM is $2x+1$.

$$\frac{1-[x/(2x+1)]}{x-[1/(2x+1)]}=\frac{1-[x/(2x+1)]}{x-[1/(2x+1)]}\cdot\frac{2x+1}{2x+1}$$
$$=\frac{1(2x+1)-[x/(2x+1)](2x+1)}{x(2x+1)-[1/(2x+1)](2x+1)}$$
$$=\frac{2x+1-x}{2x^2+x-1}=\frac{\cancel{(x+1)}^{1}}{\cancel{(x+1)}_{1}(2x-1)}=\frac{1}{2x-1}$$

19. The LCM is $x+4$.

$$\frac{x-5+[14/(x+4)]}{x+3-[2/(x+4)]}=\frac{x-5+[14/(x+4)]}{x+3-[2/(x+4)]}\cdot\frac{x+4}{x+4}=\frac{x(x-5)(x+4)+[14/(x+4)](x+4)}{x(x+3)(x+4)-[2/(x+4)](x+4)}=\frac{x^2-x-20+14}{x^2+7x+12-2}=\frac{x^2-x-6}{x^2+7x+10}$$
$$=\frac{\cancel{(x+2)}^{1}(x-3)}{\cancel{(x+2)}_{1}(x+5)}=\frac{x-3}{x+5}$$

21. The LCM is $x-6$.

$$\frac{x+3-[10/(x-6)]}{x+2-[20/(x-6)]}=\frac{x+3-[10/(x-6)]}{x+2-[20/(x-6)]}\cdot\frac{x-6}{x-6}=\frac{(x+3)(x-6)-[10/(x-6)](x-6)}{(x+2)(x-6)-[20/(x-6)](x-6)}=\frac{x^2-3x-18-10}{x^2-4x-12-20}=\frac{x^2-3x-28}{x^2-4x-32}$$
$$=\frac{\cancel{(x+4)}^{1}(x-7)}{\cancel{(x+4)}_{1}(x-8)}=\frac{x-7}{x-8}$$

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23. The LCM is $2y+3$.

$$\frac{y-6+[22/(2y+3)]}{y-5+[11/(2y+3)]}=\frac{y-6+[22/(2y+3)]}{y-5+[11/(2y+3)]}\cdot\frac{2y+3}{2y+3}=\frac{(y-6)(2y+3)+[22/(2y+3)](2y+3)}{(y-5)(2y+3)+[11/(2y+3)](2y+3)}=\frac{2y^2-9y-18+22}{2y^2-7y-15+11}$$

$$=\frac{2y^2-9y+4}{2y^2-7y-4}=\frac{(2y-1)\cancel{(y-4)}^{1}}{(2y+1)\cancel{(y-4)}_{1}}=\frac{2y-1}{2y+1}$$

25. The LCM is $2x-3$.

$$\frac{x-[2/(2x-3)]}{2x-1-[8/(2x-3)]}=\frac{x-[2/(2x-3)]}{2x-1-[8/(2x-3)]}\cdot\frac{2x-3}{2x-3}=\frac{x(2x-3)-[2/(2x-3)](2x-3)}{(2x-1)(2x-3)-[8/(2x-3)](2x-3)}=\frac{2x^2-3x-2}{4x^2-8x+3-8}$$

$$=\frac{2x^2-3x-2}{4x^2-8x-5}=\frac{\cancel{(2x+1)}^{1}(x-2)}{\cancel{(2x+1)}_{1}(2x-5)}=\frac{x-2}{2x-5}$$

27. The LCM of x and $x-1$ is $x(x-1)$.

$$\frac{(1/x)-[2/(x-1)]}{(3/x)+[1/(x-1)]}=\frac{(1/x)-[2/(x-1)]}{(3/x)+[1/(x-1)]}\cdot\frac{x(x-1)}{x(x-1)}=\frac{(1/x)(x)(x-1)-[2/(x-1)](x)(x-1)}{(3/x)(x)(x-1)+[1/(x-1)](x)(x-1)}=\frac{x-1-2x}{3(x-1)+x}=\frac{-x-1}{3x-3+x}$$

$$=\frac{-x-1}{4x-3}=-\frac{x+1}{4x-3}$$

29. The LCM of x and $2x-1$ is $x(2x-1)$.

$$\frac{[3/(2x-1)]-(1/x)}{(4/x)+[2/(2x-1)]}=\frac{[3/(2x-1)]-(1/x)}{(4/x)+[2/(2x-1)]}\cdot\frac{x(2x-1)}{x(2x-1)}=\frac{[3/(2x-1)](x)(2x-1)-(1/x)(x)(2x-1)}{(4/x)(x)(2x-1)+[2/(2x-1)](x)(2x-1)}=\frac{3x-(2x-1)}{4(2x-1)+2x}=\frac{3x-2x+1}{8x-4+2x}$$

$$=\frac{x+1}{10x-4}=\frac{x+1}{2(5x-2)}$$

Applying the Concepts

31.
$$1+\frac{1}{1+(1/2)}=1+\frac{1}{1+(1/2)}\cdot\frac{2}{2}=1+\frac{2}{2+1}$$

$$=1+\frac{2}{3}=\frac{3}{3}+\frac{2}{3}=\frac{5}{3}$$

33.
$$1-\frac{1}{1-(1/x)}=1-\frac{1}{1-(1/x)}\cdot\frac{x}{x}=1-\frac{x}{x-1}=\frac{x-1}{x-1}-\frac{x}{x-1}$$

$$=\frac{x-1-x}{x-1}=\frac{-1}{x-1}$$

35.
$$\left(\frac{y}{4}-\frac{4}{y}\right)\div\left(\frac{4}{y}-3+\frac{y}{2}\right)=\frac{(y/4)-(4/y)}{(4/y)-3+(y/2)}\cdot\frac{4y}{4y}$$

$$=\frac{y^2-16}{16-12y+2y^2}$$

$$=\frac{(y+4)\cancel{(y-4)}^{1}}{2\cancel{(y-4)}_{1}(y-2)}=\frac{y+4}{2(y-2)}$$

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Section 8.4

Objective A Exercises

1. $\frac{2x}{3}-\frac{5}{2}=-\frac{1}{2}$ The LCM is 6.

$$\frac{6}{1}\left(\frac{2x}{3}-\frac{5}{2}\right)=\frac{6}{1}\left(-\frac{1}{2}\right)$$
$$\frac{6}{1}\cdot\frac{2x}{3}-\frac{6}{1}\cdot\frac{5}{2}=-3$$
$$4x-15=-3$$
$$4x=12$$
$$x=3$$

3 checks as a solution. The solution is 3.

3. $\frac{x}{3}-\frac{1}{4}=\frac{x}{4}-\frac{1}{6}$ The LCM is 12.

$$\frac{12}{1}\left(\frac{x}{3}-\frac{1}{4}\right)=\frac{12}{1}\left(\frac{x}{4}-\frac{1}{6}\right)$$
$$\frac{12}{1}\cdot\frac{x}{3}-\frac{12}{1}\cdot\frac{1}{4}=\frac{12}{1}\cdot\frac{x}{4}-\frac{12}{1}\cdot\frac{1}{6}$$
$$4x-3=3x-2$$
$$x-3=-2$$
$$x=1$$

1 checks as a solution. The solution is 1.

5. $\frac{2x-5}{8}+\frac{1}{4}=\frac{x}{8}+\frac{3}{4}$ The LCM is 8.

$$\frac{8}{1}\left(\frac{2x-5}{8}+\frac{1}{4}\right)=\frac{8}{1}\left(\frac{x}{8}+\frac{3}{4}\right)$$
$$\frac{8}{1}\cdot\frac{2x-5}{8}+\frac{8}{1}\cdot\frac{1}{4}=\frac{8}{1}\cdot\frac{x}{8}+\frac{8}{1}\cdot\frac{3}{4}$$
$$2x-5+2=x+6$$
$$2x-3=x+6$$
$$x-3=6$$
$$x=9$$

9 checks as a solution. The solution is 9.

7. $\frac{6}{2a+1}=2$ The LCM is $2a+1$.

$$\frac{2a+1}{1}\cdot\frac{6}{2a+1}=\frac{2a+1}{1}\cdot 2$$
$$6=(2a+1)2$$
$$6=4a+2$$
$$4=4a$$
$$1=a$$

1 checks as a solution. The solution is 1.

9. $\frac{9}{2x-5}=-2$ The LCM is $2x-5$.

$$\frac{2x-5}{1}\cdot\frac{9}{2x-5}=\frac{2x-5}{1}(-2)$$
$$9=(2x-5)(-2)$$
$$9=-4x+10$$
$$-1=-4x$$
$$\frac{1}{4}=x$$

$\frac{1}{4}$ checks as a solution. The solution is $\frac{1}{4}$.

11. $2+\frac{5}{x}=7$ The LCM is x.

$$\frac{x}{1}\left(2+\frac{5}{x}\right)=\frac{x}{1}\cdot 7$$
$$2x+\frac{x}{1}\cdot\frac{5}{x}=7x$$
$$2x+5=7x$$
$$5=5x$$
$$1=x$$

1 checks as a solution. The solution is 1.

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13. $1-\frac{9}{x}=4$ The LCM is x.

$$\frac{x}{1}\left(1-\frac{9}{x}\right)=\frac{x}{1}\cdot 4$$
$$x-\frac{x}{1}\cdot\frac{9}{x}=4x$$
$$x-9=4x$$
$$-9=3x$$
$$-3=x$$

-3 checks as a solution. The solution is -3.

15. $\frac{2}{y}+5=9$ The LCM is y.

$$\frac{y}{1}\left(\frac{2}{y}+5\right)=\frac{y}{1}\cdot 9$$
$$\frac{y}{1}\cdot\frac{2}{y}+\frac{y}{1}\cdot 5=9y$$
$$2+5y=9y$$
$$2=4y$$
$$\frac{1}{2}=y$$

$\frac{1}{2}$ checks as a solution. The solution is $\frac{1}{2}$.

17. $\frac{3}{x-2}=\frac{4}{x}$ The LCM is $x(x-2)$.

$$\frac{x(x-2)}{1}\cdot\frac{3}{x-2}=\frac{x(x-2)}{1}\cdot\frac{4}{x}$$
$$3x=(x-2)4$$
$$3x=4x-8$$
$$-x=-8$$
$$x=8$$

8 checks as a solution. The solution is 8.

19. The LCM is $(3x-1)(4x+1)$.

$$\frac{2}{3x-1}=\frac{3}{4x+1}$$
$$\frac{(3x-1)(4x+1)}{1}\cdot\frac{2}{3x-1}=\frac{(3x-1)(4x+1)}{1}\cdot\frac{3}{4x+1}$$
$$(4x+1)2=(3x-1)3$$
$$8x+2=9x-3$$
$$-x+2=-3$$
$$-x=-5$$
$$x=5$$

5 checks as a solution. The solution is 5.

21. The LCM is $(2x+5)(x-1)$.

$$\frac{-3}{2x+5}=\frac{2}{x-1}$$
$$\frac{(2x+5)(x-1)}{1}\cdot\frac{-3}{2x+5}=\frac{(2x+5)(x-1)}{1}\cdot\frac{2}{x-1}$$
$$(x-1)(-3)=(2x+5)2$$
$$-3x+3=4x+10$$
$$-7x+3=10$$
$$-7x=7$$
$$x=-1$$

-1 checks as a solution. The solution is -1.

23. The LCM is $x-4$.

$$\frac{4x}{x-4}+5=\frac{5x}{x-4}$$
$$\frac{x-4}{1}\left(\frac{4x}{x-4}+5\right)=\frac{x-4}{1}\cdot\frac{5x}{x-4}$$
$$\frac{x-4}{1}\cdot\frac{4x}{x-4}+\frac{x-4}{1}\cdot 5=5x$$
$$4x+(x-4)5=5x$$
$$4x+5x-20=5x$$
$$9x-20=5x$$
$$-20=-4x$$
$$5=x$$

5 checks as a solution. The solution is 5.

25. The LCM is $a-3$.

$$2+\frac{3}{a-3}=\frac{a}{a-3}$$
$$\frac{a-3}{1}\left(2+\frac{3}{a-3}\right)=\frac{a-3}{1}\cdot\frac{a}{a-3}$$
$$\frac{a-3}{1}\cdot 2+\frac{a-3}{1}\cdot\frac{3}{a-3}=a$$
$$(a-3)2+3=a$$
$$2a-6+3=a$$
$$2a-3=a$$
$$a-3=0$$
$$a=3$$

3 does not check as a solution. The equation has no solution.

27. The LCM is $(x-1)(x+2)$.

$$\frac{x}{x-1}=\frac{8}{x+2}$$
$$\frac{(x-1)(x+2)}{1}\cdot\frac{x}{x-1}=\frac{(x-1)(x+2)}{1}\cdot\frac{8}{x+2}$$
$$(x+2)x=(x-1)8$$
$$x^2+2x=8x-8$$
$$x^2-6x+8=0$$
$$(x-4)(x-2)=0$$
$$x-4=0 \qquad x-2=0$$
$$x=4 \qquad x=2$$

Both 4 and 2 check as solutions. The solutions are 4 and 2.

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29. The LCM is $(x+4)(x-1)$.

$$\frac{2x}{x+4}=\frac{3}{x-1}$$

$$\frac{(x+4)(x-1)}{1}\cdot\frac{2x}{x+4}=\frac{(x+4)(x-1)}{1}\cdot\frac{3}{x-1}$$

$$(x-1)2x=(x+4)3$$

$$2x^2-2x=3x+12$$

$$2x^2-5x-12=0$$

$$(2x+3)(x-4)=0$$

$$2x+3=0 \qquad x-4=0$$

$$2x=-3 \qquad x=4$$

$$x=-\frac{3}{2}$$

Both $-\frac{3}{2}$ and 4 check as solutions. The solutions are $-\frac{3}{2}$ and 4.

31. The LCM is $x-2$.

$$x+\frac{6}{x-2}=\frac{3x}{x-2}$$

$$\frac{x-2}{1}\left(x+\frac{6}{x-2}\right)=\frac{x-2}{1}\cdot\frac{3x}{x-2}$$

$$\frac{x-2}{1}\cdot x+\frac{x-2}{1}\cdot\frac{6}{x-2}=3x$$

$$x^2-2x+6=3x$$

$$x^2-5x+6=0$$

$$(x-3)(x-2)=0$$

$$x-3=0 \qquad x-2=0$$

$$x=3 \qquad x=2$$

2 does not check as a solution. 3 checks as a solution. The solution is 3.

33. The LCM is $y(y-2)$.

$$\frac{8}{y}=\frac{2}{y-2}+1$$

$$\frac{y(y-2)}{1}\cdot\frac{8}{y}=\frac{y(y-2)}{1}\left(\frac{2}{y-2}+1\right)$$

$$(y-2)8=\frac{y(y-2)}{1}\cdot\frac{2}{y-2}+\frac{y(y-2)}{1}\cdot 1$$

$$8y-16=2y+y^2-2y$$

$$8y-16=y^2$$

$$0=y^2-8y+16$$

$$0=(y-4)(y-4)$$

$$y-4=0 \qquad y-4=0$$

$$y=4 \qquad y=4$$

4 checks as a solution. The solution is 4.

Objective B Exercises

37.

$$\frac{6}{x}=\frac{2}{3}$$

$$3x\cdot\frac{6}{x}=3x\cdot\frac{2}{3}$$

$$18=2x$$

$$9=x$$

The solution is 9.

39.

$$\frac{16}{9}=\frac{64}{x}$$

$$9x\cdot\frac{16}{9}=9x\cdot\frac{64}{x}$$

$$16x=576$$

$$x=36$$

The solution is 36.

41.

$$\frac{3}{5}=\frac{x-4}{10}$$

$$10\cdot\frac{3}{5}=10\cdot\frac{x-4}{10}$$

$$6=x-4$$

$$10=x$$

The solution is 10.

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43.

$$\frac{2}{11}=\frac{20}{x-3}$$

$$11(x-3)\cdot\frac{2}{11}=11(x-3)\cdot\frac{20}{x-3}$$

$$(x-3)2=220$$

$$2x-6=220$$

$$2x=226$$

$$x=113$$

The solution is 113.

45.

$$\frac{16}{x-2}=\frac{8}{x}$$

$$x(x-2)\cdot\frac{16}{x-2}=x(x-2)\cdot\frac{8}{x}$$

$$16x=(x-2)8$$

$$16x=8x-16$$

$$8x=-16$$

$$x=-2$$

The solution is –2.

47.

$$\frac{x-6}{3}=\frac{x}{5}$$

$$15\cdot\frac{x-6}{3}=15\cdot\frac{x}{5}$$

$$5(x-6)=3x$$

$$5x-30=3x$$

$$2x-30=0$$

$$2x=30$$

$$x=15$$

The solution is 15.

49.

$$\frac{9}{x+2}=\frac{3}{x-2}$$

$$(x+2)(x-2)\cdot\frac{9}{x+2}=(x+2)(x-2)\cdot\frac{3}{x-2}$$

$$(x-2)9=(x+2)3$$

$$9x-18=3x+6$$

$$6x-18=6$$

$$6x=24$$

$$x=4$$

The solution is 4.

Objective C Exercises

51. Strategy

To find $\overline{AC}$, write a proportion using the fact that in similar triangles, the ratios of corresponding sides are equal. Solve the proportion for $\overline{AC}$.

Solution

$$\frac{AC}{AB}=\frac{DF}{DE}$$

$$\frac{AC}{4}=\frac{15}{9}$$

$$36\cdot\frac{AC}{4}=\frac{15}{9}\cdot 36$$

$$9AC=60$$

$$AC=6.7$$

The length of $\overline{AC}$ is 6.7 cm.

53. Strategy

To find the height of triangle *ABC*, write a proportion using the fact that in similar triangles, the ratio of corresponding sides equals the ratio of corresponding heights. Solve the proportion for the height (h).

Solution

$$\frac{h}{BC}=\frac{FH}{FH}$$

$$\frac{h}{5}=\frac{7}{12}$$

$$60\cdot\frac{h}{5}=\frac{7}{12}\cdot 60$$

$$12h=35$$

$$h=2.9$$

The height of triangle *ABC* is 2.9 m.

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55. Strategy

To find the perimeter:

- Find side $\overline{DF}$ by writing a proportion using the fact that the ratios of corresponding sides of similar triangles are equal.
- Use the formula for the perimeter of a triangle.

Solution

$$\frac{AC}{BC}=\frac{DF}{EF}$$
$$\frac{5}{6}=\frac{DF}{9}$$
$$18\cdot\frac{5}{6}=\frac{DF}{9}\cdot 18$$
$$15=2DF$$
$$DF=7.5$$
$$P=a+b+c=7.5+9+6=22.5$$

The perimeter of triangle DEF is 22.5 ft.

57. Strategy

To find the area of triangle ABC, write a proportion using the fact that in similar triangles, the ratio of corresponding sides equals the ratio of corresponding heights. Solve the proportion for the area (a).

Solution

$$\frac{a}{AB}=\frac{A_{DEF}}{DE}$$
$$\frac{a}{12}=\frac{72}{18}$$
$$36\cdot\frac{a}{12}=\frac{72}{18}\cdot 36$$
$$3a=144$$
$$a=48$$

The area of triangle ABC is $48\ \text{m}^2$.

59. Strategy

To find $\overline{BC}$, write a proportion using the fact that in similar triangles, the ratios of corresponding sides are equal. Solve the proportion for $\overline{BC}$.

Solution

$$\frac{BD}{BC}=\frac{AE}{AC}$$
$$\frac{5}{BC}=\frac{8}{10}$$
$$10(BC)\frac{5}{BC}=\frac{8}{10}(BC)10$$
$$50=8BC$$
$$BC=6.25$$

The length of BC is 6.25 cm.

61. Strategy

To find $\overline{DA}$, write a proportion using the fact that in similar triangles, the ratios of corresponding sides are equal. Solve the proportion for $\overline{DA}$.

Solution

$$\frac{DE}{AC}=\frac{DB}{AB}$$
$$\frac{6}{10}=\frac{DB}{15}$$
$$30\cdot\frac{6}{10}=\frac{DB}{15}\cdot 30$$
$$18=2DB$$
$$DB=9$$
$$DA=AB-DB=15-9=6$$

The length of $\overline{DA}$ is 6 in.

63. Strategy

To find $\overline{OP}$, write a proportion using the fact that in similar triangles, the ratios of corresponding sides are equal. Solve the proportion for $\overline{OP}$.

Solution

$$\frac{OP}{MP}=\frac{OQ}{NQ}$$
$$\frac{OP}{39}=\frac{12}{36}$$
$$468\cdot\frac{OP}{39}=\frac{12}{36}\cdot 468$$
$$12OP=156$$
$$OP=13$$

The length of $\overline{OP}$ is 13 cm.

65. Strategy

To find the height of the flagpole, write a proportion using the fact that in similar triangles, the ratios of corresponding sides are equal. Solve the proportion for the height (h).

Solution

$$\frac{h}{30}=\frac{5\ 9/12}{12}$$
$$60\cdot\frac{h}{30}=\frac{5\ 9/12}{12}\cdot 60$$
$$2h=18\frac{3}{4}$$
$$h=14\frac{3}{8}$$

The height of the flagpole is 14.375 ft.

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67. Strategy To find the width of the ravine, write a proportion using the fact that in similar triangles, the ratios of corresponding sides are equal. Solve the proportion for the width (w).

Solution

$$\frac{60}{160} = \frac{w}{220}$$
$$8800 \cdot \frac{60}{160} = \frac{w}{220} \cdot 8800$$
$$3300 = 40w$$
$$w = 82.5$$

The width of the ravine is 82.5 ft.

Objective D Exercises

69. Strategy Write and solve a proportion using x to represent the number of voters who favored the amendment.

Solution

$$\frac{4}{7} = \frac{x}{35{,}000}$$
$$35{,}000 \cdot \frac{4}{7} = \frac{x}{35{,}000} \cdot 35{,}000$$
$$20{,}000 = x$$

20,000 people voted in favor of the amendment.

71. Strategy To find the number of miles between the two cities, write and solve a proportion using x to represent the distance between the two cities.

Solution

$$\frac{25}{3/8} = \frac{x}{2\ 5/8}$$
$$\frac{25}{3/8} = \frac{x}{21/8}$$
$$\frac{1}{8} \cdot \frac{25}{3/8} = \frac{1}{8} \cdot \frac{x}{21/8}$$
$$\frac{25}{3} = \frac{x}{21}$$
$$21 \cdot \frac{25}{3} = 21 \cdot \frac{x}{21}$$
$$175 = x$$

The distance between the two cities is 175 mi.

73. Strategy To find the number of millimeters of water, write and solve a proportion using x to represent the amount of water needed. If there are 4 parts water and 3 parts syrup, then there are 4 parts water for every $4 + 3 = 7$ parts soft drink.

Solution

$$\frac{4}{7} = \frac{x}{280}$$
$$280 \cdot \frac{4}{7} = 280 \cdot \frac{x}{280}$$
$$160 = x$$

160 ml of water are needed.

75. Strategy Write and solve a proportion using x to represent the required time.

Solution

$$\frac{5}{2} = \frac{45}{x}$$
$$5x = 90$$
$$x = 18$$

It takes 18 min to print 45 pages.

77. Strategy To determine the recommended area, write and solve a proportion using x to represent the area of the window.

Solution

$$\frac{5}{12} = \frac{x}{8 \cdot 12}$$
$$\frac{5}{12} = \frac{x}{96}$$
$$96\left(\frac{5}{12}\right) = 96\left(\frac{x}{96}\right)$$
$$40 = x$$

The recommended area of the window is 40 ft^2.

79. Strategy To find the height of the person, write and solve a proportion using x to represent the height of the person.

Solution

$$\frac{1}{54} = \frac{1.25}{x}$$
$$54x\left(\frac{1}{54}\right) = 54x\left(\frac{1.25}{x}\right)$$
$$x = 67.5$$

The person is 67.5 in.

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Applying the Concepts

81. Strategy: Write and solve a proportion using x to represent the first person's share of the winnings.

Solution

$$\frac{25}{90}=\frac{x}{4.5}$$
$$90x=112.5$$
$$x=1.25$$

The first person won \$1.25 million.

83. Strategy: Write and solve a proportion using x to represent the number of foul shots made by the player.

Solution

$$\frac{5}{1}=\frac{x}{42}$$
$$x=210$$

The player made 210 foul shots.

Section 8.5

Objective A Exercises

1.
$$A=\frac{1}{2}bh$$
$$2\cdot A=2\cdot\frac{1}{2}bh$$
$$2A=bh$$
$$\frac{2A}{b}=\frac{bh}{b}$$
$$\frac{2A}{b}=h$$

3.
$$d=rt$$
$$\frac{d}{r}=\frac{rt}{r}$$
$$\frac{d}{r}=t$$

5.
$$PV=nRT$$
$$\frac{PV}{nR}=\frac{nRT}{nR}$$
$$\frac{PV}{nR}=T$$

7.
$$P=2L+2W$$
$$P-2W=2L+2W-2W$$
$$P-2W=2L$$
$$\frac{P-2W}{2}=\frac{2L}{2}$$
$$\frac{P-2W}{2}=L$$

9.
$$A=\frac{1}{2}h(b_1+b_2)$$
$$2\cdot A=2\cdot\frac{1}{2}h(b_1+b_2)$$
$$2A=h(b_1+b_2)$$
$$2A=hb_1+hb_2$$
$$2A-hb_2=hb_1+hb_2-hb_2$$
$$2A-hb_2=hb_1$$
$$\frac{2A-hb_2}{h}=\frac{hb_1}{h}$$
$$\frac{2A-hb_2}{h}=b_1$$

11.
$$V=\frac{1}{3}Ah$$
$$3\cdot V=3\cdot\frac{1}{3}Ah$$
$$3V=Ah$$
$$\frac{3V}{A}=\frac{Ah}{A}$$
$$\frac{3V}{A}=h$$

13.
$$R=\frac{C-S}{t}$$
$$t\cdot R=t\cdot\frac{C-S}{t}$$
$$Rt=C-S$$
$$Rt-C=C-C-S$$
$$Rt-C=-S$$
$$-1(Rt-C)=-1(-S)$$
$$C-Rt=S$$

15.
$$A=P+Prt$$
$$A=P(1+rt)$$
$$\frac{A}{1+rt}=\frac{P(1+rt)}{1+rt}$$
$$\frac{A}{1+rt}=P$$

17.
$$A=Sw+w$$
$$A=w(S+1)$$
$$\frac{A}{S+1}=\frac{w(S+1)}{S+1}$$
$$\frac{A}{S+1}=w$$

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Applying the Concepts

19. **a.** $S = 2\pi rh + 2\pi r^2$

$S = 2\pi r(h + r)$

$\frac{S}{2\pi r} = h + r$

$\frac{S}{2\pi r} - r = h$

$h = \frac{S - 2\pi r^2}{2\pi r}$

b. $h = \frac{S - 2\pi r^2}{2\pi r} = \frac{12\pi - 2\pi(1)^2}{2\pi}$

$= \frac{10\pi}{2\pi} = 5$

The height is 5 in.

c. $h = \frac{S - 2\pi r^2}{2\pi r} = \frac{24\pi - 2\pi(2)^2}{2\pi(2)}$

$= \frac{24\pi - 8\pi}{4\pi} = \frac{16\pi}{4\pi} = 4$

The height is 4 in.

21. **a.** $B = \frac{F}{S - V}$

$(S - V)(B) = (S - V) \cdot \frac{F}{S - V}$

$(S - V)(B) = F$

$S - V = \frac{F}{B}$

$S = \frac{F}{B} + V$

$S = \frac{F + VB}{B}$

b. $S = \frac{F + VB}{B} = \frac{20{,}000 + (80)(200)}{200}$

$= \frac{36{,}000}{200} = 180$

The required selling price is \$180.

c. $S = \frac{F + VB}{B} = \frac{15{,}000 + (50)(600)}{600}$

$= \frac{45{,}000}{6} = 75$

The required selling price is \$75.

Section 8.6

Objective A Application Problems

1. Strategy

- Time to fill the fountain: t

	Rate	Time	Part
First sprinkler	$\frac{1}{3}$	t	$\frac{t}{3}$
Second sprinkler	$\frac{1}{6}$	t	$\frac{t}{6}$

- The sum of the parts filled by both sprinklers must equal 1.

Solution

$\frac{t}{3} + \frac{t}{6} = 1$

$6\left(\frac{t}{3} + \frac{t}{6}\right) = 1 \cdot 6$

$at + t = 6$

$3t = 6$

$t = 2$

Working together, it will take 2 h to fill the fountain.

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3. Strategy

- Time to remove the earth with both skiploaders working together: t

	Rate	Time	Part
First skiploader	$\frac{1}{12}$	t	$\frac{t}{12}$
Second skiploader	$\frac{1}{4}$	t	$\frac{t}{4}$

- The sum of the parts of the task completed by each skiploader must equal 1.

Solution

$$\frac{t}{12}+\frac{t}{4}=1$$
$$12\left(\frac{t}{12}+\frac{t}{4}\right)=12\cdot 1$$
$$t+3t=12$$
$$4t=12$$
$$t=3$$

With both skiploaders working together, it would take 3 h to remove the earth.

5. Strategy

- Time for the computers working together: t

	Rate	Time	Part
First computer	$\frac{1}{75}$	t	$\frac{t}{75}$
Second computer	$\frac{1}{50}$	t	$\frac{t}{50}$

- The sum of the parts of the task completed by each computer must equal 1.

Solution

$$\frac{t}{75}+\frac{t}{50}=1$$
$$150\left(\frac{t}{75}+\frac{t}{50}\right)=150\cdot 1$$
$$2t+3t=150$$
$$5t=150$$
$$t=30$$

With both computers working, it would take 30 h to solve the problem.

7. Strategy

- Time to cool the room with both air conditioners operating: t

	Rate	Time	Part
Small air conditioner	$\frac{1}{75}$	t	$\frac{t}{75}$
Large air conditioner	$\frac{1}{50}$	t	$\frac{t}{50}$

- The sum of the parts of the task completed by each air conditioner must equal 1.

Solution

$$\frac{t}{75}+\frac{t}{50}=1$$
$$150\left(\frac{t}{75}+\frac{t}{50}\right)=150\cdot 1$$
$$2t+3t=150$$
$$5t=150$$
$$t=30$$

It would take 30 min to cool the room with both air conditioners working.

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9. Strategy

- Time for the second pipeline to fill the tank: t

	Rate	Time	Part
First pipeline	$\frac{1}{45}$	30	$\frac{30}{45}$
Second pipeline	$\frac{1}{t}$	30	$\frac{30}{t}$

- The sum of the parts of the task completed by each pipeline must equal 1.

Solution

$$\frac{30}{45}+\frac{30}{t}=1$$
$$45t\left(\frac{30}{45}+\frac{30}{t}\right)=45t\cdot 1$$
$$30t+1350=45t$$
$$1350=15t$$
$$90=t$$

The second pipeline, working alone, would take 90 min.

11. Strategy

- Time for the second mason to complete the task: r

	Rate	Time	Part
First mason	$\frac{1}{10}$	6	$\frac{6}{10}$
Second mason	$\frac{1}{r}$	6	$\frac{6}{r}$

- The sum of the parts of the task completed by each mason must equal 1.

Solution

$$\frac{6}{10}+\frac{6}{r}=1$$
$$\frac{6}{r}=\frac{4}{10}$$
$$r\cdot\frac{6}{r}=\frac{4}{10}\cdot r$$
$$\frac{60}{4}=r$$
$$15=r$$

It will take the second mason, working alone, 15 h to complete the task.

13. Strategy

- Time for the second technician to complete the task: t

	Rate	Time	Part
First technician	$\frac{1}{4}$	2	$\frac{2}{4}$
Second technician	$\frac{1}{6}$	t	$\frac{t}{6}$

- The sum of the parts of the task completed by each worker must equal 1.

Solution

$$\frac{2}{4}+\frac{t}{6}=1$$
$$12\left(\frac{2}{4}+\frac{t}{6}\right)=12\cdot 1$$
$$6+2t=12$$
$$2t=6$$
$$t=3$$

It will take the second technician 3 h to complete the task.

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15. Strategy

- Time for one welder to complete the task: t

	Rate	Time	Part
First welder	$\frac{1}{t}$	10	$\frac{10}{t}$
Second welder	$\frac{1}{t}$	30	$\frac{30}{t}$

- The sum of the parts of the task completed by each welder must equal 1.

Solution

$$\frac{10}{t}+\frac{30}{t}=1$$
$$t\left(\frac{10}{t}+\frac{30}{t}\right)=t\cdot 1$$
$$10+30=t$$
$$40=t$$

It would have taken one welder 40 h to complete the task.

17. Strategy

- Time for one machine to complete the task: t

	Rate	Time	Part
First machine	$\frac{1}{t}$	7	$\frac{7}{t}$
Second machine	$\frac{1}{t}$	21	$\frac{21}{t}$

- The sum of the parts of the task completed by each machine must equal 1.

Solution

$$\frac{7}{t}+\frac{21}{t}=1$$
$$t\left(\frac{7}{t}+\frac{21}{t}\right)=t\cdot 1$$
$$7+21=t$$
$$28=t$$

It would have taken one machine 28 h to fill the boxes.

Objective B Application Problems

19. Strategy

- Rate in traffic: r
 Rate on expressway: $r+20$

	Distance	Rate	Time
In congested traffic	10	r	$\frac{10}{r}$
On expressway	20	$r+20$	$\frac{20}{r+20}$

- The total time for the trip is 1 h.

Solution

$$\frac{10}{r}+\frac{20}{r+20}=1$$
$$r(r+20)\left(\frac{10}{r}+\frac{20}{r+20}\right)=r(r+20)\cdot 1$$
$$10(r+20)+20r=r(r+20)$$
$$10r+200+20r=r^2+20r$$
$$30r+200=r^2+20r$$
$$0=r^2-10r-200$$
$$0=(r-20)(r+10)$$
$$r-20=0 \qquad r+10=0$$
$$r=20 \qquad r=-10$$

The solution -10 is not possible because the rate cannot be negative. The rate in congested traffic is 20 mph.

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21. Strategy

- Rate of jogger: r
 Rate of cyclist: $r+12$

	Distance	Rate	Time
Jogger	8	r	$\frac{8}{r}$
Cyclist	20	$r+12$	$\frac{20}{r+12}$

- The time of the jogger equals the time of the cyclist.

Solution

$$\frac{8}{r}=\frac{20}{r+12}$$
$$r(r+12)\cdot\frac{8}{r}=r(r+12)\cdot\frac{20}{r+12}$$
$$8(r+12)=20r$$
$$8r+96=20r$$
$$96=12r$$
$$8=r$$
$$20=r+12$$

The rate of the jogger is 8 mph. The rate of the cyclist is 20 mph.

23. Strategy

- Rate of the helicopter: r
 Rate of the jet: $4r$

	Distance	Rate	Time
Helicopter	180	r	$\frac{180}{r}$
Jet	1080	$4r$	$\frac{1080}{4r}$

- The total time for the trip was 5 h.

Solution

$$\frac{180}{r}+\frac{1080}{4r}=5$$
$$4r\left(\frac{180}{r}+\frac{1080}{4r}\right)=4r\cdot 5$$
$$720+1080=20r$$
$$1800=20r$$
$$90=r$$
$$4r=90\cdot 4=360$$

The rate of the jet is 360 mph.

25. Strategy

- The rate of a runner: r
 The rate of a bicyclist: $r+7$

	Distance	Rate	Time
Runner	16	r	$\frac{16}{r}$
Bicyclist	30	$r+7$	$\frac{30}{r+7}$

- The time the runner travels is equal to the time the bicyclist travels.

Solution

$$\frac{16}{r}=\frac{30}{r+7}$$
$$r(r+7)\frac{16}{r}=\frac{30}{r+7}(r+7)(r)$$
$$16r+112=30r$$
$$112=14r$$
$$8=r$$

The rate of the runner is 8 mph.

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27. Strategy

- Rate of the cyclist: r
 Rate of the car: $r + 36$

	Distance	Rate	Time
Cyclist	96	r	$\frac{96}{r}$
Car	384	$r + 36$	$\frac{384}{r + 36}$

- The time of the cyclist equals the time of the car.

Solution

$$\frac{96}{r} = \frac{384}{r+36}$$

$$r(r+36)\cdot\frac{96}{r} = r(r+36)\cdot\frac{384}{r+36}$$

$$96(r+36) = 384r$$

$$96r + 3456 = 384r$$

$$3456 = 288r$$

$$12 = r$$

$$48 = r + 36$$

The rate of the car is 48 mph.

29. Strategy

- Rate of the wind: r

	Distance	Rate	Time
With the wind	600	$180 + r$	$\frac{600}{180 + r}$
Against the wind	480	$180 - r$	$\frac{480}{180 - r}$

- The time traveling with the wind equals the time traveling against the wind.

Solution

$$\frac{600}{180+r} = \frac{480}{180-r}$$

$$(180-r)(180+r)\frac{600}{180+r} = (180-r)(180+r)\frac{480}{180-r}$$

$$108{,}000 - 600r = 86{,}400 + 480r$$

$$21{,}600 = 1080r$$

$$20 = r$$

The rate of the wind is 20 mph.

31. Strategy

- Rate of the Gulf Stream: r

	Distance	Rate	Time
With Gulf Stream	170	$28 + r$	$\frac{170}{28 + r}$
Against Gulf Stream	110	$28 - r$	$\frac{110}{28 - r}$

- The time with the Gulf Stream equals the time against the Gulf Stream.

Solution

$$\frac{170}{28+r} = \frac{110}{28-r}$$

$$170(28-r) = 110(28+r)$$

$$4760 - 170r = 3080 + 110r$$

$$1680 = 280r$$

$$6 = r$$

The rate of the Gulf Stream is 6 mph.

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33. Strategy

- Constant rate of trucker: r

	Distance	Rate	Time
Constant rate	330	r	$\frac{330}{r}$
Reduced rate	30	$r-25$	$\frac{30}{r-25}$

- The total time for the trip is 7 h.

Solution

$$\frac{330}{r}+\frac{30}{r-25}=7$$

$$r(r-25)\left(\frac{330}{r}+\frac{30}{r-25}\right)=r(r-25)\cdot 7$$

$$330(r-25)+30r=7r(r-25)$$

$$330r-8250+30r=7r^2-175r$$

$$360r-8250=7r^2-175r$$

$$0=7r^2-535r+8250$$

$$0=(7r-150)(r-55)$$

$$0=7r-150 \qquad 0=r-55$$

$$150=7r \qquad 55=r$$

$$\frac{150}{7}=r$$

$$21\frac{3}{7}=r$$

The solution $21\frac{3}{7}$ mph is not possible because the rate would be negative when the trucker had to reduce the speed by 25 mph. The rate of the trucker for the first 330 mi is 55 mph.

Applying the Concepts

35. Strategy

- Time for less-experienced apprentice to construct wall: t

	Rate	Time	Part
Mason	$\frac{1}{10}$	5	$\frac{5}{10}$
Experienced apprentice	$\frac{1}{15}$	5	$\frac{5}{15}$
Less-experienced apprentice	$\frac{1}{t}$	5	$\frac{5}{t}$

- The sum of the task completed by each mason must equal 1.

Solution

$$\frac{5}{10}+\frac{5}{15}+\frac{5}{t}=1$$

$$30t\left(\frac{5}{10}+\frac{5}{15}+\frac{5}{t}\right)=30t\cdot 1$$

$$15t+10t+150=30t$$

$$150=5t$$

$$30=t$$

If working alone, it would take the less-experienced mason 30 h to complete the job.

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37. Strategy

- Usual speed: r
 Speed in bad weather: $r-10$

	Distance	Rate	Time
Usual	150	r	$\frac{150}{r}$
Bad weather	150	$r-10$	$\frac{150}{r-10}$

- The time during the bad weather is $\frac{1}{2}$ h more than the usual time.

Solution

$$\frac{150}{r-10}=\frac{150}{r}+\frac{1}{2}$$

$$2r(r-10)\cdot\frac{150}{r-10}=2r(r-10)\left(\frac{150}{r}+\frac{1}{2}\right)$$

$$300r=300(r-10)+r(r-10)$$

$$300r=300r-3000+r^2-10r$$

$$0=r^2-10r-3000$$

$$0=(r-60)(r+50)$$

$r-60=0$ $\qquad$ $r+50=0$

$r=60$ $\qquad$ $r=-50$

The solution –50 is not possible because rate cannot be negative. The bus usually travels 60 mph.

Section 8.7

Objective A Application Problems

3. Strategy

To find the profit:

- Write the basic direct variation equation, replace the variables by the given values, and solve for k.
- Write the direct variation equation, replacing k by its value. Substitute 5000 for s and solve for P.

Solution

$$P=ks$$
$$4000=k(250)$$
$$16=k$$
$$P=16s=16(5000)=80{,}000$$

When the company sells 5000 products, the profit is \$80,000.

5. Strategy

To find the pressure:

- Write the basic direct variation equation, replace the variables by the given values, and solve for k.
- Write the direct variation equation, replacing k by its value. Substitute 15 for d and solve for p.

Solution

$$p=kd$$
$$4.5=k(10)$$
$$0.45=k$$
$$p=0.45d=0.45(15)=6.75$$

The pressure is 6.75 pounds per square inch.

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7. Strategy

To find the distance:

- Write the basic direct variation equation, replace the variables by the given values, and solve for k.
- Write the direct variation equation, replacing k by its value. Substitute 10 for t and solve for d.

Solution

$d = k(t)^2$

$144 = k(3)^2$

$16 = k$

$d = 16t = 16(10)^2 = 1600$

The object will fall 1600 ft in 10 s.

9. Strategy

To find the distance:

- Write the basic direct variation equation, replace the variables by the given values, and solve for k.
- Write the direct variation equation, replacing k by its value. Substitute 3 for t and solve for s.

Solution

$s = kt^2$

$6 = k(1)^2$

$6 = k$

$s = 6t^2 = 6(3)^2 = 6(9) = 54$

In 3 s, the ball will roll 54 ft.

11. Strategy

To find the time:

- Write the basic inverse variation equation, replace the variables by the given values, and solve for k.
- Write the inverse variation equation, replacing k by its value. Replace r with 65 and solve for t.

Solution

$t = \frac{k}{r}$

$5 = \frac{k}{65}$

$275 = k$

$t = \frac{275}{r} = \frac{275}{65} = 4.2$

It would take 4.2 hours to travel between the two cities at a rate of 65 mph.

13. Strategy

To find the pressure:

- Write the basic combined variation equation, replace the variables by the given values, and solve for k.
- Write the combined variation equation, replacing k by its value. Substitute 75 for d and 1.2 for D, and solve for p.

Solution

$p = kdD$

$150 = k(100)(1.2)$

$150 = 120k$

$1.25 = k$

$p = 1.25dD = 1.25(75)(1.2) = 112.5$

The pressure is 112.5 pounds per square inch.

15. Strategy

To find the repulsive force:

- Write the basic inverse variation equation, replace the variables by the given values, and solve for k.
- Write the inverse variation equation, replacing k by its value. Substitute 2 for d and solve for f.

Solution

$f = \frac{k}{d^2}$

$20 = \frac{k}{4^2}$

$20 = \frac{k}{16}$

$320 = k$

$f = \frac{320}{d^2} = \frac{320}{2^2} = \frac{320}{4} = 80$

The repulsive force is 80 lb when the distance is 2 in.

Applying the Concepts

17. Strategy

To find the effect on y replace x with $2x$.

Solution

$y = kx \qquad y = 2kx$

If x is doubled, then y is doubled.

19. inversely

21. inversely

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Chapter Review

1. $\dfrac{6a^2b^7}{25x^3y}+\dfrac{12a^3b^4}{5x^2y^2}=\dfrac{6a^2b^7}{25x^3y}\cdot\dfrac{5x^2y^2}{12a^3b^4}=\dfrac{b^3y}{10ax}$

2. $\dfrac{x+7}{15x}+\dfrac{x-2}{20x}=\dfrac{60x}{60x}\left(\dfrac{x+7}{15x}+\dfrac{x-2}{20x}\right)$

$=\dfrac{4x+28+3x-6}{60x}=\dfrac{7x+22}{60x}$

3. $\dfrac{x-[16/(5x-2)]}{3x-4-[88/(5x-2)]}=\dfrac{5x-2\{x-[16/(5x-2)]\}}{5x-2\{3x-4-[88/(5x-2)]\}}$

$=\dfrac{5x^2-2x-16}{15x^2-20x-6x-80}$

$=\dfrac{5x^2-2x-16}{15x^2-26x-80}$

$=\dfrac{\cancel{(5x+8)}^{1}(x-2)}{\underset{1}{\cancel{(5x+8)}}(3x-10)}=\dfrac{x-2}{3x-10}$

4. $\dfrac{x^2+x-30}{15+2x-x^2}=\dfrac{(x+6)\cancel{(x-5)}^{1}}{\underset{1}{\cancel{(x-5)}}(x+3)}=-\dfrac{x+6}{x+3}$

5. $\dfrac{16x^5y^3}{24xy^{10}}=\dfrac{2x^4}{3y^7}$

6. $\dfrac{20}{x+2}=\dfrac{5}{16}$

$16(x+2)\dfrac{20}{x+2}=16(x+2)\dfrac{5}{16}$

$320=5x+10$

$310=5x$

$x=62$

7. $\dfrac{10-23y+12y^2}{6y^2-y-5}\div\dfrac{4y^2-13y+10}{18y^2+3y-10}=\dfrac{12y^2-23y+10}{6y^2-y-5}\cdot\dfrac{18y^2+3y-10}{4y^2-13y+10}=\dfrac{(3y-2)\cancel{(4y-5)}^{1}}{(y-1)\underset{1}{\cancel{(6y+5)}}}\cdot\dfrac{\cancel{(6y+5)}^{1}(3y-2)}{\underset{1}{\cancel{(4y-5)}}(y-2)}=\dfrac{(3y-2)^2}{(y-1)(y-2)}$

8. $\dfrac{8ab^2}{15x^3y}\cdot\dfrac{5xy^4}{16a^2b}=\dfrac{by^3}{6ax^2}$

9. $\dfrac{1-(1/x)}{1-[(8x-7)/x^2]}=\dfrac{x^2[1-(1/x)]}{x^2\{1-[(8x-7)/x^2]\}}=\dfrac{x^2-x}{x^2-8x+7}$

$=\dfrac{x\cancel{(x-1)}^{1}}{(x-7)\underset{1}{\cancel{(x-1)}}}=\dfrac{x}{x-7}$

10. $\dfrac{x}{12x^2+16x-3}=\dfrac{x}{(2x+3)(6x-1)}$

$\dfrac{4x^2}{6x^2+7x-3}=\dfrac{4x^2}{(2x+3)(3x-1)}$

$\text{LCM}=(2x+3)(6x-1)(3x-1)$

$\dfrac{3x^2-x}{(2x+3)(6x-1)(3x-1)},\dfrac{24x^3-4x^2}{(2x+3)(6x-1)(3x-1)}$

11. $T=2(ab+bc+ca)=2ab+2bc+2ca$

$T-abc=2a(b+c)$

$\dfrac{T-2bc}{2(b+c)}=a$

12. $\dfrac{5}{7}+\dfrac{x}{2}=2-\dfrac{x}{7}$

$14\left(\dfrac{5}{7}+\dfrac{x}{2}\right)=14\left(2-\dfrac{x}{7}\right)$

$10+7x=28-2x$

$9x=18$

$x=2$

13. $i=\dfrac{100m}{c}$

$c(i)=\dfrac{100m}{c}(c)$

$ci=100m$

$c=\dfrac{100m}{i}$

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14.
$$\frac{x+8}{x+4}=1+\frac{5}{x+4}$$
$$(x+4)\left(\frac{x+8}{x+4}\right)=(x+4)\left(1+\frac{5}{x+4}\right)$$
$$x+8=x+4+5$$
$$0=1$$
There is no solution.

15.
$$\frac{20x^2-45x}{6x^3+4x^2}\div\frac{40x^3-90x^2}{12x^2+8x}=\frac{20x^2-45x}{6x^3+4x^2}\cdot\frac{12x^2+8x}{40x^3-90x^2}$$
$$=\frac{5x(4x-9)}{2x^2(3x+2)}\cdot\frac{4x(3x+2)}{10x^2(4x-9)}$$
$$=\frac{1}{x^2}$$

16.
$$\frac{2y}{5y-7}+\frac{3}{7-5y}=\frac{2y}{5y-7}-\frac{3}{5y+7}$$
$$=\frac{(5y-7)(5y+7)}{(5y-7)(5y+7)}\left(\frac{2y}{5y-7}-\frac{3}{5y+7}\right)$$
$$=\frac{2y(5y+7)-3(5y-7)}{(5y-7)(5y+7)}$$
$$=\frac{10y^2-y+21}{(5y-7)(5y+7)}=\frac{(2y-3)(5y+7)}{(5y-7)(5y+7)}$$
$$=\frac{2y-3}{5y-7}$$

17.
$$\frac{5x+3}{2x^2+5x-3}-\frac{3x+4}{2x^2+5x-3}=\frac{2x-1}{2x^2+5x-3}$$
$$=\frac{2x-1}{(2x-1)(x+3)}=\frac{1}{x+3}$$

18.
$$10x^2-11x+3=(5x-3)(2x-1)$$
$$20x^2-17x+3=(5x-3)(4x-1)$$
$$\text{LCM}=(5x-3)(2x-1)(4x-1)$$

19.
$$4x+9y=18$$
$$9y=-4x+18$$
$$y=-\frac{4}{9}x+2$$

20.
$$\frac{24x^2-94x+15}{12x^2-49x+15}\cdot\frac{24x^2+7x-5}{4-27x+18x^2}=\frac{(6x-1)(4x-15)}{(3x-1)(4x-15)}\cdot\frac{(8x+5)(3x-1)}{(3x-4)(6x-1)}=\frac{8x+5}{3x-4}$$

21.
$$\frac{20}{2x+3}=\frac{17x}{2x+3}-5$$
$$(2x+3)\frac{20}{2x+3}=(2x+3)\left(\frac{17x}{2x+3}-5\right)$$
$$20=17x-10x-15$$
$$35=7x$$
$$5=x$$

22.
$$\frac{x-1}{x+2}+\frac{3x-2}{5-x}+\frac{5x^2+15x-11}{x^2-3x-10}$$
$$\text{LCM}=(x-5)(x+2)$$
$$=\frac{(x-5)(x-1)-(3x-2)(x+2)+5x^2+15x-11}{(x+2)(x-5)}$$
$$=\frac{x^2-6x+5-3x^2-4x+4+5x^2+15x-11}{(x+2)(x-5)}$$
$$=\frac{3x^2+5x-2}{(x+2)(x-5)}=\frac{(3x-1)(x+2)}{(x+2)(x-5)}=\frac{3x-1}{x-5}$$

23.
$$\frac{6}{x-7}=\frac{8}{x-6}$$
$$(x-6)(x-7)\frac{6}{x-7}=(x-6)(x-7)\frac{8}{x-6}$$
$$6x-36=8x-56$$
$$20=2x$$
$$10=x$$

24.
$$\frac{3}{20}=\frac{x}{80}$$
$$80\cdot\frac{3}{20}=\frac{x}{80}\cdot 80$$
$$12=x$$

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25. Strategy

To find the perimeter:

- Find side AC by writing a proportion using the fact that the ratios of corresponding sides of similar triangles are equal.
- Use the formula for the perimeter of a triangle.

Solution

$$\frac{AC}{BC}=\frac{DF}{EF}$$
$$\frac{AC}{6}=\frac{12}{9}$$
$$18\cdot\frac{AC}{6}=\frac{12}{9}\cdot 18$$
$$3AC=24$$
$$AC=8$$
$$P=a+b+c=10+8+6=24$$

The perimeter of triangle ABC is 24 in.

26. Strategy

- Time to fill the pool: t

	Rate	Time	Part
First hose	$\frac{1}{15}$	t	$\frac{t}{15}$
Second hose	$\frac{1}{10}$	t	$\frac{t}{10}$

- The sum of the parts of the task completed by each hose must equal 1.

Solution

$$\frac{t}{15}+\frac{t}{10}=1$$
$$30\left(\frac{t}{15}+\frac{t}{10}\right)=30\cdot 1$$
$$2t+3t=30$$
$$5t=30$$
$$t=6$$

It would take 6 h to fill the pool using both hoses.

27. Strategy

- Rate of the bus: r
 Rate of the car: $r+10$

	Distance	Rate	Time
Car	315	$r+10$	$\frac{315}{r+10}$
Bus	245	r	$\frac{245}{r}$

- The time of the car equals the time of the bus.

Solution

$$\frac{315}{r+10}=\frac{245}{r}$$
$$r(r+10)\frac{315}{r+10}=r(r+10)\frac{245}{r}$$
$$315r=245r+2450$$
$$70r=2450$$
$$r=35$$
$$r+10=45$$

The rate of the car is 45 mph.

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28. Strategy

- Rate of the wind: r

	Distance	Rate	Time
With the wind	2100	$400+r$	$\frac{2100}{400+r}$
Against the wind	1900	$400-r$	$\frac{1900}{400-r}$

- The time traveling with the wind equals the time traveling against the wind.

Solution

$$\frac{2100}{400+r}=\frac{1900}{400-r}$$
$$(400+r)(400-r)\frac{2100}{400+r}=(400+r)(400-r)\frac{1900}{400-r}$$
$$840{,}000-2100r==760{,}000+1900r$$
$$80{,}000=4000r$$
$$20=r$$

The rate of the wind is 20 mph.

29. Strategy

To find the ERA write a proportion using the number of runs allowed in 100 innings. Solve for the ERA for 9 innings.

Solution

$$\frac{\text{ERA}}{9}=\frac{15}{100}$$
$$900\cdot\frac{\text{ERA}}{9}=900\cdot\frac{15}{100}$$
$$100\text{ERA}=135$$
$$\text{ERA}=1.35$$

The pitcher's ERA is 1.35.

30. Strategy

To find the current:

- Write the basic inverse variation equation, replace the variables by the given values, and solve for k.
- Write the inverse variation equation, replacing k by its value. Substitute 100 for R and solve for I.

Solution

$$I=\frac{k}{R}$$
$$4=\frac{k}{50}$$
$$200=k$$
$$I=\frac{200}{R}=\frac{200}{100}=2$$

The current is 2 amps when the resistance is 100 ohms.

Chapter Test

1. $\dfrac{16x^5y}{24x^2y^4}=\dfrac{2x^3}{3y^3}$

2. $\dfrac{x^2+4x-5}{1-x^2}=\dfrac{(x+5)(x-1)}{(1+x)(1-x)}=-\dfrac{x+5}{x+1}$

3. $\dfrac{x^3y^4}{x^2-4x+4}\cdot\dfrac{x^2-x-2}{x^6y^4}=\dfrac{x^3y^4}{(x-2)(x-2)}\cdot\dfrac{(x-2)(x+1)}{x^6y^4}$
$=\dfrac{x+1}{x^3(x-2)}$

4. $\dfrac{x^2+2x-3}{x^2+6x+9}\cdot\dfrac{2x^2-11x+5}{2x^2+3x-5}=\dfrac{(x-3)(x-1)}{(x+3)(x+3)}\cdot\dfrac{(2x-1)(x-5)}{(2x+5)(x-1)}$
$=\dfrac{(2x-1)(x-5)}{(x+3)(2x+5)}$

5. $\dfrac{x^2+3x+2}{x^2+5x+4}\div\dfrac{x^2-x-6}{x^2+2x-15}=\dfrac{x^2+3x+2}{x^2+5x+4}\cdot\dfrac{x^2+2x-15}{x^2-x-6}$
$=\dfrac{(x+2)(x+1)}{(x+4)(x+1)}\cdot\dfrac{(x+5)(x-3)}{(x-3)(x+2)}$
$=\dfrac{x+5}{x+4}$

6. $6x-3=3(2x-1)$
$2x^2+x-1=(2x-1)(x+1)$
$\text{LCM}=3(2x-1)(x+1)$

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7. $\dfrac{3}{x^2-2x} \quad \dfrac{x}{x^2-4}$

$x(x-2) \quad (x+2)(x-2)$

$\text{LCM} = x(x-2)(x+2)$

$$\frac{3(x+2)}{x(x-2)(x+2)} = \frac{3x+6}{x(x-2)(x+2)}$$

$$\frac{x(x)}{x(x-2)(x+2)} = \frac{x^2}{x(x-2)(x+2)}$$

8.
$$\frac{2x}{x^2+3x-10} - \frac{4}{x^2+3x-10} = \frac{2x-4}{x^2+3x-10}$$
$$= \frac{2(x-2)}{(x+5)(x-2)} = \frac{2}{x+5}$$

9.
$$\frac{2}{2x-1} - \frac{3}{3x+1} = \frac{2}{2x-1}\cdot\frac{3x+1}{3x+1} - \frac{3}{3x+1}\cdot\frac{2x-1}{2x-1}$$
$$= \frac{2(3x+1)-3(2x-1)}{(2x-1)(3x+1)}$$
$$= \frac{6x+2-6x+3}{(2x-1)(3x+1)} = \frac{5}{(2x-1)(3x+1)}$$

10. $\dfrac{x}{x+3} - \dfrac{2x-5}{x^2+x-6}$

$\text{LCM} = (x+3)(x-2)$

$$\frac{x}{x+3} = \frac{x(x-2)}{(x+3)(x-2)} = \frac{x^2-2x}{(x+3)(x-2)}$$

$$\frac{x}{x+3} - \frac{2x-5}{x^2+x-6} = \frac{x^2-2x}{(x+3)(x-2)} - \frac{2x-5}{(x+3)(x-2)}$$
$$= \frac{x^2-2x-(2x-5)}{(x+3)(x-2)}$$
$$= \frac{x^2-2x-2x+5}{(x+3)(x-2)} = \frac{x^2-4x+5}{(x+3)(x-2)}$$

11.
$$\frac{1+(1/x)-(12/x^2)}{1+(2/x)-(8/x^2)} = \frac{1+(1/x)-(12/x^2)}{1+(2/x)-(8/x^2)}\cdot\frac{x^2}{x^2}$$
$$= \frac{x^2+x-12}{x^2+2x-8} = \frac{(x+4)(x-3)}{(x+4)(x-2)} = \frac{x-3}{x-2}$$

12. $\dfrac{6}{x} - 2 = 1$

$$\frac{6}{x} = \frac{3}{1}$$
$$3x = 6$$
$$x = 2$$

13. $\dfrac{2x}{x+1} - 3 = \dfrac{-2}{x+1}$

$$(x+1)\left(\frac{2x}{x+1} - 3\right) = (x+1)\left(\frac{-2}{x+1}\right)$$
$$2x - 3(x+1) = -2$$
$$2x - 3x - 3 = -2$$
$$-x = 1$$
$$x = -1$$

-1 does not check. The equation has no solution.

14. $\dfrac{3}{x+4} = \dfrac{5}{x+6}$

$$5(x+4) = 3(x+6)$$
$$5x + 20 = 3x + 18$$
$$2x = -2$$
$$x = -1$$

15. Strategy

To find the area:

- Find the base (DF) of triangle DEF by writing a proportion using the fact that in similar triangles, the ratio of corresponding heights equals the ratio of corresponding sides.
- Use the formula for the area of a triangle. $h = 9$

Solution

$$\frac{5}{8} = \frac{9}{DF}$$
$$5(DF) = 8(9)$$
$$5(DF) = 72$$
$$DF = 14.4$$
$$A = \frac{1}{2}bh = \frac{1}{2}(14.4)(9) = 64.8$$

The area of triangle DEF is $64.8\ \text{m}^2$.

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16.
$$d = s + rt$$
$$d - s = rt$$
$$\frac{d-s}{r} = \frac{rt}{r}$$
$$\frac{d-s}{r} = t$$

17. **Strategy** To find the number of rolls of wallpaper, write and solve a proportion using x to represent the number of rolls.

Solution
$$\frac{2}{45} = \frac{x}{315}$$
$$315\left(\frac{2}{45}\right) = 315\left(\frac{x}{315}\right)$$
$$14 = x$$
The office requires 14 rolls of wallpaper.

18. **Strategy**

- Unknown time for both landscapers working together: t

	Rate	Time	Part
First landscaper	$\frac{1}{30}$	t	$\frac{t}{30}$
Second landscaper	$\frac{1}{15}$	t	$\frac{t}{15}$

- The sum of the part of the task completed by the first landscaper and the part of the task completed by the second landscaper is 1.

$$\frac{t}{30} + \frac{t}{15} = 1$$

Solution
$$\frac{t}{30} + \frac{t}{15} = 1$$
$$3\left(\frac{t}{30} + \frac{t}{15}\right) = 1(30)$$
$$t + 2t = 30$$
$$3t = 30$$
$$t = 10$$
Working together, the landscapers can complete the task in 10 min.

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19. Strategy

- Rate of hiker: r
 Rate of cyclist: $r+7$

	Distance	Rate	Time
Hiker	6	r	$\frac{6}{r}$
Cyclist	20	$r+7$	$\frac{20}{r+7}$

- The time the hiker hikes equals the time the cyclist cycles.

$$\frac{6}{r}=\frac{20}{r+7}$$

Solution

$$\frac{6}{r}=\frac{20}{r+7}$$
$$\frac{6}{r}[r(r+7)]=\frac{20}{r+7}[r(r+7)]$$
$$6(r+7)=20r$$
$$6r+42=20r$$
$$42=14r$$
$$3=r$$
$$10=r+7$$

The rate of the cyclist is 10 mph.

20. Strategy

To find the resistance:

- Write the basic combined variation equation, replace the variables with the given values, and solve for k.
- Write the combined variation equation, replacing k with its value. Substitute 8000 for l and d for $\frac{1}{2}$, and solve for r.

Solution

$$r=\frac{kl}{d^2}$$
$$3.2=\frac{k\cdot 16{,}000}{(1/4)^2}$$
$$0.0000125=k$$
$$r=\frac{0.0000125l}{d^2}=\frac{0.0000125(8000)}{(1/2)^2}$$
$$=0.4$$

The resistance is 0.4 ohm.

Cumulative Review

1.
$$\left(\frac{2}{3}\right)^2\div\left(\frac{3}{2}-\frac{2}{3}\right)+\frac{1}{2}=\left(\frac{2}{3}\right)^2\div\left(\frac{9}{6}-\frac{4}{6}\right)+\frac{1}{2}$$
$$=\frac{4}{9}\div\left(\frac{5}{6}\right)+\frac{1}{2}=\frac{4}{9}\cdot\frac{6}{5}+\frac{1}{2}$$
$$=\frac{2\cdot 2\cdot 2\cdot 3}{3\cdot 3\cdot 5}+\frac{1}{2}=\frac{8}{15}+\frac{1}{2}$$
$$=\frac{16}{30}+\frac{15}{30}=\frac{31}{30}$$

2.
$$-a^2+(a-b)^2=-(-2)^2+(-2-3)^2=-4+(-5)^2$$
$$=-4+25=21$$

3.
$$-2x-(-3y)+7x-5y=-2x+3y+7x-5y$$
$$=-2x+7x+3y-5y=5x-2y$$

4.
$$2[3x-7(x-3)-8]=2[3x-7x+21-8]$$
$$=2[-4x+13]=-8x+26$$

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5. $4-\frac{2}{3}x=7$

$$-\frac{2}{3}x=3$$

$$-\frac{3}{2}\left(-\frac{2}{3}x\right)=3\left(-\frac{3}{2}\right)$$

$$x=-\frac{9}{2}$$

The solution is $-\frac{9}{2}$.

6. $3[x-2(x-3)]=2(3-2x)$

$$3[x-2x+6]=6-4x$$

$$3[-x+6]=6-4x$$

$$-3x+18=6-4x$$

$$x=-12$$

The solution is –12.

7. $P\cdot B=A$

$$16\frac{2}{3}\%\cdot 60=A$$

$$\frac{1}{6}\cdot 60=A$$

$$10=A$$

8. $x-3(1-2x)\geq 1-4(3-2x)$

$$x-3+6x\geq 1-12+8x$$

$$7x-3\geq 8x-11$$

$$-x-3\geq -11$$

$$-x\geq -8$$

$$(-1)(-x)\leq(-1)(-8)$$

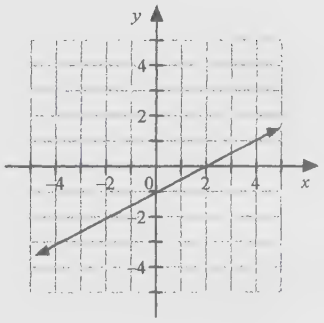
$$x\leq 8$$

$\{x\,|\,x\leq 8\}$

9. $V=LWH=10(5)(4)=50(4)=200$

The volume is 200 ft^3.

10. $x-2y=2$



11. $P(x)=\frac{x-1}{2x-3}$

$$P(-2)=\frac{-2-1}{2(-2)-3}=\frac{-3}{-4-3}=\frac{-3}{-7}=\frac{3}{7}$$

12. $3x-2y=6$

$$-2y=-3x+6$$

$$y=\frac{3}{2}x-3$$

$$y-y_1=m(x-x_1)$$

$$y-(-1)=\frac{3}{2}[x-(-2)]$$

$$y+1=\frac{3}{2}(x+2)$$

$$y+1=\frac{3}{2}x+3$$

$$y=\frac{3}{2}x+2$$

13. $\begin{vmatrix}6 & 5\\ 2 & -3\end{vmatrix}=6(-3)-5\cdot 2=-18-10$

$$=-28$$

14. $(a^2b^5)(ab^2)=a^3b^7$

15. $\frac{(2a^{-2}b^3)^{-2}}{(4a)^{-1}}=\frac{2^{-2}a^4b^{-6}}{4^{-1}a^{-1}}$

$$=2^{-2}4a^{4-(-1)}b^{-6}$$

$$=\frac{4a^5}{4b^6}=\frac{a^5}{b^6}$$

16. $0.000000035=3.5\times 10^{-8}$

17. $(2a^2-3a+1)(-2a^2)=-4a^4+6a^3-2a^2$

18. $(a-3b)(a+4b)=a^2+4ab-3ab-12b^2$

$$=a^2+ab-12b^2$$

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19.
$$\begin{array}{r} x^2+2x+4 \\ x-2\overline{)\,x^3+0x^2+0x-8} \\ \underline{x^3-2x^2} \\ 2x^2+0x \\ \underline{2x^2-4x} \\ 4x-8 \\ \underline{4x-8} \\ 0 \end{array}$$
$(x^3-8)\div(x-2)=x^2+2x+4$

20. y^2-7y+6
$1(6)=6$
Factors of 6 whose sum is –7: –6 and –1
$y^2-6y-y+6$
$y(y-6)-1(y-6)=(y-6)(y-1)$

21. $12x^2-x-1$
$12\cdot(-1)=-12$
Factors of –12 whose sum is –1: –4 and 3
$12x^2-3x+3x-1$
$4x(3x-1)+1(3x-1)=(3x-1)(4x+1)$

22. $2a^3+7a^2-15a$
The GCF is a: $a(2a^2+7a-15)$
$2\cdot(-15)=-30$
Factors of –30 whose sum is 7: 10 and –3
$a(2a^2+10a-3a-15)$
$a[2a(a+5)-3(a+5)]=a(a+5)(2a-3)$

23. $4b^2-100$
The GCF is 4: $4(b^2-25)$
$4[b^2-(5)^2]=4(b+5)(b-5)$

24. $(x+3)(2x-5)=0$
$x+3=0 \qquad 2x-5=0$
$x=-3 \qquad 2x=5$
$x=\frac{5}{2}$

25. $\frac{12x^4y^2}{18xy^7}=\frac{2x^3}{3y^5}$

26. $\frac{x^2-7x+10}{25-x^2}=\frac{(x-5)(x-2)}{(5-x)(5+x)}=-\frac{x-2}{x+5}$

27. $\frac{x^2-x-56}{x^2+8x+7}\div\frac{x^2-13x+40}{x^2-4x-5}=\frac{x^2-x-56}{x^2+8x+7}\cdot\frac{x^2-4x-5}{x^2-13x+40}=\frac{(x-8)(x+7)}{(x+7)(x+1)}\cdot\frac{(x-5)(x+1)}{(x-8)(x-5)}=1$

28. The GCF is $(2x-1)(x+1)$
$$\frac{2}{2x-1}-\frac{1}{x+1}=\frac{2}{2x-1}\cdot\frac{x+1}{x+1}-\frac{1}{x+1}\cdot\frac{2x-1}{2x-1}$$
$$=\frac{2(x+1)-1(2x-1)}{(2x-1)(x+1)}$$
$$=\frac{2x+2-2x+1}{(2x-1)(x+1)}=\frac{3}{(2x-1)(x+1)}$$

29. $\frac{1-(2/x)-(15/x^2)}{1-(25/x^2)}=\frac{1-(2/x)-(15/x^2)}{1-(25/x^2)}\cdot\frac{x^2}{x^2}$
$$=\frac{x^2-2x-15}{x^2-25}=\frac{(x-5)(x+3)}{(x-5)(x+5)}=\frac{x+3}{x+5}$$

30.
$$\frac{3x}{x-3}-2=\frac{10}{x-3}$$
$$(x-3)\left(\frac{3x}{x-3}-2\right)=(x-3)\left(\frac{10}{x-3}\right)$$
$$3x-2(x-3)=10$$
$$3x-2x+6=10$$
$$x+6=10$$
$$x=4$$

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31. Strategy

- Percent of silver in the alloy: x

	Amount	Percent	Quantity
40% silver	60	0.40	0.40(60)
Silver alloy	120	x	$120x$
Mixture	180	0.60	0.60(180)

- The sum of the quantities before mixing is equal to the quantity after mixing.

Solution

$$0.40(60) + 120x = 0.60(180)$$
$$24 + 120x = 108$$
$$120x = 84$$
$$x = 0.70$$

The silver alloy is 70% silver.

32. Strategy Write and solve a proportion using x to represent the cost for a \$5000 policy.

Solution

$$\frac{32}{1000} = \frac{x}{5000}$$
$$\frac{\cancel{5000}}{1} \cdot \frac{32}{\cancel{1000}_{1}} = \frac{x}{\cancel{5000}_{1}} \cdot \frac{\cancel{5000}}{1}$$
$$80 = x$$

The cost of a \$5000 policy is \$160.

33. Strategy

- Time for both pipes working together: t

	Rate	Time	Part
First pipe	$\frac{1}{9}$	t	$\frac{t}{9}$
Second pipe	$\frac{1}{18}$	t	$\frac{t}{18}$

- The sum of the parts of the task completed by each pipe must equal 1..

Solution

$$\frac{t}{9} + \frac{t}{18} = 1$$
$$18\left(\frac{t}{9} + \frac{t}{18}\right) = 18 \cdot 1$$
$$2t + t = 18$$
$$3t = 18$$
$$t = 6$$

It would take both pipes, working together, 6 min to fill the tank.

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Chapter 9

Rational Exponents and Radicals

Prep Test

1. $48 \div 16 = 3$
$48 = 16 \cdot 3$

2. $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$

3. $6\left(\frac{3}{2}\right) = 9$

4. $\frac{1}{2} - \frac{2}{3} + \frac{1}{4} = \frac{6}{12} - \frac{8}{12} + \frac{3}{12} = \frac{1}{12}$

5. $(3 - 7x) - (4 - 2x)$
$= 3 - 7x - 4 + 2x$
$= -5x - 1$

6. $\frac{3x^5y^6}{12x^4y} = \frac{xy^5}{4}$

7. $(3x - 2)^2$
$= (3x - 2)(3x - 2)$
$= 9x^2 - 12x + 4$

8. $(2 + 4x)(5 - 3x)$
$= 10 + 14x - 12x^2$
$= -12x^2 + 14x + 10$

9. $(6x - 1)(6x - 1) = 36x^2 - 1$

10 $x^2 - 14x - 5 = 10$
$x^2 - 14x - 15 = 0$
$(x - 15)(x + 1) = 0$
$x - 15 = 0 \quad x + 1 = 0$
$x = 15 \quad x = -1$
The solutions are –1 and 15.

Go Figure

There are less than 10 tables. If you seat 5 people at each table, and there are two people at the last table, you may have as many as 47 guests or as few as 7 guests. However, since we also know that if we seat 3 people at each table and have 9 people with nowhere to sit, we know that neither 47 nor 7 is the solution.
The number of guests can be written as the expression $5x + 2$ from the first sentence and $3y + 9$, from the second sentence, where x and y represent the number of tables and are integers less than 10. We want to find the value of the expressions when they are equal.
One way to approach the solution is to use trial and error and try all values of tables until a solution is determined:

$5(9) + 2 = 47$	$3(9) + 9 = 36$
$5(8) + 2 = 42$	$3(8) + 9 = 33$
$5(7) + 2 = 37$	$3(7) + 9 = 30$
$5(6) + 2 = 32$	$3(6) + 9 = 27$
$5(5) + 2 = 27$	

There are 27 guests.

Section 9.1

Objective A Exercises

1. $8^{1/3} = \left(2^3\right)^{1/3} = 2$

3. $9^{3/2} = \left(3^2\right)^{3/2} = 3^3 = 27$

5. $27^{-2/3} = \left(3^3\right)^{-2/3} = 3^{-2}$
$= \frac{1}{3^2} = \frac{1}{9}$

7. $32^{2/5} = \left(2^5\right)^{2/5} = 2^2 = 4$

9. $(-25)^{5/2}$
The base of the exponential expression is a negative number, and the denominator of the exponent is an even number. Therefore, $(-25)^{5/2}$ is not a real number.

11. $\left(\frac{25}{49}\right)^{-3/2} = \left(\frac{5^2}{7^2}\right)^{-3/2} = \left[\left(\frac{5}{7}\right)^2\right]^{-3/2} = \left(\frac{5}{7}\right)^{-3} = \frac{5^{-3}}{7^{-3}}$
$= \frac{7^3}{5^3} = \frac{343}{125}$

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13. $x^{1/2}x^{1/2} = x$

15. $y^{-1/4}y^{3/4} = y^{-1/4+3/4} = y^{1/2}$

17. $x^{-2/3} \cdot x^{3/4} = x^{-8/12+9/12} = x^{1/12}$

19. $a^{1/3} \cdot a^{3/4} \cdot a^{-1/2} = a^{4/12+9/12-6/12} = a^{7/12}$

21. $\dfrac{a^{1/2}}{a^{3/2}} = a^{1/2-3/2} = a^{-1} = \dfrac{1}{a}$

23. $\dfrac{y^{-3/4}}{y^{1/4}} = y^{-3/4-1/4} = y^{-1} = \dfrac{1}{y}$

25. $\dfrac{y^{2/3}}{y^{-5/6}} = y^{4/6+5/6} = y^{9/6} = y^{3/2}$

27. $\left(x^{2}\right)^{-1/2} = x^{-1} = \dfrac{1}{x}$

29. $\left(x^{-2/3}\right)^{6} = x^{-4} = \dfrac{1}{x^{4}}$

31. $\left(a^{-1/2}\right)^{-2} = a$

33. $\left(x^{-3/8}\right)^{-4/5} = x^{3/10}$

35. $\left(a^{1/2} \cdot a\right)^{2} = \left(a^{3/2}\right)^{2} = a^{3}$

37. $\left(x^{-1/2}x^{3/4}\right)^{-2} = \left(x^{1/4}\right)^{-2} = x^{-1/2} = \dfrac{1}{x^{1/2}}$

39. $\left(y^{-1/2}y^{2/3}\right)^{2/3} = \left(y^{1/6}\right)^{2/3} = y^{1/9}$

41. $\left(x^{8}y^{2}\right)^{1/2} = x^{4}y$

43. $\left(x^{4}y^{2}z^{6}\right)^{3/2} = x^{6}y^{3}z^{9}$

45. $\left(x^{-3}y^{6}\right)^{-1/3} = xy^{-2} = \dfrac{x}{y^{2}}$

47. $\left(x^{-2}y^{1/3}\right)^{-3/4} = x^{3/2}y^{-1/4} = \dfrac{x^{3/2}}{y^{1/4}}$

49. $\left(\dfrac{x^{1/2}}{y^{2}}\right)^{4} = \dfrac{x^{2}}{y^{8}}$

51. $\dfrac{x^{1/4} \cdot x^{-1/2}}{x^{2/3}} = \dfrac{x^{-1/4}}{x^{2/3}} = x^{-11/12} = \dfrac{1}{x^{11/12}}$

53. $\left(\dfrac{y^{2/3} \cdot y^{-5/6}}{y^{1/9}}\right)^{9} = \left(\dfrac{y^{-1/6}}{y^{1/9}}\right)^{9} = \left(y^{-5/18}\right)^{9} = y^{-5/2} = \dfrac{1}{y^{5/2}}$

55. $\left(\dfrac{b^{2} \cdot b^{-3/4}}{b^{-1/2}}\right)^{-1/2} = \left(\dfrac{b^{5/4}}{b^{-1/2}}\right)^{-1/2} = \left(b^{7/4}\right)^{-1/2} = b^{-7/8} = \dfrac{1}{b^{7/8}}$

57. $\left(a^{2/3}b^{2}\right)^{6}\left(a^{3}b^{3}\right)^{1/3} = \left(a^{4}b^{12}\right)(ab) = a^{5}b^{13}$

59. $\left(16m^{-2}n^{4}\right)^{-1/2}\left(mn^{1/2}\right) = \left(2^{4}\right)^{-1/2} mn^{-2} \cdot mn^{1/2} = 2^{-2}m^{2}n^{-3/2} = \dfrac{m^{2}}{2^{2}n^{3/2}} = \dfrac{m^{2}}{4n^{3/2}}$

61. $\left(\dfrac{x^{1/2}y^{-3/4}}{y^{2/3}}\right)^{-6} = \left(x^{1/2}y^{-17/12}\right)^{-6} = x^{-3}y^{17/2} = \dfrac{y^{17/2}}{x^{3}}$

63. $\left(\dfrac{2^{-6}b^{-3}}{a^{-1/2}}\right)^{-2/3} = \dfrac{2^{4}b^{2}}{a^{1/3}} = \dfrac{16b^{2}}{a^{1/3}}$

65. $y^{3/2}\left(y^{1/2} - y^{-1/2}\right) = y^{4/2} - y^{2/2} = y^{2} - y$

67. $a^{-1/4}\left(a^{5/4} - a^{9/4}\right) = a^{1} - a^{2} = a - a^{2}$

69. $x^{n} \cdot x^{3n} = x^{4n}$

71. $x^{n} \cdot x^{n/2} = x^{3n/2}$

73. $\dfrac{y^{n/2}}{y^{-n}} = y^{3n/2}$

75. $\left(x^{2n}\right)^{n} = x^{2n^{2}}$

77. $\left(x^{n/4}y^{n/8}\right)^{8} = x^{2n}y^{n}$

79. $\left(x^{n/5}y^{n/10}\right)^{20} = x^{4n}y^{2n}$

Objective B Exercises

81. $3^{1/4} = \sqrt[4]{3}$

83. $a^{3/2} = \left(a^{3}\right)^{1/2} = \sqrt{a^{3}}$

85. $(2t)^{5/2} = \sqrt{(2t)^{5}} = \sqrt{32t^{5}}$

87. $-2x^{2/3} = -2\left(x^{2}\right)^{1/3} = -2\sqrt[3]{x^{2}}$

89. $\left(a^{2}b\right)^{2/3} = \sqrt[3]{\left(a^{2}b\right)^{2}} = \sqrt[3]{a^{4}b^{2}}$

91. $\left(a^{2}b^{4}\right)^{3/5} = \sqrt[5]{\left(a^{2}b^{4}\right)^{3}} = \sqrt[5]{a^{6}b^{12}}$

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93. $(4x-3)^{3/4} = \sqrt[4]{(4x-3)^3}$ **95.** $x^{-2/3} = \dfrac{1}{x^{2/3}} = \dfrac{1}{\sqrt[3]{x^2}}$ **97.** $\sqrt{14} = 14^{1/2}$ **99.** $\sqrt[3]{x} = x^{1/3}$

101. $\sqrt[3]{x^4} = x^{4/3}$ **103.** $\sqrt[5]{b^3} = b^{3/5}$ **105.** $\sqrt[3]{2x^2} = (2x^2)^{1/3}$ **107.** $-\sqrt{3x^5} = -(3x^5)^{1/2}$

109. $3x\sqrt[3]{y^2} = 3xy^{2/3}$ **111.** $\sqrt{a^2-2} = (a^2-2)^{1/2}$

Objective C Exercises

113. $\sqrt{x^{16}} = x^8$ **115.** $-\sqrt{x^8} = -x^4$ **117.** $\sqrt[3]{x^3y^9} = xy^3$ **119.** $-\sqrt[3]{x^{15}y^3} = -x^5y$

121. $\sqrt{16a^4b^{12}} = \sqrt{2^4a^4b^{12}}$
$= 2^2a^2b^6$
$= 4a^2b^6$

123. $\sqrt{-16x^4y^2}$
The square root of a negative number is not a real number, since the square of a real number must be positive or zero.
Therefore, $\sqrt{-16x^4y^2}$ is not a real number unless x or $y = 0$.

125. $\sqrt[3]{27x^9} = \sqrt[3]{3^3x^9} = 3x^3$ **127.** $\sqrt[3]{-64x^9y^{12}} = \sqrt[3]{(-4)^3x^9y^{12}} = -4x^3y^4$ **129.** $-\sqrt[4]{x^8y^{12}} = -x^2y^3$

131. $\sqrt[5]{x^{20}y^{10}} = x^4y^2$ **133.** $\sqrt[4]{81x^4y^{20}} = \sqrt[4]{3^4x^4y^{20}} = 3xy^5$ **135.** $\sqrt[5]{32a^5b^{10}} = \sqrt[5]{2^5a^5b^{10}} = 2ab^2$

Applying the Concepts

137. **a.** $\sqrt{(-2)^2} = -2$, false
$\sqrt{(-2)^2} = \sqrt{4} = 2$

b. $\sqrt[3]{(-3)^3} = -3$, true

c. $\sqrt[n]{a} = a^{1/n}$, true

d. $\sqrt[n]{a^n + b^n} = a + b$, false
$\sqrt[n]{a^n + b^n} = (a^n + b^n)^{1/n}$

e. $(a^{1/2} + b^{1/2})^2 = a + b$, false
$(a^{1/2} + b^{1/2})^2 = a + 2a^{1/2}b^{1/2} + b$

f. $\sqrt[m]{a^n} = a^{mn}$, false
$\sqrt[m]{a^n} = a^{n/m}$

Section 9.2

Objective A Exercises

1. $\sqrt{x^4y^3z^5} = \sqrt{x^4y^2z^4(yz)} = \sqrt{x^4y^2z^4}\sqrt{yz} = x^2yz^2\sqrt{yz}$

3. $\sqrt{8a^3b^8} = \sqrt{2^3a^3b^8} = \sqrt{2^2a^2b^8(2a)} = \sqrt{2^2a^2b^8}\sqrt{2a}$
$= 2ab^4\sqrt{2a}$

5. $\sqrt{45x^2y^3z^5} = \sqrt{3^2 \cdot 5x^2y^3z^5} = \sqrt{3^2x^2y^2z^4(5yz)}$
$= \sqrt{3^2x^2y^2z^4}\sqrt{5yz} = 3xyz^2\sqrt{5yz}$

7. $\sqrt{-9x^3}$
The square root of a negative number is not a real number, since the square of a real number must be positive.
Therefore, $\sqrt{-9x^3}$ is not a real number.

9. $\sqrt[3]{a^{16}b^8} = \sqrt[3]{a^{15}b^6(ab^2)} = \sqrt[3]{a^{15}b^6}\sqrt[3]{ab^2} = a^5b^2\sqrt[3]{ab^2}$

11. $\sqrt[3]{-125x^2y^4} = \sqrt[3]{(-5)^3x^2y^4} = \sqrt[3]{(-5)^3y^3(x^2y)}$
$= \sqrt[3]{(-5)^3y^3}\sqrt[3]{x^2y} = -5y\sqrt[3]{x^2y}$

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13. $\sqrt[3]{a^4b^5c^6} = \sqrt[3]{a^3b^3c^6(ab^2)} = \sqrt[3]{a^3b^3c^6}\sqrt[3]{ab^2} = abc^2\sqrt[3]{ab^2}$

15. $\sqrt[4]{16x^9y^5} = \sqrt[4]{2^4x^9y^5} = \sqrt[4]{2^4x^8y^4(xy)} = \sqrt[4]{2^4x^8y^4}\sqrt[4]{xy}$
$= 2x^2y\sqrt[4]{xy}$

Objective B Exercises

17. $2\sqrt{x} - 8\sqrt{x} = -6\sqrt{x}$

19. $\sqrt{8} - \sqrt{32} = \sqrt{2^3} - \sqrt{2^5}$
$= \sqrt{2^2}\sqrt{2} - \sqrt{2^4}\sqrt{2}$
$= 2\sqrt{2} - 2^2\sqrt{2}$
$= 2\sqrt{2} - 4\sqrt{2} = -2\sqrt{2}$

21. $\sqrt{18b} + \sqrt{75b} = \sqrt{2\cdot 3^2 b} + \sqrt{3\cdot 5^2 b} = \sqrt{3^2}\sqrt{2b} + \sqrt{5^2}\sqrt{3b}$
$= 3\sqrt{2b} + 5\sqrt{3b}$

23. $3\sqrt{8x^2y^3} - 2x\sqrt{32y^3} = 3\sqrt{2^3x^2y^3} - 2x\sqrt{2^5y^3} = 3\sqrt{2^2x^2y^2}\sqrt{2y} - 2x\sqrt{2^4y^2}\sqrt{2y} = 3\cdot 2xy\sqrt{2y} - 2x\cdot 2^2y\sqrt{2y}$
$= 6xy\sqrt{2y} - 8xy\sqrt{2y} = -2xy\sqrt{2y}$

25. $2a\sqrt{27ab^5} + 3b\sqrt{3a^3b} = 2a\sqrt{3^3ab^5} + 3b\sqrt{3a^3b} = 2a\sqrt{3^2b^4}\sqrt{3ab} + 3b\sqrt{a^2}\sqrt{3ab} = 2a\cdot 3b^2\sqrt{3ab} + 3ab\sqrt{3ab}$
$= 6ab^2\sqrt{3ab} + 3ab\sqrt{3ab}$

27. $\sqrt[3]{16} - \sqrt[3]{54} = \sqrt[3]{2^4} - \sqrt[3]{2\cdot 3^3} = \sqrt[3]{2^3}\sqrt[3]{2} - \sqrt[3]{3^3}\sqrt[3]{2}$
$= 2\sqrt[3]{2} - 3\sqrt[3]{2} = -\sqrt[3]{2}$

29. $2b\sqrt[3]{16b^2} + \sqrt[3]{128b^5} = 2b\sqrt[3]{2^4b^2} + \sqrt[3]{2^7b^5} = 2b\sqrt[3]{2^3}\sqrt[3]{2b^2} + \sqrt[3]{2^6b^3}\sqrt[3]{2b^2} = 2b\cdot 2\sqrt[3]{2b^2} + 2^2b\sqrt[3]{2b^2} = 4b\sqrt[3]{2b^2} + 4b\sqrt[3]{2b^2} = 8b\sqrt[3]{2b^2}$

31. $3\sqrt[4]{32a^5} - a\sqrt[4]{162a} = 3\sqrt[4]{2^5a^5} - a\sqrt[4]{2\cdot 3^4a} = 3\sqrt[4]{2^4a^4}\sqrt[4]{2a} - a\sqrt[4]{3^4}\sqrt[4]{2a} = 3\cdot 2a\sqrt[4]{2a} - 3a\sqrt[4]{2a} = 6a\sqrt[4]{2a} - 3a\sqrt[4]{2a} = 3a\sqrt[4]{2a}$

33. $2\sqrt{50} - 3\sqrt{125} + \sqrt{98} = 2\sqrt{2\cdot 5^2} - 3\sqrt{5^3} + \sqrt{2\cdot 7^2} = 2\sqrt{5^2}\sqrt{2} - 3\sqrt{5^2}\sqrt{5} + \sqrt{7^2}\sqrt{2} = 2\cdot 5\sqrt{2} - 3\cdot 5\sqrt{5} + 7\sqrt{2}$
$= 10\sqrt{2} - 15\sqrt{5} + 7\sqrt{2} = 17\sqrt{2} - 15\sqrt{5}$

35. $\sqrt{9b^3} - \sqrt{25b^3} + \sqrt{49b^3} = \sqrt{3^2b^3} - \sqrt{5^2b^3} + \sqrt{7^2b^3} = \sqrt{3^2b^2}\sqrt{b} - \sqrt{5^2b^2}\sqrt{b} + \sqrt{7^2b^2}\sqrt{b} = 3b\sqrt{b} - 5b\sqrt{b} + 7b\sqrt{b} = 5b\sqrt{b}$

37. $2x\sqrt{8xy^2} - 3y\sqrt{32x^3} + \sqrt{4x^3y^3} = 2x\sqrt{2^3xy^2} - 3y\sqrt{2^5x^3} + \sqrt{2^2x^3y^3} = 2x\sqrt{2^2y^2}\sqrt{2x} - 3y\sqrt{2^4x^2}\sqrt{2x} + \sqrt{2^2x^2y^2}\sqrt{xy}$
$= 2x\cdot 2y\sqrt{2x} - 3y\cdot 2^2x\sqrt{2x} + 2xy\sqrt{xy} = 4xy\sqrt{2x} - 12xy\sqrt{2x} + 2xy\sqrt{xy}$
$= -8xy\sqrt{2x} + 2xy\sqrt{xy}$

39. $\sqrt[3]{54xy^3} - 5\sqrt[3]{2xy^3} + y\sqrt[3]{128x} = \sqrt[3]{2\cdot 3^3xy^3} - 5\sqrt[3]{2xy^3} + y\sqrt[3]{2^7x} = \sqrt[3]{3^3y^3}\sqrt[3]{2x} - 5\sqrt[3]{y^3}\sqrt[3]{2x} + y\sqrt[3]{2^6}\sqrt[3]{2x}$
$= 3y\sqrt[3]{2x} - 5y\sqrt[3]{2x} + 2^2y\sqrt[3]{2x} = 3y\sqrt[3]{2x} - 5y\sqrt[3]{2x} + 4y\sqrt[3]{2x} = 2y\sqrt[3]{2x}$

41. $2a\sqrt[4]{32b^5} - 3b\sqrt[4]{162a^4b} + \sqrt[4]{2a^4b^5} = 2a\sqrt[4]{2^5b^5} - 3b\sqrt[4]{2\cdot 3^4a^4b} + \sqrt[4]{2a^4b^5} = 2a\sqrt[4]{2^4b^4}\sqrt[4]{2b} - 3b\sqrt[4]{3^4a^4}\sqrt[4]{2b} + \sqrt[4]{a^4b^4}\sqrt[4]{2b}$
$= 2a\cdot 2b\sqrt[4]{2b} - 3b\cdot 3a\sqrt[4]{2b} + ab\sqrt[4]{2b} = 4ab\sqrt[4]{2b} - 9ab\sqrt[4]{2b} + ab\sqrt[4]{2b} = -4ab\sqrt[4]{2b}$

Objective C Exercises

43. $\sqrt{8}\sqrt{32} = \sqrt{256} = \sqrt{2^8} = 2^4 = 16$

45. $\sqrt[3]{4}\sqrt[3]{8} = \sqrt[3]{32} = \sqrt[3]{2^5} = \sqrt[3]{2^3}\sqrt[3]{2^2} = 2\sqrt[3]{4}$

47. $\sqrt{x^2y^5}\sqrt{xy} = \sqrt{x^3y^6} = \sqrt{x^2y^6}\sqrt{x}$
$= xy^3\sqrt{x}$

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49. $\sqrt{2x^2y}\sqrt{32xy} = \sqrt{64x^3y^2} = \sqrt{2^6x^3y^2} = \sqrt{2^6x^2y^2}\sqrt{x}$
$= 2^3xy\sqrt{x} = 8xy\sqrt{x}$

51. $\sqrt[3]{x^2y}\sqrt[3]{16x^4y^2} = \sqrt[3]{16x^6y^3} = \sqrt[3]{2^4x^6y^3} = \sqrt[3]{2^3x^6y^3}\sqrt[3]{2}$
$= 2x^2y\sqrt[3]{2}$

53. $\sqrt[4]{12ab^3}\sqrt[4]{4a^5b^2} = \sqrt[4]{48a^6b^5} = \sqrt[4]{2^4\cdot 3a^6b^5}$
$= \sqrt[4]{2^4a^4b^4}\sqrt[4]{3a^2b} = 2ab\sqrt[4]{3a^2b}$

55. $\sqrt{3}\left(\sqrt{27}-\sqrt{3}\right) = \sqrt{81}-\sqrt{9} = \sqrt{3^4}-\sqrt{3^2}$
$= 3^2-3 = 9-3 = 6$

57. $\sqrt{x}\left(\sqrt{x}-\sqrt{2}\right) = \sqrt{x^2}-\sqrt{2x} = x-\sqrt{2x}$

59. $\sqrt{2x}\left(\sqrt{8x}-\sqrt{32}\right) = \sqrt{16x^2}-\sqrt{64x} = \sqrt{2^4x^2}-\sqrt{2^6x}$
$= 2^2x-2^3\sqrt{x} = 4x-8\sqrt{x}$

61. $\left(\sqrt{x}-3\right)^2 = \left(\sqrt{x}\right)^2-3\sqrt{x}-3\sqrt{x}+9 = x-6\sqrt{x}+9$

63. $\left(4\sqrt{5}+2\right)^2 = \left(4\sqrt{5}\right)^2+8\sqrt{5}+8\sqrt{5}+4 = 16\cdot 5+16\sqrt{5}+4$
$= 80+16\sqrt{5}+4 = 84+16\sqrt{5}$

65. $2\sqrt{14xy}\cdot 4\sqrt{7x^2y}\cdot 3\sqrt{8xy^2} = 24\sqrt{784x^4y^4}$
$= 24\sqrt{2^4\cdot 7^2x^4y^4}$
$= 24\cdot 2^2\cdot 7x^2y^2 = 672x^2y^2$

67. $\sqrt[3]{2a^2b}\sqrt[3]{4a^3b^2}\sqrt[3]{8a^5b^6} = \sqrt[3]{64a^{10}b^9} = \sqrt[3]{2^6a^{10}b^9}$
$= \sqrt[3]{2^6a^9b^9}\sqrt[3]{a} = 2^2a^3b^3\sqrt[3]{a}$
$= 4a^3b^3\sqrt[3]{a}$

69. $\left(\sqrt{5}-5\right)\left(2\sqrt{5}+2\right) = 2\sqrt{5^2}+2\sqrt{5}-10\sqrt{5}-10$
$= 2\cdot 5-8\sqrt{5}-10 = 10-8\sqrt{5}-10 = -8\sqrt{5}$

71. $\left(\sqrt{x}-y\right)\left(\sqrt{x}+y\right) = \sqrt{x^2}-y^2 = x-y^2$

73. $\left(2\sqrt{3x}-\sqrt{y}\right)\left(2\sqrt{3x}+\sqrt{y}\right) = 4\sqrt{3^2x^2}-\sqrt{y^2} = 4\cdot 3x-y = 12x-y$

Objective D Exercises

77. $\dfrac{\sqrt{60y^4}}{\sqrt{12y}} = \sqrt{\dfrac{60y^4}{12y}} = \sqrt{5y^3} = \sqrt{y^2}\sqrt{5y} = y\sqrt{5y}$

79. $\dfrac{\sqrt{65ab^4}}{\sqrt{5ab}} = \sqrt{\dfrac{65ab^4}{5ab}} = \sqrt{13b^3} = \sqrt{b^2}\sqrt{13b} = b\sqrt{13b}$

81. $\dfrac{1}{\sqrt{2}} = \dfrac{1}{\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\sqrt{2}}{\sqrt{2^2}} = \dfrac{\sqrt{2}}{2}$

83. $\dfrac{2}{\sqrt{3y}} = \dfrac{2}{\sqrt{3y}}\cdot\dfrac{\sqrt{3y}}{\sqrt{3y}} = \dfrac{2\sqrt{3y}}{\sqrt{3^2y^2}}$
$= \dfrac{2\sqrt{3y}}{3y}$

85. $\dfrac{9}{\sqrt{3a}} = \dfrac{9}{\sqrt{3a}}\cdot\dfrac{\sqrt{3a}}{\sqrt{3a}} = \dfrac{9\sqrt{3a}}{\sqrt{3^2a^2}} = \dfrac{9\sqrt{3a}}{3a}$
$= \dfrac{3\sqrt{3a}}{a}$

87. $\sqrt{\dfrac{y}{2}} = \dfrac{\sqrt{y}}{\sqrt{2}} = \dfrac{\sqrt{y}}{\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\sqrt{2y}}{\sqrt{2^2}} = \dfrac{\sqrt{2y}}{2}$

89. $\dfrac{5}{\sqrt[3]{9}} = \dfrac{5}{\sqrt[3]{3^2}}\cdot\dfrac{\sqrt[3]{3}}{\sqrt[3]{3}} = \dfrac{5\sqrt[3]{3}}{\sqrt[3]{3^3}} = \dfrac{5\sqrt[3]{3}}{3}$

91. $\dfrac{5}{\sqrt[3]{3y}} = \dfrac{5}{\sqrt[3]{3y}}\cdot\dfrac{\sqrt[3]{3^2y^2}}{\sqrt[3]{3^2y^2}} = \dfrac{5\sqrt[3]{3^2y^2}}{\sqrt[3]{3^3y^3}} = \dfrac{5\sqrt[3]{9y^2}}{3y}$

93. $\dfrac{\sqrt{15a^2b^5}}{\sqrt{30a^5b^3}} = \sqrt{\dfrac{15a^2b^5}{30a^5b^3}} = \sqrt{\dfrac{b^2}{2a^3}} = \dfrac{b}{\sqrt{a^2}\sqrt{2a}}$
$= \dfrac{b}{a\sqrt{2a}}\cdot\dfrac{\sqrt{2a}}{\sqrt{2a}} = \dfrac{b\sqrt{2a}}{a\sqrt{2^2a^2}} = \dfrac{b\sqrt{2a}}{a\cdot 2a} = \dfrac{b\sqrt{2a}}{2a^2}$

95. $\dfrac{\sqrt{12x^3y}}{\sqrt{20x^4y}} = \sqrt{\dfrac{12x^3y}{20x^4y}} = \sqrt{\dfrac{3}{5x}} = \dfrac{\sqrt{3}}{\sqrt{5x}}\cdot\dfrac{\sqrt{5x}}{\sqrt{5x}} = \dfrac{\sqrt{5x}}{\sqrt{5^2x^2}}$
$= \dfrac{\sqrt{15x}}{5x}$

97. $\dfrac{-2}{1-\sqrt{2}} = \dfrac{-2}{1-\sqrt{2}}\cdot\dfrac{1+\sqrt{2}}{1+\sqrt{2}} = \dfrac{-2-2\sqrt{2}}{1^2-\left(\sqrt{2}\right)^2} = \dfrac{-2-2\sqrt{2}}{1-2}$
$= \dfrac{-2-2\sqrt{2}}{-1} = 2+2\sqrt{2}$

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99. $\dfrac{-4}{3-\sqrt{2}} = \dfrac{-4}{3-\sqrt{2}} \cdot \dfrac{3+\sqrt{2}}{3+\sqrt{2}} = \dfrac{-12-4\sqrt{2}}{3^2-\left(\sqrt{2}\right)^2} = \dfrac{-12-4\sqrt{2}}{9-2}$

$= \dfrac{-12-4\sqrt{2}}{7} = \dfrac{-12-4\sqrt{2}}{7}$

101. $\dfrac{5}{2-\sqrt{7}} = \dfrac{5}{2-\sqrt{7}} \cdot \dfrac{2+\sqrt{7}}{2+\sqrt{7}} = \dfrac{10+5\sqrt{7}}{2^2-\left(\sqrt{7}\right)^2} = \dfrac{10+5\sqrt{7}}{4-7}$

$= \dfrac{10+5\sqrt{7}}{-3} = -\dfrac{10+5\sqrt{7}}{3}$

103. $\dfrac{-7}{\sqrt{x}-3} = -\dfrac{7}{\sqrt{x}-3} \cdot \dfrac{\sqrt{x}+3}{\sqrt{x}+3} = -\dfrac{7\sqrt{x}+21}{\left(\sqrt{x}\right)^2-3^2}$

$= -\dfrac{7\sqrt{x}+21}{x-9}$

105. $\dfrac{\sqrt{3}+\sqrt{4}}{\sqrt{2}+\sqrt{3}} = \dfrac{\sqrt{3}+\sqrt{2^2}}{\sqrt{2}+\sqrt{3}} = \dfrac{\sqrt{3}+2}{\sqrt{2}+\sqrt{3}} \cdot \dfrac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}$

$= \dfrac{\sqrt{6}-\sqrt{3^2}+2\sqrt{2}-2\sqrt{3}}{\left(\sqrt{2}\right)^2-\left(\sqrt{3}\right)^2}$

$= \dfrac{\sqrt{6}-3+2\sqrt{2}-2\sqrt{3}}{2-3} = \dfrac{\sqrt{6}-3+2\sqrt{2}-2\sqrt{3}}{-1}$

$= -\sqrt{6}+3-2\sqrt{2}+2\sqrt{3}$

107. $\dfrac{2+3\sqrt{5}}{1-\sqrt{5}} = \dfrac{2+3\sqrt{5}}{1-\sqrt{5}} \cdot \dfrac{1+\sqrt{5}}{1+\sqrt{5}} = \dfrac{2+2\sqrt{5}+3\sqrt{5}+3\left(\sqrt{5}\right)^2}{1-\left(\sqrt{5}\right)^2}$

$= \dfrac{2+5\sqrt{5}+3(5)}{1-5} = \dfrac{2+5\sqrt{5}+15}{-4} = \dfrac{17+5\sqrt{5}}{-4}$

$= -\dfrac{17+5\sqrt{5}}{4}$

109. $\dfrac{2\sqrt{a}-\sqrt{b}}{4\sqrt{a}+3\sqrt{b}} = \dfrac{2\sqrt{a}-\sqrt{b}}{4\sqrt{a}+3\sqrt{b}} \cdot \dfrac{4\sqrt{a}-3\sqrt{b}}{4\sqrt{a}-3\sqrt{b}}$

$= \dfrac{8\left(\sqrt{a}\right)^2-6\sqrt{ab}-4\sqrt{ab}+3\left(\sqrt{b}\right)^2}{16\left(\sqrt{a}\right)^2-9\left(\sqrt{b}\right)^2}$

$= \dfrac{8a-10\sqrt{ab}+3b}{16a-9b}$

111. $\dfrac{3\sqrt{y}-y}{\sqrt{y}+2y} = \dfrac{3\sqrt{y}-y}{\sqrt{y}+2y} \cdot \dfrac{\sqrt{y}-2y}{\sqrt{y}-2y}$

$= \dfrac{3\left(\sqrt{y}\right)^2-6y\sqrt{y}-y\sqrt{y}+2y^2}{\left(\sqrt{y}\right)^2-4y^2}$

$= \dfrac{3y-7y\sqrt{y}+2y^2}{y-4y^2} = \dfrac{y\left(3-7\sqrt{y}+2y\right)}{y(1-4y)}$

$= \dfrac{3-7\sqrt{y}+2y}{1-4y}$

Applying the Concepts

113. **a.** $\sqrt[2]{3} \cdot \sqrt[3]{4} = \sqrt[5]{12}$, false
$\sqrt[2]{3} \cdot \sqrt[3]{4} = \sqrt[6]{432}$

b. $\sqrt{3} \cdot \sqrt{3} = 3$, true

c. $\sqrt[3]{x} \cdot \sqrt[3]{x} = x$, false
$\sqrt[3]{x} \cdot \sqrt[3]{x} = x^{1/3}x^{1/3} = x^{2/3} = \sqrt[3]{x^2}$

d. $\sqrt{x}+\sqrt{y} = \sqrt{x+y}$, false
$\sqrt{x}+\sqrt{y}$ is in an irreducible form.

e. $\sqrt[2]{2}+\sqrt[3]{3} = \sqrt[5]{2+3}$, false
$\sqrt[2]{2}+\sqrt[3]{3}$ is in an irreducible form.

f. $8\sqrt[5]{a}-2\sqrt[5]{a} = 6\sqrt[5]{a}$, true

115. $\dfrac{\sqrt[4]{(a+b)^3}}{\sqrt{a+b}} = \dfrac{(a+b)^{3/4}}{(a+b)^{1/2}} = (a+b)^{1/4} = \sqrt[4]{a+b}$

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Section 9.3

Objective A Exercises

3. $\sqrt{-4} = i\sqrt{4} = i\sqrt{2^2} = 2i$

5. $\sqrt{-98} = i\sqrt{98} = i\sqrt{2 \cdot 7^2} = 7i\sqrt{2}$

7. $\sqrt{-27} = i\sqrt{27} = i\sqrt{3^2 \cdot 3} = 3i\sqrt{3}$

9. $\sqrt{16} + \sqrt{-4} = \sqrt{16} + i\sqrt{4} = \sqrt{2^4} + i\sqrt{2^2} = 4 + 2i$

11. $\sqrt{12} - \sqrt{-18} = \sqrt{12} - i\sqrt{18} = \sqrt{2^2 \cdot 3} - i\sqrt{3^2 \cdot 2}$
$= 2\sqrt{3} - 3i\sqrt{2}$

13. $\sqrt{160} - \sqrt{-147} = \sqrt{160} - i\sqrt{147} = \sqrt{2^4 \cdot 2 \cdot 5} - i\sqrt{7^2 \cdot 3}$
$= 4\sqrt{10} - 7i\sqrt{3}$

Objective B Exercises

15. $(2 + 4i) + (6 - 5i) = 8 - i$

17. $(-2 - 4i) - (6 - 8i) = -8 + 4i$

19. $\left(8 - \sqrt{-4}\right) - \left(2 + \sqrt{-16}\right) = \left(8 - i\sqrt{4}\right) - \left(2 + i\sqrt{16}\right)$
$= \left(8 - i\sqrt{2^2}\right) - \left(2 + i\sqrt{2^4}\right)$
$= (8 - 2i) - (2 + 4i) = 6 - 6i$

21. $\left(12 - \sqrt{-50}\right) + \left(7 - \sqrt{-18}\right) = \left(12 - i\sqrt{50}\right) + \left(7 - i\sqrt{18}\right)$
$= \left(12 - i\sqrt{5^2 \cdot 2}\right) + \left(7 - i\sqrt{2^2 \cdot 2}\right)$
$= \left(12 - 5i\sqrt{2}\right) + \left(7 - 2i\sqrt{2}\right)$
$= 19 - 7i\sqrt{2}$

23. $\left(\sqrt{8} + \sqrt{-18}\right) + \left(\sqrt{32} - \sqrt{-72}\right) = \left(\sqrt{8} + i\sqrt{18}\right) + \left(\sqrt{32} - i\sqrt{72}\right) = \left(\sqrt{2^2 \cdot 2} + i\sqrt{3^2 \cdot 2}\right) + \left(\sqrt{2^4 \cdot 2} - i\sqrt{2^2 \cdot 3^2 \cdot 2}\right)$
$= \left(2\sqrt{2} + 3i\sqrt{2}\right) + \left(4\sqrt{2} - 6i\sqrt{2}\right) = 6\sqrt{2} - 3i\sqrt{2}$

Objective C Exercises

25. $(7i)(-9i) = -63i^2 = -63(-1) = 63$

27. $\sqrt{-2}\sqrt{-8} = i\sqrt{2} \cdot i\sqrt{8} = i^2\sqrt{16} = -\sqrt{2^4} = -4$

29. $\sqrt{-3}\sqrt{-6} = i\sqrt{3} \cdot i\sqrt{6} = i^2\sqrt{18} = -\sqrt{3^2 \cdot 2} = -3\sqrt{2}$

31. $2i(6 + 2i) = 12i + 4i^2 = 12i + 4(-1) = -4 + 12i$

33. $\sqrt{-2}\left(\sqrt{8} + \sqrt{-2}\right) = i\sqrt{2}\left(\sqrt{8} + i\sqrt{2}\right) = i\sqrt{16} + i^2\sqrt{2^2}$
$= i\sqrt{2^4} - \sqrt{2^2} = 4i - 2 = -2 + 4i$

35. $(5 - 2i)(3 + i) = 15 + 5i - 6i - 2i^2 = 15 - i - 2i^2$
$= 15 - i - 2(-1) = 17 - i$

37. $(6 + 5i)(3 + 2i) = 18 + 12i + 15i + 10i^2 = 18 + 27i + 10i^2$
$= 18 + 27i + 10(-1) = 8 + 27i$

39. $(1 - i)\left(\frac{1}{2} + \frac{1}{2}i\right) = \frac{1}{2} + \frac{1}{2}i - \frac{1}{2}i - \frac{1}{2}i^2 = \frac{1}{2} - \frac{1}{2}i^2$
$= \frac{1}{2} - \frac{1}{2}(-1) = \frac{1}{2} + \frac{1}{2} = 1$

41. $\left(\frac{6}{5} + \frac{3}{5}i\right)\left(\frac{2}{3} - \frac{1}{3}i\right) = \frac{4}{5} - \frac{2}{5}i + \frac{2}{5}i - \frac{1}{5}i^2 = \frac{4}{5} - \frac{1}{5}i^2$
$= \frac{4}{5} - \frac{1}{5}(-1) = \frac{4}{5} + \frac{1}{5} = 1$

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Objective D Exercises

43. $\frac{3}{i}=\frac{3}{i}\cdot\frac{i}{i}=\frac{3i}{i^2}=\frac{3i}{-1}=-3i$

45. $\frac{2-3i}{-4i}=\frac{2-3i}{-4i}\cdot\frac{i}{i}=\frac{2i-3i^2}{-4i^2}=\frac{2i-3(-1)}{-4(-1)}=\frac{3+2i}{4}$
$=\frac{3}{4}+\frac{1}{2}i$

47. $\frac{4}{5+i}=\frac{4}{5+i}\cdot\frac{5-i}{5-i}=\frac{20-4i}{25+1}=\frac{20-4i}{26}=\frac{10-2i}{13}$
$=\frac{10}{13}-\frac{2}{13}i$

49. $\frac{2}{2-i}=\frac{2}{2-i}\cdot\frac{2+i}{2+i}=\frac{4+2i}{4+1}=\frac{4+2i}{5}=\frac{4}{5}+\frac{2}{5}i$

51. $\frac{1-3i}{3+i}=\frac{1-3i}{3+i}\cdot\frac{3-i}{3-i}=\frac{3-i-9i+3i^2}{9+1}=\frac{3-10i+3i^2}{10}$
$=\frac{3-10i+3(-1)}{10}=\frac{-10i}{10}=-i$

53. $\frac{\sqrt{-10}}{\sqrt{8}-\sqrt{-2}}=\frac{i\sqrt{10}}{\sqrt{8}-i\sqrt{2}}=\frac{i\sqrt{10}}{\sqrt{2^2\cdot 2}-i\sqrt{2}}=\frac{i\sqrt{10}}{2\sqrt{2}-i\sqrt{2}}\cdot\frac{2\sqrt{2}+i\sqrt{2}}{2\sqrt{2}+i\sqrt{2}}=\frac{2i\sqrt{20}+i^2\sqrt{20}}{\left(2\sqrt{2}\right)^2+\left(\sqrt{2}\right)^2}=\frac{2i\sqrt{2^2\cdot 5}-\sqrt{2^2\cdot 5}}{8+2}$
$=\frac{2i\cdot 2\sqrt{5}-2\sqrt{5}}{10}=\frac{-2\sqrt{5}+4i\sqrt{5}}{10}=\frac{-\sqrt{5}+2i\sqrt{5}}{5}=-\frac{\sqrt{5}}{5}+\frac{2\sqrt{5}}{5}i$

55. $\frac{2-3i}{3+i}=\frac{2-3i}{3+i}\cdot\frac{3-i}{3-i}=\frac{6-2i-9i+3i^2}{9+1}=\frac{6-11i+3i^2}{10}$
$=\frac{6-11i+3(-1)}{10}=\frac{3-11i}{10}=\frac{3}{10}-\frac{11}{10}i$

57. $\frac{5+3i}{3-i}=\frac{5+3i}{3-i}\cdot\frac{3+i}{3+i}=\frac{15+5i+9i+3i^2}{9+1}$
$=\frac{15=14i+3i^2}{10}=\frac{15+14i+3(-1)}{10}$
$=\frac{12+14i}{10}=\frac{6+7i}{5}=\frac{6}{5}+\frac{7}{5}i$

Applying the Concepts

59. **a.** $2x^2+18=0$; plug in $3i$.
$2(3i)^2+18=2(-9)+18=0$
$0=0$
Yes, $3i$ is a solution.

b. $x^2-6x+10=0$; plug in $3+i$.
$(3+i)^2-6(3+i)+10=9+6i+i^2-18-6i+10=-9+(-1)+10=0$
$0=0$
Yes, $3+i$ is a solution.

Section 9.4

Objective A Exercises

1. $\sqrt{x}=5$
$\left(\sqrt{x}\right)^2=5^2$
$x=25$
The solution is 25.

Check:
$\sqrt{x}=5$
$\sqrt{25}\mid 5$
$5=5$

3. $\sqrt[3]{a}=3$
$\left(\sqrt[3]{a}\right)^3=3^3$
$a=27$
The solution is 27.

Check:
$\sqrt[3]{a}=3$
$\sqrt[3]{27}\mid 3$
$3=3$

5. $\sqrt{3x}=12$
$\left(\sqrt{3x}\right)^2=12^2$
$3x=144$
$x=48$
The solution is 48.

Check:
$\sqrt{3x}=12$
$\sqrt{3(48)}\mid 12$
$\sqrt{144}\mid 12$
$12=12$

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7. $\sqrt[3]{4x} = -2$

$\left(\sqrt[3]{4x}\right)^3 = (-2)^3$

$4x = -8$

$x = -2$

Check:

$\sqrt[3]{4x} = -2$

$\sqrt[3]{4(-2)} \mid -2$

$\sqrt[3]{-8} \mid -2$

$-2 = -2$

The solution is –2.

9. $\sqrt{2x} = -4$

$\left(\sqrt{2x}\right)^2 = (-4)^2$

$2x = 16$

$x = 8$

Check:

$\sqrt{2x} = -4$

$\sqrt{2(8)} \mid -4$

$\sqrt{16} \mid -4$

$4 \neq -4$

8 does not check as a solution.
The equation has no solution.

11. $\sqrt{3x-2} = 5$

$\left(\sqrt{3x-2}\right)^2 = 5^2$

$3x - 2 = 25$

$3x = 27$

$x = 9$

Check:

$\sqrt{3x-2} = 5$

$\sqrt{3(9)-2} \mid 5$

$\sqrt{27-2} \mid 5$

$\sqrt{25} \mid 5$

$5 = 5$

The solution is 9.

13. $\sqrt{3-2x} = 7$

$\left(\sqrt{3-2x}\right)^2 = 7^2$

$3 - 2x = 49$

$-2x = 46$

$x = -23$

Check:

$\sqrt{3-2x} = 7$

$\sqrt{3-2(-23)} \mid 7$

$\sqrt{3+46} \mid 7$

$\sqrt{49} \mid 7$

$7 = 7$

The solution is –23.

15. $7 = \sqrt{1-3x}$

$(7)^2 = \left(\sqrt{1-3x}\right)^2$

$49 = 1 - 3x$

$48 = -3x$

$-16 = x$

Check:

$7 = \sqrt{1-3x}$

$7 \mid \sqrt{1-3(-16)}$

$7 \mid \sqrt{1+48}$

$7 \mid \sqrt{49}$

$7 = 7$

The solution is –16.

17. $\sqrt[3]{4x-1} = 2$

$\left(\sqrt[3]{4x-1}\right)^3 = 2^3$

$4x - 1 = 8$

$4x = 9$

$x = \frac{9}{4}$

Check:

$\sqrt[3]{4x-1} = 2$

$\sqrt[3]{4\left(\frac{9}{4}\right)-1} \mid 2$

$\sqrt[3]{9-1} \mid 2$

$\sqrt[3]{8} \mid 2$

$2 = 2$

The solution is $\frac{9}{4}$.

19. $\sqrt[3]{1-2x} = -3$

$\left(\sqrt[3]{1-2x}\right)^3 = (-3)^3$

$1 - 2x = -27$

$-2x = -28$

$x = 14$

Check:

$\sqrt[3]{1-2x} = -3$

$\sqrt[3]{1-2(14)} \mid -3$

$\sqrt[3]{1-28} \mid -3$

$\sqrt[3]{-27} \mid -3$

$-3 = -3$

The solution is 14.

21. $\sqrt[3]{9x+1} = 4$

$\left(\sqrt[3]{9x+1}\right)^3 = (4)^3$

$9x + 1 = 64$

$9x = 63$

$x = 7$

Check:

$\sqrt[3]{9x+1} = 4$

$\sqrt[3]{9(7)+1} \mid 4$

$\sqrt[3]{63+1} \mid 4$

$\sqrt[3]{64} \mid 4$

$4 = 4$

The solution is 7.

23. $\sqrt{4x-3} - 5 = 0$

$\sqrt{4x-3} = 5$

$\left(\sqrt{4x-3}\right)^2 = 5^2$

$4x - 3 = 25$

$4x = 28$

$x = 7$

Check:

$\sqrt{4x-3} - 5 = 0$

$\sqrt{4(7)-3} - 5 \mid 0$

$\sqrt{28-3} - 5 \mid 0$

$\sqrt{25} - 5 \mid 0$

$5 - 5 \mid 0$

$0 = 0$

The solution is 7.

25. $\sqrt[3]{x-3} + 5 = 0$

$\sqrt[3]{x-3} = -5$

$\left(\sqrt[3]{x-3}\right)^3 = (-5)^3$

$x - 3 = -125$

$x = -122$

Check:

$\sqrt[3]{x-3} + 5 = 0$

$\sqrt[3]{-122-3} + 5 \mid 0$

$\sqrt[3]{-125} + 5 \mid 0$

$-5 + 5 \mid 0$

$0 = 0$

The solution is –122.

27. $\sqrt[3]{2x-6} = 4$

$\left(\sqrt[3]{2x-6}\right)^3 = 4^3$

$2x - 6 = 64$

$2x = 70$

$x = 35$

Check:

$\sqrt[3]{2x-6} = 4$

$\sqrt[3]{2(35)-6} \mid 4$

$\sqrt[3]{70-6} \mid 4$

$\sqrt[3]{64} \mid 4$

$4 = 4$

The solution is 35.

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29. $\sqrt{x^2+7x+11}=1$

$\left(\sqrt{x^2+7x+11}\right)^2=1^2$

$x^2+7x+11=1$

$x^2+7x+10=0$

$(x+5)(x+2)=0$

$x+5=0 \quad x+2=0$

$x=-5 \quad x=-2$

The solutions are –5 and –2.

Check:

$\sqrt{x^2+7x+11} = 1$

$\sqrt{(-5)^2+7(-5)+11} \mid 1$

$\sqrt{25-35+11} \mid 1$

$\sqrt{1} \mid 1$

$1=1$

$\sqrt{x^2+7x+11} = 1$

$\sqrt{(-2)^2+7(-2)+11} \mid 1$

$\sqrt{4-14+11} \mid 1$

$\sqrt{1} \mid 1$

$1=1$

31. $\sqrt[4]{2x-9}=3$

$\left(\sqrt[4]{2x-9}\right)^4=3^4$

$2x-9=81$

$2x=90$

$x=45$

The solution is 45.

Check:

$\sqrt[4]{2x-9} = 3$

$\sqrt[4]{2(45)-9} \mid 3$

$\sqrt[4]{90-9} \mid 3$

$\sqrt[4]{81} \mid 3$

$3=3$

33. $\sqrt{3x-5}-5=3$

$\sqrt{3x-5}=8$

$\left(\sqrt{3x-5}\right)^2=8^2$

$3x-5=64$

$3x=69$

$x=23$

The solution is 23.

Check:

$\sqrt{3x-5}-5 = 3$

$\sqrt{3(23)-5}-5 \mid 3$

$\sqrt{69-5}-5 \mid 3$

$\sqrt{64}-5 \mid 3$

$8-5 \mid 3$

$3=3$

35. $\sqrt[3]{x-4}+7=5$

$\sqrt[3]{x-4}=-2$

$\left(\sqrt[3]{x-4}\right)^3=(-2)^3$

$x-4=-8$

$x=-4$

The solution is 4.

Check:

$\sqrt[3]{x-4} = 5$

$\sqrt[3]{-4-4}+7 \mid 5$

$\sqrt[3]{-8}+7 \mid 5$

$-2+7 \mid 5$

$5=5$

37. $\sqrt{3x-5}-2=3$

$\left(\sqrt{3x-5}\right)^2=(5)^2$

$3x-5=25$

$3x=30$

$x=10$

The solution is 10.

Check:

$\sqrt{3x-5}-2 = 3$

$\sqrt{3(10)-5}-2 \mid 3$

$\sqrt{30-5}-2 \mid 3$

$\sqrt{25}-2 \mid 3$

$5-2 \mid 3$

$3=3$

39. $\sqrt{7x+2}-10=-7$

$\left(\sqrt{7x+2}\right)^2=(3)^2$

$7x+2=9$

$7x=7$

$x=1$

The solution is 1.

Check:

$\sqrt{7x+2}-10 = -7$

$\sqrt{7(1)+2}-10 \mid -7$

$\sqrt{7+2}-10 \mid -7$

$\sqrt{9}-10 \mid -7$

$3-10 \mid -7$

$-7=-7$

41. $\sqrt[3]{1-3x}+5=3$

$\left(\sqrt[3]{1-3x}\right)^3=(-2)^3$

$1-3x=-8$

$-3x=-9$

$x=3$

The solution is 3.

Check:

$\sqrt[3]{1-3x}+5 = 3$

$\sqrt[3]{1-3(3)}+5 \mid 3$

$\sqrt[3]{1-9}+5 \mid 3$

$\sqrt[3]{-8}+5 \mid 3$

$-2+5 \mid 3$

$3=3$

43. $7-\sqrt{3x+1}=-1$

$\left(-\sqrt{3x+1}\right)^2=(-8)^2$

$3x+1=64$

$3x=63$

$x=21$

The solution is 21.

Check:

$7-\sqrt{3x+1} = -1$

$7-\sqrt{3(21)+1} \mid -1$

$7-\sqrt{63+1} \mid -1$

$7-\sqrt{64} \mid -1$

$7-8 \mid -1$

$-1=-1$

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45. $\sqrt{2x+4} = 3 - \sqrt{2x}$

$$\left(\sqrt{2x+4}\right)^2 = \left(3-\sqrt{2x}\right)^2$$
$$2x+4 = 9 - 6\sqrt{2x} + 2x$$
$$6\sqrt{2x} = 5$$
$$\left(6\sqrt{2x}\right)^2 = (5)^2$$
$$36(2x) = 25$$
$$72x = 25$$
$$x = \frac{25}{72}$$

The solution is $\frac{25}{72}$

Check:

$$\sqrt{2\left(\frac{25}{72}\right)+4} \mid 3-\sqrt{2\left(\frac{25}{72}\right)}$$
$$\sqrt{\frac{25}{36}+4} \mid 3-\sqrt{\frac{25}{36}}$$
$$\sqrt{\frac{169}{36}} \mid 3-\frac{5}{6}$$
$$\frac{13}{6} = \frac{13}{6}$$

47. $\sqrt{x^2-4x-1}+3 = x$

$$\sqrt{x^2-4x-1} = x-3$$
$$\left(\sqrt{x^2-4x-1}\right)^2 = (x-3)^2$$
$$x^2-4x-1 = x^2-6x+9$$
$$2x = 10$$
$$x = 5$$

The solution is 5.

Check:

$$\sqrt{x^2-4x-1}+3 = x$$
$$\sqrt{5^2-4(5)-1}+3 \mid 5$$
$$\sqrt{25-20-1}+3 \mid 5$$
$$\sqrt{4}+3 \mid 5$$
$$2+3 \mid 5$$
$$5 = 5$$

49. $\sqrt{x^2-2x+1} = 3$

$$\left(\sqrt{x^2-2x+1}\right)^2 = 3^2$$
$$x^2-2x+1 = 9$$
$$x^2-2x-8 = 0$$
$$(x-4)(x+2) = 0$$
$$x-4=0 \quad x+2=0$$
$$x = 4 \qquad x = -2$$

The solutions are 4 and –2.

Check:

$$\sqrt{x^2-2x+1} = 3$$
$$\sqrt{4^2-2(4)+1} \mid 3$$
$$\sqrt{16-8+1} \mid 3$$
$$\sqrt{9} \mid 3$$
$$3 = 3$$

$$\sqrt{x^2-2x+1} = 3$$
$$\sqrt{(-2)^2-2(-2)+1} \mid 3$$
$$\sqrt{4+4+1} \mid 3$$
$$\sqrt{9} \mid 3$$
$$3 = 3$$

51. $\sqrt{4x+1} - \sqrt{2x+4} = 1$

$$\left(\sqrt{4x+1}\right)^2 = \left(1+\sqrt{2x-4}\right)^2$$
$$4x+1 = 1+2\sqrt{2x+4}+2x+4$$
$$4x+1 = 5+2x+2\sqrt{2x+4}$$
$$(2x-4)^2 = \left(2\sqrt{2x+4}\right)^2$$
$$4x^2-16x+16 = 4(2x+4)$$
$$4x^2-16x+16 = 8x+16$$
$$4x^2-24x = 0$$
$$4x(x-6) = 0$$
$$x = 0 \quad x-6=0$$
$$x = 6$$

The solution is 6.

Check:

$$\sqrt{4x+1} - \sqrt{2x+4} = 1$$
$$\sqrt{4(0)+1} - \sqrt{2(0)+4} \mid 1$$
$$\sqrt{1} - \sqrt{4} \mid 1$$
$$1-2 \mid 1$$
$$-1 \neq 1$$

$$\sqrt{4x+1} - \sqrt{2x+4} = 1$$
$$\sqrt{4(6)+1} - \sqrt{2(6)+4} \mid 1$$
$$\sqrt{24+1} - \sqrt{12+4} \mid 1$$
$$\sqrt{25} - \sqrt{16} \mid 1$$
$$5-4 \mid 1$$
$$1 = 1$$

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53. $\sqrt{5x+4}-\sqrt{3x+1}=1$

$\left(\sqrt{5x+4}\right)^2=\left(1+\sqrt{3x+1}\right)^2$

$5x+4=1+2\sqrt{3x+1}+3x+1$

$5x+4=2+3x+2\sqrt{3x+1}$

$(2x+2)^2=\left(2\sqrt{3x+1}\right)^2$

$4x^2+8x+4=4(3x+1)$

$4x^2+8x+4=12x+4$

$4x^2-4x=0$

$4x(x-1)=0$

$x=0 \quad x-1=0$

$x=1$

The solutions are 0 and 1.

Check:

$\sqrt{5x+4}-\sqrt{3x+1}$	=	1
$\sqrt{5(0)+4}-\sqrt{3(0)+1}$	\|	1
$\sqrt{4}-\sqrt{1}$	\|	1
$2-1$	\|	1
	$1=1$	

$\sqrt{5x+4}-\sqrt{3x+1}$	=	1
$\sqrt{5(1)+4}-\sqrt{3(1)+1}$	\|	1
$\sqrt{5+4}-\sqrt{3+1}$	\|	1
$\sqrt{9}-\sqrt{4}$	\|	1
$3-2$	\|	1
	$1=1$	

55. $\sqrt[3]{x^2+4}-2=0$

$\sqrt[3]{x^2+4}=2$

$\left(\sqrt[3]{x^2+4}\right)^3=2^3$

$x^2+4=8$

$x^2-4=0$

$(x+2)(x-2)=0$

$x+2=0 \quad x-2=0$

$x=-2 \qquad x=2$

The solutions are –2 and 2.

Check:

$\sqrt[3]{x^2+4}-2$	=	0
$\sqrt[3]{(-2)^2+4}-2$	\|	0
$\sqrt[3]{4+4}-2$	\|	0
$\sqrt[3]{8}-2$	\|	0
$2-2$	\|	0
	$0=0$	

$\sqrt[3]{x^2+4}-2$	=	0
$\sqrt[3]{2^2+4}-2$	\|	0
$\sqrt[3]{4+4}-2$	\|	0
$\sqrt[3]{8}-2$	\|	0
$2-2$	\|	0
	$0=0$	

57. $\sqrt[4]{x^2+x-1}-1=0$

$\sqrt[4]{x^2+x-1}=1$

$\left(\sqrt[4]{x^2+x-1}\right)^4=1^4$

$x^2+x-1=1$

$x^2+x-2=0$

$(x+2)(x-1)=0$

$x+2=0 \quad x-1=0$

$x=-2 \qquad x=1$

The solutions are –2 and 1.

Check:

$\sqrt[4]{x^2+x-1}-1$	=	0
$\sqrt[4]{(-2)^2+(-2)-1}-1$	\|	0
$\sqrt[4]{4-2-1}-1$	\|	0
$\sqrt[4]{1}-1$	\|	0
$1-1$	\|	0
	$0=0$	

$\sqrt[4]{x^2+x-1}-1$	=	0
$\sqrt[4]{1^2+1-1}-1$	\|	0
$\sqrt[4]{1+1-1}-1$	\|	0
$\sqrt[4]{1}-1$	\|	0
$1-1$	\|	0
	$0=0$	

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59. $3\sqrt{x-2}+2=x$

$3\sqrt{x-2}=x-2$

$\left(3\sqrt{x-2}\right)^2=(x-2)^2$

$9(x-2)=x^2-4x+4$

$9x-18=x^2-4x+4$

$0=x^2-13x+22$

$0=(x-2)(x-11)$

$x-2=0 \quad x-11=0$

$x=2 \qquad x=11$

The solutions are 2 and 11.

Check:

$3\sqrt{x-2}+2$	=	x
$3\sqrt{2-2}+2$	\|	2
$3\sqrt{0}+2$	\|	2
$3(0)+2$	\|	2
$0+2$	\|	2
$2=2$		

$3\sqrt{x-2}+2$	=	x
$3\sqrt{11-2}+2$	\|	11
$3\sqrt{9}+2$	\|	11
$3(3)+2$	\|	11
$9+2$	\|	11
$11=11$		

Objective B Application Problems

61. Strategy: To find the width, use the Pythagorean Theorem. The hypotenuse is the diagonal. The length is one leg. The unknown width is the other leg.

Solution

$c^2=a^2+b^2$

$10^2=8^2+b^2$

$100=64+b^2$

$35=b^2$

$(36)^{1/2}=\left(b^2\right)^{1/2}$

$\sqrt{36}=b$

$6=b$

The width is 6 ft.

63. Strategy: To find the distance, use the Pythagorean Theorem. The hypotenuse is the length of the ladder. The distance along the ground from the building to the ladder is the unknown leg.

Solution

$c^2=a^2+b^2$

$26^2=24^2+b^2$

$676=576+b^2$

$100=b^2$

$\sqrt{100}=b$

$10=b$

The bottom of the ladder is 10 ft from the wall.

65. Strategy: To find the distance the object has fallen, replace v in the equation with the given value and solve for d.

Solution

$v=\sqrt{64d}$

$400=\sqrt{64d}$

$(400)^2=\left(\sqrt{64d}\right)^2$

$160{,}000=64d$

$2500=d$

The object has fallen 2500 ft.

67. Strategy: To find the distance above the water, replace d in the equation with the given value and solve for h.

Solution

$d=\sqrt{1.5h}$

$3.6=\sqrt{1.5h}$

$(3.6)^2=\left(\sqrt{1.5h}\right)^2$

$12.96=1.5h$

$8.64=h$

The periscope must be 8.64 ft above the water.

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69. Strategy To find the length of the pendulum, replace T in the equation with the given value and solve for L.

Solution
$$T = 2\pi\sqrt{\frac{L}{32}}$$
$$2.4 = 2\pi\sqrt{\frac{L}{32}}$$
$$\frac{2.4}{2\pi} = \sqrt{\frac{L}{32}}$$
$$\left(\frac{2.4}{2\pi}\right)^2 = \left(\sqrt{\frac{L}{32}}\right)^2$$
$$\left(\frac{2.4}{2\pi}\right)^2 = \frac{L}{32}$$
$$4.67 \approx L$$
The length of the pendulum is 4.67 ft.

71. Strategy To find the distance, replace the variables v and a in the equation by their given value and solve for s.

Solution
$$v = \sqrt{2as}$$
$$60 = \sqrt{20s}$$
$$60^2 = \left(\sqrt{20s}\right)^2$$
$$3600 = 20s$$
$$180 = s$$
The distance is 180 m.

73. Strategy To find how much wider the HDTV is, use the Pythagorean Theorem to find the width of each television and subtract the difference.

Diagonal: d, Height: h, Width: w

HDTV	*Regular TV*
$d^2 = h^2 + w^2$	$d^2 = h^2 + w^2$
$33^2 = 16.2^2 + w^2$	$27^2 = 16.2^2 + w^2$
$1089 = 262.44 + w^2$	$729 = 262.44 + w^2$
$826.56 = w^2$	$466.56 = w^2$
$\sqrt{826.56} = w$	$\sqrt{466.56} = w$
$w = 28.75$	$w = 21.6$

$w_{HDTV} - w_{Regular} = 28.75 - 21.6 = 7.15$

The HDTV screen is 7.15 in. wider.

Applying the Concepts

75. **a.** $x^2 + 3 = 7$
$$x^2 = 4$$
$$x = \pm 2$$
x is an integer, a rational number, and a real number.

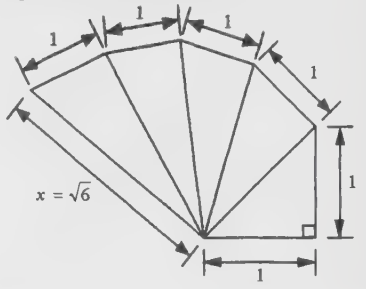
b. $x^2 + 1 = 0$
$$x^2 = -1$$
$$x = \pm i$$
x is an imaginary number.

c. $x^{3/4} = 8$
$$\left(x^{3/4}\right)^{4/3} = (8)^{4/3}$$
$$x = \sqrt[3]{8^4}$$
$$= \sqrt[3]{4096}$$
$$x = 16$$
x is an integer, a rational number, and a real number.

77. $a^2 + b^2 = c^2$
$$a^2 = c^2 - b^2$$
$$a = \pm\sqrt{c^2 - b^2}$$

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79. Use the Pythagorean Theorem five times to determine the lengths of the diagonals for each triangle.



$c = \sqrt{a^2 + b^2}$

$c = \sqrt{1^2 + 1^2} = \sqrt{2}$

$c = \sqrt{\left(\sqrt{2}\right)^2 + 1^2} = \sqrt{3}$

$c = \sqrt{\left(\sqrt{3}\right)^2 + 1^2} = \sqrt{4} = 2$

$c = \sqrt{(2)^2 + 1^2} = \sqrt{5}$

$x = \sqrt{\left(\sqrt{5}\right)^2 + 1^2} = \sqrt{6}$

The length of x equals $\sqrt{6}$.

Chapter Review

1. $\left(16x^{-4}y^{12}\right)^{1/4}\left(100x^{6}y^{-2}\right)^{1/2} = 16^{1/4}x^{-1}y^{3}100^{1/2}x^{3}y^{-1}$
$= 2(10)x^{2}y^{2} = 20x^{2}y^{2}$

2. $\sqrt[4]{3x-5} = 2$
$\left(\sqrt[4]{3x-5}\right)^4 = (2)^4$
$3x - 5 = 16$
$3x = 21$
$x = 7$

3. $(6-5i)(4+3i) = 24 + 18i - 20i - 15i^2 = 24 - 2i - 15(-1)$
$= 39 - 2i$

4. $7y\sqrt[3]{x^2} = 7x^{2/3}y$

5. $\left(\sqrt{3}+8\right)\left(\sqrt{3}-2\right) = \sqrt{3^2} - 2\sqrt{3} + 8\sqrt{3} - 16$
$= 3 + 6\sqrt{3} - 16 = 6\sqrt{3} - 13$

6. $\sqrt{4x+9} + 10 = 11$
$\sqrt{4x+9} = 1$
$\left(\sqrt{4x+9}\right)^2 = 1^2$
$4x + 9 = 1$
$4x = -8$
$x = -2$

7. $\dfrac{x^{-3/2}}{x^{7/2}} = x^{3\ 7/2\ 2} = x^{-10/2} = x^{-5} = \dfrac{1}{x^5}$

8. $\dfrac{8}{\sqrt{3y}} \cdot \dfrac{\sqrt{3y}}{\sqrt{3y}} = \dfrac{8\sqrt{3y}}{\sqrt{(3y)^2}} = \dfrac{8\sqrt{3y}}{3y}$

9. $\sqrt[3]{-8a^6b^{12}} = \sqrt[3]{(-2)^3a^6b^{12}} = -2a^2b^4$

10. $\sqrt{50a^4b^3} - ab\sqrt{18a^2b} = \sqrt{5^2 \cdot 2a^4b^3} - ab\sqrt{3^2 \cdot 2a^2b}$
$= 5a^2b\sqrt{2b} - 3a^2b\sqrt{2b}$
$= 2a^2b\sqrt{2b}$

11. $\dfrac{x+2}{\sqrt{x}+\sqrt{2}} \cdot \dfrac{\sqrt{x}-\sqrt{2}}{\sqrt{x}-\sqrt{2}} = \dfrac{x\sqrt{x} - x\sqrt{2} + 2\sqrt{x} - 2\sqrt{2}}{\sqrt{x^2} - \sqrt{2^2}}$
$= \dfrac{x\sqrt{x} - x\sqrt{2} + 2\sqrt{x} - 2\sqrt{2}}{x-2}$

12. $\dfrac{5+2i}{3i} \cdot \dfrac{i}{i} = \dfrac{5i + 2i^2}{3i^2} = \dfrac{5i + 2(-1)}{3(-1)} = \dfrac{5i-2}{-3} = \dfrac{2}{3} - \dfrac{5}{3}i$

13. $\sqrt{18a^3b^6} = \sqrt{3^2 \cdot 2a^3b^6} = 3ab^3\sqrt{2a}$

14. $\left(\sqrt{50} + \sqrt{-72}\right) - \left(\sqrt{162} - \sqrt{-8}\right) = \left(\sqrt{5^2 \cdot 2} + i\sqrt{6^2 \cdot 2}\right) - \left(\sqrt{9^2 \cdot 2} - i\sqrt{2^2 \cdot 2}\right) = \left(5\sqrt{2} + 6i\sqrt{2}\right) - \left(9\sqrt{2} - 2i\sqrt{2}\right) = -4\sqrt{2} + 8i\sqrt{2}$

15. $3x^3\sqrt{54x^8y^{10}} - 2x^2y^3\sqrt{16x^5y^7} = 3x^3\sqrt{3^2 \cdot 2x^8y^{10}} - 2x^2y^3\sqrt{2^3 \cdot 2x^5y^7} = 9x^3y^3\sqrt[3]{2x^2y} - 4x^3y^3\sqrt[3]{2x^2y} = 5x^3y^3\sqrt[3]{2x^2y}$

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16. $\sqrt[3]{16x^4y^3}\sqrt[3]{4xy^5} = \sqrt[3]{64x^5y^6} = \sqrt[3]{4^3x^5y^6} = 4xy^2\sqrt[3]{x^2}$

17. $i(3-7i) = 3i - 7i^2 = 3i - 7(-1) = 7 + 3i$

18. $3x^{3/4} = 3\sqrt[4]{x^3}$

19. $\sqrt[5]{-64a^8b^{12}} = \sqrt[5]{(-2)^5 \cdot 2a^8b^{12}}$
$= -2ab^2\sqrt[5]{2a^3b^2}$

20. $\dfrac{5+9i}{1-i}\cdot\dfrac{1+i}{1+i} = \dfrac{5+5i+9i+9i^2}{1-i^2}$
$= \dfrac{5+14i+9(-1)}{1-(-1)}$
$= \dfrac{-4+14i}{2} = -2+7i$

21. $\sqrt{-12}\sqrt{-6} = i\sqrt{12}\cdot i\sqrt{6} = i^2\sqrt{72} = (-1)\sqrt{6^2\cdot 2} = -6\sqrt{2}$

22.
$\sqrt{x-5}+\sqrt{x+6} = 11$
$\sqrt{x-5} = 11-\sqrt{x+6}$
$\left(\sqrt{x-5}\right)^2 = \left(11-\sqrt{x+6}\right)^2$
$x-5 = 121-22\sqrt{x+6}+x+6$
$x-5 = 127+x-22\sqrt{x+6}$
$-136 = -22\sqrt{x+6}$
$6 = \sqrt{x+6}$
$(6)^2 = \left(\sqrt{x+6}\right)^2$
$36 = x+6$
$30 = x$

The solution is 30.

Check:
$\sqrt{x-5}+\sqrt{x+6} = 11$
$\sqrt{30-5}+\sqrt{30+6} = 11$
$\sqrt{25}+\sqrt{36} = 11$
$5+6 = 11$
$11 = 11$

23. $\sqrt[4]{81a^8b^{12}} = \sqrt[4]{3^4a^8b^{12}} = 3a^2b^3$

24. $\sqrt{-50} = i\sqrt{50} = i\sqrt{5^2\cdot 2} = 5i\sqrt{2}$

25. $(-8+3i)-(4-7i) = -12+10i$

26. $\left(5-\sqrt{6}\right)^2 = 25-10\sqrt{6}+\sqrt{6^2} = 25-10\sqrt{6}+6 = 31-10\sqrt{6}$

27. $4x\sqrt{12x^2y}+\sqrt{3x^4y}-x^2\sqrt{27y} = 4x\sqrt{2^2\cdot 3x^2y}+\sqrt{3x^4y}-x^2\sqrt{3^2\cdot 3y} = 8x^2\sqrt{3y}+x^2\sqrt{3y} = -3x^2\sqrt{3y} = 6x^2\sqrt{3y}$

28. Strategy: To find the power generated, replace v in the formula and solve for P.

Solution:
$v = 4.05\sqrt[3]{P}$
$20 = 4.05\sqrt[3]{P}$
$4.94 = \sqrt[3]{P}$
$(4.94)^3 = \left(\sqrt[3]{P}\right)^3$
$120 = P$

The amount of power generated is 120 watts.

29. Strategy: To find the distance, replace v and a in the formula and solve for s.

Solution:
$v = \sqrt{2as}$
$88 = \sqrt{2(16)s}$
$88 = \sqrt{32s}$
$(88)^2 = \left(\sqrt{2(16)s}\right)^2$
$7744 = 32s$
$242 = s$

The distance required to reach a velocity of 88 ft/s is 242 ft.

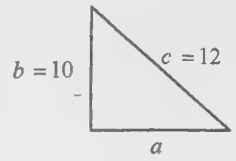
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30. Strategy To find the distance, replace b and c in the Pythagorean Theorem and solve for a.

Solution

$$a^2 + b^2 = c^2$$
$$a^2 + (10)^2 = (12)^2$$
$$a^2 + 100 = 144$$
$$a^2 = 44$$
$$\sqrt{a^2} = \sqrt{44}$$
$$a = 6.63$$

The bottom of the ladder is 6.63 ft from the building.



Chapter Test

1. $\frac{1}{2}\sqrt[4]{x^3} = \frac{1}{2}x^{3/4}$

2. $\sqrt[3]{54x^7y^3} - x\sqrt[3]{128x^4y^3} - x^2\sqrt[3]{2xy^3} = \sqrt[3]{3^3x^6y^3(2x)} - x\sqrt[3]{4^3x^3y^3(2x)} - x^2\sqrt[3]{y^3(2x)} = 3x^2y\sqrt[3]{2x} - 4x^2y\sqrt[3]{2x} - x^2y\sqrt[3]{2x}$
$= -2x^2y\sqrt[3]{2x}$

3. $3y^{2/5} = 3\sqrt[5]{y^2}$

4. $(2+5i)(4-2i) = 8 - 4i + 20i + 20i - 10i^2$
$= 8 + 16i - 10(-1) = 18 + 16i$

5. $\left(2\sqrt{x} + \sqrt{y}\right)^2 = 4\sqrt{x^2} + 4\sqrt{xy} + \sqrt{y^2} = 4x + 4\sqrt{xy} + y$

6. $\frac{r^{2/3}r^{-1}}{r^{-1/2}} = \frac{r^{-1/3}}{r^{-1/2}} = r^{1/6}$

7.
$$\sqrt{x+12} - \sqrt{x} = 2$$
$$\sqrt{x+12} = 2 + \sqrt{x}$$
$$\left(\sqrt{x+12}\right)^2 = \left(2 + \sqrt{x}\right)^2$$
$$x + 12 = 4 + 4\sqrt{x} + x$$
$$12 = 4 + 4\sqrt{x}$$
$$8 = 4\sqrt{x}$$
$$2 = \sqrt{x}$$
$$(2)^2 = \left(\sqrt{x}\right)^2$$
$$4 = x$$

The solution is 4.

Check:
$$\sqrt{x+12} - \sqrt{x} = 2$$
$$\sqrt{4+12} - \sqrt{4} = 2$$
$$\sqrt{16} - \sqrt{4} = 2$$
$$4 - 2 = 2$$
$$2 = 2$$

8. $\sqrt[3]{8x^3y^6} = \sqrt[3]{2^3x^3y^6} = 2xy^2$

9. $\sqrt{3x}\left(\sqrt{x} - \sqrt{25x}\right) = \sqrt{3x^2} - \sqrt{75x^2} = \sqrt{x^2(3)} - \sqrt{5^2x^2(3)}$
$= x\sqrt{3} - 5x\sqrt{3} = -4x\sqrt{3}$

10. $(5-2i) - (8-4i) = -3 + 2i$

11. $\sqrt{32x^4y^7} = \sqrt{2^4x^4y^6(2y)} = 4x^2y^3\sqrt{2y}$

12. $\left(2\sqrt{3}+4\right)\left(3\sqrt{3}-1\right) = 6\sqrt{3^2} - 2\sqrt{3} + 12\sqrt{3} - 4$
$= 18 + 10\sqrt{3} - 4 = 14 + 10\sqrt{3}$

13. $(2+i) + (2-i) = 4$

14. $\frac{4-2\sqrt{5}}{2-\sqrt{5}} = \frac{4-2\sqrt{5}}{2-\sqrt{5}} \cdot \frac{2+\sqrt{5}}{2+\sqrt{5}} = \frac{8+4\sqrt{5}-4\sqrt{5}-2\sqrt{5^2}}{2^2-\sqrt{5^2}}$
$= \frac{8-2\cdot 5}{4-5} = \frac{8-10}{-1} = \frac{-2}{-1} = 2$

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15. $\sqrt{18a^3} + a\sqrt{50a} = \sqrt{3^2a^2(2a)} + a\sqrt{5^2(2a)}$
$= 3a\sqrt{2a} + 5a\sqrt{2a} = 8a\sqrt{2a}$

16. $(\sqrt{a} - 3\sqrt{b})(2\sqrt{a} + 5\sqrt{b}) = 2\sqrt{a^2} + 5\sqrt{ab} - 6\sqrt{ab} - 15\sqrt{b^2}$
$= 2a - \sqrt{ab} - 15b$

17. $\dfrac{(2x^{1/3}y^{-2/3})^6}{(x^{-4}y^8)^{1/4}} = \dfrac{2^6x^2y^{-4}}{x^{-1}y^2} = 2^6x^3y^{-6} = \dfrac{64x^3}{y^6}$

18. $\dfrac{\sqrt{x}}{\sqrt{x} - \sqrt{y}} = \dfrac{\sqrt{x}}{\sqrt{x} - \sqrt{y}} \cdot \dfrac{\sqrt{x} + \sqrt{y}}{\sqrt{x} + \sqrt{y}} = \dfrac{\sqrt{x^2} + \sqrt{xy}}{\sqrt{x^2} - \sqrt{y^2}} = \dfrac{x + \sqrt{xy}}{x - y}$

19. $\dfrac{2+3i}{1-2i} = \dfrac{2+3i}{1-2i} \cdot \dfrac{1+2i}{1+2i} = \dfrac{2+4i+3i+6i^2}{1-4i^2}$
$= \dfrac{2+7i+6(-1)}{1-4(-1)} = \dfrac{-4+7i}{5} = -\dfrac{4}{5} + \dfrac{7}{5}i$

20. $\sqrt[3]{2x-2} + 4 = 2$
$\sqrt[3]{2x-2} = -2$
$(\sqrt[3]{2x-2})^3 = (-2)^3$
$2x - 2 = -8$
$2x = -6$
$x = -3$
The solution is -3.

Check:
$\sqrt[3]{2x-2} + 4 = 2$
$\sqrt[3]{2(-3)-2} + 4 = 2$
$\sqrt[3]{-8} + 4 = 2$
$-2 + 4 = 2$
$2 = 2$

21. $\left(\dfrac{4a^4}{b^2}\right)^{-3/2} = \dfrac{4^{-3/2}a^{-6}}{b^{-3}} = (2^2)^{-3/2}a^{-6}b^3 = 2^{-3}a^{-6}b^3 = \dfrac{b^3}{8a^6}$

22. $\sqrt[3]{27a^4b^3c^7} = \sqrt[3]{3^3a^3b^3c^6(ac)} = 3abc^2\sqrt[3]{ac}$

23. $\dfrac{\sqrt{32x^5y}}{\sqrt{2xy^3}} = \sqrt{\dfrac{32x^5y}{2xy^3}} = \sqrt{\dfrac{16x^4}{y^2}} = \sqrt{\dfrac{4^2x^4}{y^2}} = \dfrac{4x^2}{y}$

24. $(\sqrt{-8})(\sqrt{-2}) = i\sqrt{8} \cdot i\sqrt{2} = i^2\sqrt{16} = -4$

25. Strategy To find the distance, replace v in the formula and solve for d.

Solution
$v = \sqrt{64d}$
$192 = \sqrt{64d}$
$(192)^2 = (\sqrt{64d})^2$
$36{,}864 = 64d$
$576 = d$
The object has fallen 576 ft.

Cumulative Review

1. $2^3 \cdot 3 - 4(3 - 4 \cdot 5) = 2^3 \cdot 3 - 4(3 - 20) = 2^3 \cdot 3 - 4(-17)$
$= 8 \cdot 3 - 4(-17) = 24 - 4(-17)$
$= 24 + 68 = 92$

2. $4a^2b - a^3 = 4(-2)^2(3) - (-2)^3 = 4(4)(3) - (-8)$
$= 16(3) - (-8) = 48 - (-8)$
$= 48 + 8 = 56$

3. $-3(4x - 1) - 2(1 - x) = -12x + 3 - 2 + 2x$
$= -10x + 1$

4. $5 - \dfrac{2}{3}x = 4$
$5 - 5 - \dfrac{2}{3}x - 5 = 4 - 5$
$-\dfrac{2}{3}x = -1$
$\left(-\dfrac{3}{2}\right)\left(-\dfrac{2}{3}\right)x = -1\left(-\dfrac{3}{2}\right)$
$x = \dfrac{3}{2}$
The solution is $\dfrac{3}{2}$.

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5. $2[4-2(3-2x)]=4(1-x)$

$2[4-6+4x]=4-4x$

$2[-2+4x]=4-4x$

$-4+8x=4-4x$

$-4+8x+4x=4-4x+4x$

$12x-4=4$

$12x-4+4=4+4$

$12x=8$

$\left(\frac{1}{12}\right)12x=\frac{1}{12}(8)$

$x=\frac{2}{3}$

The solution is $\frac{2}{3}$.

6. $6x-3(2x+2)>3-3(x+2)$

$6x-6x-6>3-3x-6$

$-6>-3-3x$

$-6+3x>-3-3x+3x$

$3x-6>-3$

$3x-6+6>-3+6$

$3x>3$

$\left(\frac{1}{3}\right)3x>\frac{1}{3}(3)$

$x>1$

$\{x|x>1\}$

7. $2+|4-3x|=5$

$|4-3x|=3$

$4-3x=3$ $\quad 4-3x=-3$

$-3x=-1$ $\quad -3x=-7$

$x=\frac{1}{3}$ $\quad x=\frac{7}{3}$

The solutions are $\frac{1}{3}$ and $\frac{7}{3}$.

8. $|2x+3|\le 9$

$-9\le 2x+3\le 9$

$-9-3\le 2x+3-3\le 9-3$

$-12\le 2x\le 6$

$\frac{1}{2}(-12)\le\frac{1}{2}(2x)\le\frac{1}{2}(6)$

$-6\le x\le 3$

$\{x|-6\le x\le 3\}$

9. $A=\frac{1}{2}bh=\frac{1}{2}(25)(15)=12.5(15)=187.5$

The area is $187.5\ \text{cm}^2$.

10. $\begin{vmatrix}1&2&-3\\0&-1&2\\3&1&-2\end{vmatrix}=1\cdot\begin{vmatrix}-1&2\\1&-2\end{vmatrix}-2\begin{vmatrix}0&2\\3&-2\end{vmatrix}-3\begin{vmatrix}0&-1\\3&1\end{vmatrix}=1(2-2)-2(0-6)-3(0+3)=1\cdot 0-2(-6)-3\cdot 3=3$

11. To find the y-intercept, let $x=0$.

$3(0)-2y=-6$

$y=3$ The y-intercept is $(0, 3)$.

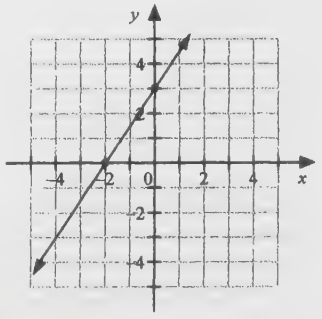
To find the slope, find the x-intercept and use it to find the slope.

$3x-2(0)=-6$

$x=-2$ The x-intercept is $(-2, 0)$.

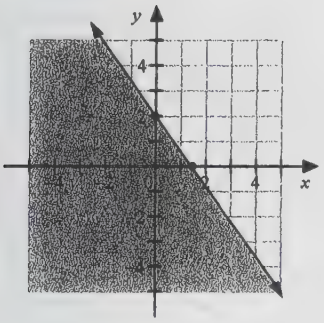
$m=\frac{y_2-y_1}{x_2-x_1}=\frac{3-0}{0-(-2)}=\frac{3}{2}$

The slope is $\frac{3}{2}$, and the y-intercept is $(0, 3)$.



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12.



13. First find the slope of the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-3}{-1-2} = \frac{-1}{-3} = \frac{1}{3}$$

Use the point-slope formula to find the equation of the line.

$$y - y_1 = m(x - x_1)$$

$$y - 3 = \frac{1}{3}(x - 2)$$

$$y - 3 = \frac{1}{3}x - \frac{2}{3}$$

$$y = \frac{1}{3}x + \frac{7}{3}$$

The equation of the line is $y = \frac{1}{3}x + \frac{7}{3}$.

14.

$$2x - y = 4$$
$$-2x + 3y = 5$$

$$D = \begin{vmatrix} 2 & -1 \\ -2 & 3 \end{vmatrix} = 4$$

$$D_x = \begin{vmatrix} 4 & -1 \\ 5 & 3 \end{vmatrix} = 17$$

$$D_4 = \begin{vmatrix} 2 & 4 \\ -2 & 5 \end{vmatrix} = 18$$

$$x = \frac{D_x}{D} = \frac{17}{4}$$

$$y = \frac{D_y}{D} = \frac{18}{4} = \frac{9}{2}$$

The solution is $\left(\frac{17}{4}, \frac{9}{2}\right)$.

15.

$$(2^{-1}x^2y^{-6})(2^{-1}y^{-4})^{-2} = (2^{-1}x^2y^{-6})(2^2y^8)$$
$$= 2^{-1+2}x^2y^{-6+8} = 2^1x^2y^2 = 2x^2y^2$$

16.

$$81x^2 - y^2 = (9x+y)(9x-y)$$

17.

$$x^5 + 2x^3 - 3x = x(x^4 + 2x^2 - 3) = x(x^2+3)(x^2-1)$$
$$= x(x^2+3)(x+1)(x-1)$$

18.

$$P = \frac{R-C}{n}$$
$$P \cdot n = \frac{R-C}{n} \cdot n$$
$$nP = R - C$$
$$nP + C = R$$
$$C = R - nP$$

19.

$$\left(\frac{x^{-2/3}y^{1/2}}{y^{-1/3}}\right)^6 = \frac{x^{-4}y^3}{y^{-2}} = x^{-4}y^{3-(-2)} = x^{-4}y^5 = \frac{y^5}{x^4}$$

20.

$$\sqrt{40x^3} - x\sqrt{90x} = \sqrt{2^2x^2(10x)} - x\sqrt{3^2(10x)}$$
$$= 2x\sqrt{10x} - 3x\sqrt{10x} = -x\sqrt{10x}$$

21.

$$(\sqrt{3}-2)(\sqrt{3}-5) = \sqrt{3^2} - 5\sqrt{3} - 2\sqrt{3} + 10$$
$$= 3 - 7\sqrt{3} + 10 = 13 - 7\sqrt{3}$$

22.

$$\frac{4}{\sqrt{6}-\sqrt{2}} = \frac{4}{\sqrt{6}-\sqrt{2}} \cdot \frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}} = \frac{4\sqrt{6}+4\sqrt{2}}{\sqrt{6^2}-\sqrt{2^2}}$$
$$= \frac{4\sqrt{6}+4\sqrt{2}}{6-2} = \frac{4\sqrt{6}+4\sqrt{2}}{4} = \sqrt{6}+\sqrt{2}$$

23.

$$\frac{2i}{3-i} = \frac{2i}{3-i} \cdot \frac{3+i}{3+i} = \frac{6i+2i^2}{9+1} = \frac{6i+2(-1)}{10}$$
$$= \frac{-2+6i}{10} = -\frac{1}{5} + \frac{3}{5}i$$

24.

$$\sqrt[3]{3x-4} + 5 = 1$$
$$\sqrt[3]{3x-4} = -4$$
$$\left(\sqrt[3]{3x-4}\right)^3 = (-4)^3$$
$$3x - 4 = -64$$
$$3x = -60$$
$$x = -20$$

The solution is –20.

Check:

$$\sqrt[3]{3x-4} + 5 = 1$$
$$\sqrt[3]{3(-20)-4} + 5 \mid 1$$
$$\sqrt[3]{64} + 5 \mid 1$$
$$-4 + 5 \mid 1$$
$$1 = 1$$

25.

$$\frac{BC}{EF} = \frac{AC}{DE}$$
$$\frac{8}{12} = \frac{18}{DE}$$
$$8(DE) = 12(18)$$
$$8(DE) = 216$$
$$DE = 27$$

The length of side DE is 27 m.

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26. Strategy

- Amount invested at 8.4%: x

	Principal	Rate	Interest
Amount invested at 7.2%	2500	0.072	0.072(2500)
Amount invested at 8.4%	x	0.084	$0.084x$

- The total amount of interest earned is \$516.

Solution

$$0.072(2500)+0.084x = 516$$
$$180+0.084x = 516$$
$$0.084x = 336$$
$$x = 4000$$

\$4000 must be invested at 8.4%.

27. Strategy

- Unknown rate of the car: x
 Unknown rate of the plane: $5x$

	Distance	Rate	Time
Car	25	x	$\frac{25}{x}$
Plane	625	$5x$	$\frac{625}{5x}$

- The total time of the trip was 3 h.

Solution

$$\frac{25}{x}+\frac{625}{5x}=3$$
$$5x\left(\frac{25}{x}+\frac{625}{5x}\right)=3(5x)$$
$$125+625=15x$$
$$750=15x$$
$$50=x$$
$$5x=5(50)=250$$

The rate of the plane is 250 mph.

28. Strategy

To find the time it takes light to travel from the earth to the moon, use the formula $RT = D$, substituting for R and D and solving for T.

Solution

$$RT = D$$
$$1.86\times10^5\cdot T = 232{,}500$$
$$1.86\times10^5\cdot T = 2.325\times10^5$$
$$T = 1.25\times10^{10}$$
$$T = 1.25$$

It takes 1.25 s for light to travel to Earth from the moon.

29. Slope $m = \frac{y_2-y_1}{x_2-x_1}=\frac{400-0}{5000-0}=\frac{400}{5000}=0.08$

The slope represents the simple interest rate on an investment. The interest rate is 8%.

30. Strategy

To find the height of the periscope, replace d in the given equation and solve for h.

Solution

$$d=\sqrt{1.5h}$$
$$7=\sqrt{1.5h}$$
$$7^2\left(\sqrt{1.5h}\right)^2$$
$$49=1.5h$$
$$32.7\approx h$$

The periscope must be 32.7 ft above the water.

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Chapter 10
Quadratic Equations

Prep Test

1. $\sqrt{18} = \sqrt{2 \cdot 9}$
$= 3\sqrt{2}$

2. $\sqrt{-9} = 3i$

3. $\frac{3x-2}{x-1} - 1 = \frac{3x-2}{x-1} - \frac{x-1}{x-1}$
$= \frac{3x-2-x+1}{x-1}$
$= \frac{2x-1}{x-1}$

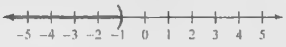
4. $b^2 - 4ac$
$(-4)^2 - 4(2)(1) = 16 - 8 = 8$

5. $4x^2 + 28x + 49 = (2x+7)^2$
Yes

6. $4x^2 - 4x + 1 = (2x-1)^2$

7. $9x^2 - 4 = (3x+2)(3x-2)$

8. $\{x \mid x < -1\} \cap \{x \mid x < 4\}$



9. $x(x-1) = x+15$
$x^2 - x = x+15$
$x^2 - 2x - 15 = 0$
$(x-5)(x+3) = 0$
$x-5=0 \quad x+3=0$
$x=5 \quad x=-3$
The solutions are –3 and 5.

10. $\frac{4}{x-3} = \frac{16}{x}$
$x(x-3)\frac{4}{x-3} = x(x-3)\frac{16}{x}$
$4x = 16x - 48$
$-12x = -48$
$x = 4$
The solution is 4.

Go Figure

The repeating decimal 0.999… has digits 9. The repeating decimal 0.999… is equivalent to 9/9 which equals 1, and $\sqrt{1} = 1$. Therefore the value of A is 9.

Section 10.1

Objective A Exercises

3. $2x^2 - 4x = 5$
$2x^2 - 4x - 5 = 0$
$a = 2, b = -4, c = -5$

5. $5x = 4x^2 + 6$
$4x^2 - 5x + 6 = 0$
$a = 4, b = -5, c = 6$

7. $x^2 - 4x = 0$
$x(x-4) = 0$
$x = 0 \quad x-4=0$
$x = 4$
The solutions are 0 and 4.

9. $t^2 - 25 = 0$
$(t-5)(t+5) = 0$
$t-5=0 \quad t+5=0$
$t=5 \quad t=-5$
The solutions are 5 and –5.

11. $s^2 - s - 6 = 0$
$(s-3)(s+2) = 0$
$s-3=0 \quad s+2=0$
$s=3 \quad s=-2$
The solutions are 3 and –2.

13. $y^2 - 6y + 9 = 0$
$(y-3)(y-3) = 0$
$y-3=0 \quad y-3=0$
$y=3 \quad y=3$
The solution is 3.

15. $9z^2 - 18z = 0$
$9z(z-2) = 0$
$9z = 0 \quad z-2=0$
$z=0 \quad z=2$
The solutions are 0 and 2.

17. $r^2 - 3r = 10$
$r^2 - 3r - 10 = 0$
$(r-5)(r+2) = 0$
$r-5=0 \quad r+2=0$
$r=5 \quad r=-2$
The solutions are 5 and –2.

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19. $v^2+10=7v$
$v^2-7v+10=0$
$(v-2)(v-5)=0$
$v-2=0 \qquad v-5=0$
$v=2 \qquad v=5$
The solutions are 2 and 5.

21. $2x^2-9x-18=0$
$(x-6)(2x+3)=0$
$x-6=0 \qquad 2x+3=0$
$x=6 \qquad 2x=-3$
$x=-\frac{3}{2}$
The solutions are 6 and $-\frac{3}{2}$.

23. $4z^2-9z+2=0$
$(z-2)(4z-1)=0$
$z-2=0 \qquad 4z-1=0$
$z=2 \qquad 4z=1$
$z=\frac{1}{4}$
The solutions are 2 and $\frac{1}{4}$.

25. $3w^2+11w=4$
$3w^2+11w-4=0$
$(3w-1)(w+4)=0$
$3w-1=0 \qquad w+4=0$
$3w=1 \qquad w=-4$
$w=\frac{1}{3}$
The solutions are $\frac{1}{3}$ and –4.

27. $6x^2=23x+18$
$6x^2-23x-18=0$
$(2x-9)(3x+2)=0$
$2x-9=0 \qquad 3x+2=0$
$2x=9 \qquad 3x=-2$
$x=\frac{9}{2} \qquad x=-\frac{2}{3}$
The solutions are $\frac{9}{2}$ and $-\frac{2}{3}$.

29. $4-15u-4u^2=0$
$(1-4u)(4+u)=0$
$1-4u=0 \qquad 4+u=0$
$-4u=-1 \qquad u=-4$
$u=\frac{1}{4}$
The solutions are $\frac{1}{4}$ and –4.

31. $x+18=x(x-6)$
$x+18=x^2-6x$
$0=x^2-7x-18$
$0=(x-9)(x+2)$
$x-9=0 \qquad x+2=0$
$x=9 \qquad x=-2$
The solutions are 9 and –2.

33. $4s(s+3)=s-6$
$4s^2+12s=s-6$
$4s^2+11s+6=0$
$(s+2)(4s+3)=0$
$s+2=0 \qquad 4s+3=0$
$s=-2 \qquad 4s=-3$
$s=-\frac{3}{4}$
The solutions are –2 and $-\frac{3}{4}$.

35. $u^2-2u+4=(2u-3)(u+2)$
$u^2-2u+4=2u^2+u-6$
$0=u^2+3u-10$
$0=(u-2)(u+5)$
$u-2=0 \qquad u+5=0$
$u=2 \qquad u=-5$
The solutions are 2 and –5.

37. $(3x-4)(x+4)=x^2-3x-28$
$3x^2+8x-16=x^2-3x-28$
$2x^2+11x+12=0$
$(x+4)(2x+3)=0$
$x+4=0 \qquad 2x+3=0$
$x=-4 \qquad 2x=-3$
$x=-\frac{3}{2}$
The solutions are –4 and $-\frac{3}{2}$.

39. $x^2-9bx+14b^2=0$
$(x-2b)(x-7b)=0$
$x-2b=0 \qquad x-7b=0$
$x=2b \qquad x=7b$
The solutions are $2b$ and $7b$.

41. $x^2-6cx-7c^2=0$
$(x-7c)(x+c)=0$
$x-7c=0 \qquad x+c=0$
$x=7c \qquad x=-c$
The solutions are $7c$ and $-c$.

43. $2x^2+3bx+b^2=0$
$(2x+b)(x+b)=0$
$2x+b=0 \qquad x+b=0$
$2x=-b \qquad x=-b$
$x=-\frac{b}{2}$
The solutions are $-\frac{b}{2}$ and $-b$.

45. $3x^2-14ax+8a^2=0$
$(x-4a)(3x-2a)=0$
$x-4a=0 \qquad 3x-2a=0$
$x=4a \qquad 3x=2a$
$x=\frac{2a}{3}$
The solutions are $4a$ and $\frac{2a}{3}$.

47. $3x^2-8ax-3a^2=0$
$(3x+a)(x-3a)=0$
$3x+a=0 \qquad x-3a=0$
$3x=-a \qquad x=3a$
$x=-\frac{a}{3}$
The solutions are $-\frac{a}{3}$ and $3a$.

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49. $4x^2+8xy+3y^2=0$

$(2x+3y)(2x+y)=0$

$2x+3y=0 \qquad 2x+y=0$

$2x=-3y \qquad 2x=-y$

$x=-\frac{3y}{2} \qquad x=-\frac{y}{2}$

The solutions are $-\frac{3y}{2}$ and $-\frac{y}{2}$.

51. $6x^2+11ax+4a^2=0$

$(2x+a)(3x+4a)=0$

$2x=-a \qquad 3x=-4a$

$x=-\frac{a}{2} \qquad x=-\frac{4a}{3}$

The solutions are $-\frac{a}{2}$ and $-\frac{4a}{3}$.

Objective B Exercises

53. $(x-r_1)(x-r_2)=0$

$(x-2)(x-5)=0$

$x^2-7x+10=0$

55. $(x-r_1)(x-r_2)=0$

$[x-(-2)][x-(-4)]=0$

$(x+2)(x+4)=0$

$x^2+6x+8=0$

57. $(x-r_1)(x-r_2)=0$

$(x-6)[x-(-1)]=0$

$(x-6)(x+1)=0$

$x^2-5x-6=0$

59. $(x-r_1)(x-r_2)=0$

$(x-3)[x-(-3)]=0$

$(x-3)(x+3)=0$

$x^2-9=0$

61. $(x-r_1)(x-r_2)=0$

$(x-4)(x-4)=0$

$x^2-8x+16=0$

63. $(x-r_1)(x-r_2)=0$

$(x-0)(x-5)=0$

$x(x-5)=0$

$x^2-5x=0$

65. $(x-r_1)(x-r_2)=0$

$(x-0)(x-3)=0$

$x(x-3)=0$

$x^2-3x=0$

67. $(x-r_1)(x-r_2)=0$

$(x-3)\left(x-\frac{1}{2}\right)=0$

$x^2-\frac{7}{2}x+\frac{3}{2}=0$

$2\left(x^2-\frac{7}{2}x+\frac{3}{2}\right)=2\cdot 0$

$2x^2-7x+3=0$

69. $(x-r_1)(x-r_2)=0$

$\left[x-\left(-\frac{3}{4}\right)\right](x-2)=0$

$\left(x+\frac{3}{4}\right)(x-2)=0$

$x^2-\frac{5}{4}x-\frac{3}{2}=0$

$4\left(x^2-\frac{5}{4}x-\frac{3}{2}\right)=4\cdot 0$

$4x^2-5x-6=0$

71. $(x-r_1)(x-r_2)=0$

$\left[x-\left(-\frac{5}{3}\right)\right][x-(-2)]=0$

$\left(x+\frac{5}{3}\right)(x+2)=0$

$x^2+\frac{11}{3}x+\frac{10}{3}=0$

$3\left(x^2+\frac{11}{3}x+\frac{10}{3}\right)=3\cdot 0$

$3x^2+11x+10=0$

73. $(x-r_1)(x-r_2)=0$

$\left[x-\left(-\frac{2}{3}\right)\right]\left(x-\frac{2}{3}\right)=0$

$\left(x+\frac{2}{3}\right)\left(x-\frac{2}{3}\right)=0$

$x^2-\frac{4}{9}=0$

$9\left(x^2-\frac{4}{9}\right)=9\cdot 0$

$9x^2-4=0$

75. $(x-r_1)(x-r_2)=0$

$\left(x-\frac{1}{2}\right)\left(x-\frac{1}{3}\right)=0$

$x^2-\frac{5}{6}x+\frac{1}{6}=0$

$6\left(x^2-\frac{5}{6}x+\frac{1}{6}\right)=6\cdot 0$

$6x^2-5x+1=0$

77. $(x-r_1)(x-r_2)=0$

$\left(x-\frac{6}{5}\right)\left[x-\left(-\frac{1}{2}\right)\right]=0$

$\left(x-\frac{6}{5}\right)\left(x+\frac{1}{2}\right)=0$

$x^2-\frac{7}{10}x-\frac{3}{5}=0$

$10\left(x^2-\frac{7}{10}x-\frac{3}{5}\right)=10\cdot 0$

$10x^2-7x-6=0$

79. $(x-r_1)(x-r_2)=0$

$\left[x-\left(-\frac{1}{4}\right)\right]\left[x-\left(-\frac{1}{2}\right)\right]=0$

$\left(x+\frac{1}{4}\right)\left(x+\frac{1}{2}\right)=0$

$x^2+\frac{3}{4}x+\frac{1}{8}=0$

$8\left(x^2+\frac{3}{4}x+\frac{1}{8}\right)=8\cdot 0$

$8x^2+6x+1=0$

81. $(x-r_1)(x-r_2)=0$

$\left(x-\frac{3}{5}\right)\left[x-\left(-\frac{1}{10}\right)\right]=0$

$\left(x-\frac{3}{5}\right)\left(x+\frac{1}{10}\right)=0$

$x^2-\frac{1}{2}x-\frac{3}{50}=0$

$50\left(x^2-\frac{1}{2}x-\frac{3}{50}\right)=50\cdot 0$

$50x^2-25x-3=0$

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Objective C Exercises

83. $y^2 = 49$

$\sqrt{y^2} = \pm\sqrt{49}$

$y = \pm 7$

The solutions are 7 and -7.

85. $z^2 = -4$

$\sqrt{z^2} = \pm\sqrt{-4}$

$z = \pm 2i$

The solutions are $2i$ and $-2i$.

87. $s^2 - 4 = 0$

$s^2 = 4$

$\sqrt{s^2} = \pm\sqrt{4}$

$s = \pm 2$

The solutions are 2 and -2.

89. $4x^2 - 81 = 0$

$4x^2 = 81$

$x^2 = \frac{81}{4}$

$\sqrt{x^2} = \pm\sqrt{\frac{81}{4}}$

$x = \pm\frac{9}{2}$

The solutions are $\frac{9}{2}$ and $-\frac{9}{2}$.

91. $y^2 + 49 = 0$

$y^2 = -49$

$\sqrt{y^2} = \pm\sqrt{-49}$

$y = \pm 7i$

The solutions are $7i$ and $-7i$.

93. $v^2 - 48 = 0$

$v^2 = 48$

$\sqrt{v^2} = \pm\sqrt{48}$

$v = \pm 4\sqrt{3}$

The solutions are $4\sqrt{3}$ and $-4\sqrt{3}$.

95. $r^2 - 75 = 0$

$r^2 = 75$

$\sqrt{r^2} = \pm\sqrt{75}$

$r = \pm 5\sqrt{3}$

The solutions are $5\sqrt{3}$ and $-5\sqrt{3}$.

97. $z^2 + 18 = 0$

$z^2 = -18$

$\sqrt{z^2} = \pm\sqrt{-18}$

$z = \pm 3i\sqrt{2}$

The solutions are $3i\sqrt{2}$ and $-3i\sqrt{2}$.

99. $(x-1)^2 = 36$

$\sqrt{(x-1)^2} = \pm\sqrt{36}$

$x - 1 = \pm 6$

$x - 1 = 6 \qquad x - 1 = -6$

$x = 7 \qquad x = -5$

The solutions are 7 and -5.

101. $3(y+3)^2 = 27$

$(y+3)^2 = 9$

$\sqrt{(y+3)^2} = \pm\sqrt{9}$

$y + 3 = \pm 3$

$y + 3 = 3 \qquad y + 3 = -3$

$y = 0 \qquad y = -6$

The solutions are 0 and -6.

103. $5(z+2)^2 = 125$

$(z+2)^2 = 25$

$\sqrt{(z+2)^2} = \pm\sqrt{25}$

$z + 2 = \pm 5$

$z + 2 = 5 \qquad z + 2 = -5$

$z = 3 \qquad z = -7$

The solutions are 3 and -7.

105. $\left(v - \frac{1}{2}\right)^2 = \frac{1}{4}$

$\sqrt{\left(v - \frac{1}{2}\right)^2} = \pm\sqrt{\frac{1}{4}}$

$v - \frac{1}{2} = \pm\frac{1}{2}$

$v - \frac{1}{2} = \frac{1}{2} \qquad v - \frac{1}{2} = -\frac{1}{2}$

$v = 1 \qquad v = 0$

The solutions are 1 and 0.

107. $(x+5)^2 - 6 = 0$

$(x+5)^2 = 6$

$\sqrt{(x+5)^2} = \pm\sqrt{6}$

$x + 5 = \pm\sqrt{6}$

$x + 5 = \sqrt{6} \qquad x + 5 = -\sqrt{6}$

$x = -5 + \sqrt{6} \qquad x = -5 - \sqrt{6}$

The solutions are $-5 + \sqrt{6}$ and $-5 - \sqrt{6}$.

109. $(v-3)^2 + 45 = 0$

$(v-3)^2 = -45$

$\sqrt{(v-3)^2} = \pm\sqrt{-45}$

$v - 3 = \pm 3i\sqrt{5}$

$v - 3 = 3i\sqrt{5} \qquad v - 3 = -3i\sqrt{5}$

$v = 3 + 3i\sqrt{5} \qquad v = 3 - 3i\sqrt{5}$

The solutions are $3 + 3i\sqrt{5}$ and $3 - 3i\sqrt{5}$.

111. $\left(u - \frac{2}{3}\right)^2 - 18 = 0$

$\left(u - \frac{2}{3}\right)^2 = 18$

$\sqrt{\left(u - \frac{2}{3}\right)^2} = \pm\sqrt{18}$

$u - \frac{2}{3} = \pm 3\sqrt{2}$

$u - \frac{2}{3} = 3\sqrt{2} \qquad u - \frac{2}{3} = -3\sqrt{2}$

$u = \frac{2}{3} + 3\sqrt{2} \qquad u = \frac{2}{3} - 3\sqrt{2}$

$u = \frac{2 + 9\sqrt{2}}{3} \qquad u = \frac{2 - 9\sqrt{2}}{3}$

The solutions are $\frac{2 - 9\sqrt{2}}{3}$ and $\frac{2 + 9\sqrt{2}}{3}$.

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Applying the Concepts

113.
$$(x-r_1)(x-r_2)=0$$
$$\left(x-\sqrt{2}\right)\left[x-\left(-\sqrt{2}\right)\right]=0$$
$$\left(x-\sqrt{2}\right)\left(x+\sqrt{2}\right)=0$$
$$x^2-2=0$$

115.
$$(x-r_1)(x-r_2)=0$$
$$\left(x-3\sqrt{2}\right)\left[x-\left(-3\sqrt{2}\right)\right]=0$$
$$\left(x-3\sqrt{2}\right)\left(x+3\sqrt{2}\right)=0$$
$$x^2-18=0$$

117.
$$4a^2x^2=36b^2$$
$$x^2=\frac{36b^2}{4a^2}$$
$$x^2=\frac{9b^2}{a^2}$$
$$\sqrt{x^2}=\pm\sqrt{\frac{9b^2}{a^2}}$$
$$x=\pm\sqrt{\frac{9b^2}{a^2}}=\pm\frac{3b}{a}$$
Since $a>0$ and $b>0$, the solutions are $\frac{3b}{a}$ and $-\frac{3b}{a}$.

119.
$$(2x-1)^2=(2x+3)^2$$
$$\sqrt{(2x-1)^2}=\pm\sqrt{(2x+3)^2}$$
$$2x-1=\pm\sqrt{(2x+3)^2}=\pm(2x+3)$$
$2x-1=2x+3$ $\qquad$ $2x-1=-(2x+3)$
$-1=3$ $\qquad$ $4x=-2$
meaningless solution $\qquad$ $x=-\frac{1}{2}$
The solution is $-\frac{1}{2}$.

Section 10.2

Objective A Exercises

3.
$$x^2-4x-5=0$$
$$x^2-4x=5$$
Complete the square.
$$x^2-4x+4=5+4$$
$$(x-2)^2=9$$
$$\sqrt{(x-2)^2}=\pm\sqrt{9}$$
$$x-2=\pm3$$
$x-2=3$ $\quad$ $x-2=-3$
$x=5$ $\quad$ $x=-1$
The solutions are 5 and –1.

5.
$$v^2+8v-9=0$$
$$v^2+8v=9$$
Complete the square.
$$v^2+8v+16=9+16$$
$$(v+4)^2=25$$
$$\sqrt{(v+4)^2}=\pm\sqrt{25}$$
$$v+4=\pm5$$
$v+4=5$ $\quad$ $v+4=-5$
$v=1$ $\quad$ $v=-9$
The solutions are 1 and –9.

7.
$$z^2-6z+9=0$$
$$z^2-6z=-9$$
Complete the square.
$$z^2-6z+9=-9+9$$
$$(z-3)^2=0$$
$$\sqrt{(z-3)^2}=\pm\sqrt{0}$$
$$z-3=0$$
$$z=3$$
The solution is 3.

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9. $r^2+4r-7=0$

$r^2+4r=7$

Complete the square.

$r^2+4r+4=7+4$

$(r+2)^2=11$

$\sqrt{(r+2)^2}=\pm\sqrt{11}$

$r+2=\pm\sqrt{11}$

$r+2=\sqrt{11}$ $\quad r+2=-\sqrt{11}$

$r=-2+\sqrt{11}$ $\quad r=-2-\sqrt{11}$

The solutions are $-2+\sqrt{11}$ and $-2-\sqrt{11}$.

11. $x^2-6x+7=0$

$x^2-6x=-7$

Complete the square.

$x^2-6x+9=-7+9$

$(x-3)^2=2$

$\sqrt{(x-3)^2}=\pm\sqrt{2}$

$x-3=\pm\sqrt{2}$

$x-3=\sqrt{2}$ $\quad x-3=-\sqrt{2}$

$x=3+\sqrt{2}$ $\quad x=3-\sqrt{2}$

The solutions are $3+\sqrt{2}$ and $3-\sqrt{2}$.

13. $z^2-2z+2=0$

$z^2-2z=-2$

Complete the square.

$z^2-2z+1=-2+1$

$(z-1)^2=-1$

$\sqrt{(z-1)^2}=\pm\sqrt{-1}$

$z-1=\pm i$

$z-1=i$ $\quad z-1=-i$

$z=1+i$ $\quad z=1-i$

The solutions are $1+i$ and $1-i$.

15. $s^2-5s-24=0$

$s^2-5s=24$

Complete the square.

$s^2-5s+\frac{25}{4}=24+\frac{25}{4}$

$\left(s-\frac{5}{2}\right)^2=\frac{121}{4}$

$\sqrt{\left(s-\frac{5}{2}\right)^2}=\pm\sqrt{\frac{121}{4}}$

$s-\frac{5}{2}=\pm\frac{11}{2}$

$s-\frac{5}{2}=\frac{11}{2}$ $\quad s-\frac{5}{2}=-\frac{11}{2}$

$s=\frac{5}{2}+\frac{11}{2}$ $\quad s=\frac{5}{2}-\frac{11}{2}$

$s=\frac{16}{2}=8$ $\quad s=-\frac{6}{2}=-3$

The solutions are 8 and –3.

17. $x^2+5x-36=0$

$x^2+5x=36$

Complete the square.

$x^2+5x+\frac{25}{4}=36+\frac{25}{4}$

$\left(x+\frac{5}{2}\right)^2=\frac{169}{4}$

$\sqrt{\left(x+\frac{5}{2}\right)^2}=\pm\sqrt{\frac{169}{4}}$

$x+\frac{5}{2}=\pm\frac{13}{2}$

$x+\frac{5}{2}=\frac{13}{2}$ $\quad x+\frac{5}{2}=-\frac{13}{2}$

$x=-\frac{5}{2}+\frac{13}{2}$ $\quad x=-\frac{5}{2}-\frac{13}{2}$

$x=\frac{8}{2}=4$ $\quad x=-\frac{18}{2}=-9$

The solutions are 4 and –9.

19. $p^2-3p+1=0$

$p^2-3p=-1$

Complete the square.

$p^2-3p+\frac{9}{4}=-1+\frac{9}{4}$

$\left(p-\frac{3}{2}\right)^2=\frac{5}{4}$

$\sqrt{\left(p-\frac{3}{2}\right)^2}=\pm\sqrt{\frac{5}{4}}$

$p-\frac{3}{2}=\pm\frac{\sqrt{5}}{2}$

$p-\frac{3}{2}=\frac{\sqrt{5}}{2}$ $\quad p-\frac{3}{2}=-\frac{\sqrt{5}}{2}$

$p=\frac{3}{2}+\frac{\sqrt{5}}{2}$ $\quad p=\frac{3}{2}-\frac{\sqrt{5}}{2}$

The solutions are $\frac{3+\sqrt{5}}{2}$ and $\frac{3-\sqrt{5}}{2}$.

21. $t^2-t-1=0$

$t^2-t=1$

Complete the square.

$t^2-t+\frac{1}{4}=1+\frac{1}{4}$

$\left(t-\frac{1}{2}\right)^2=\frac{5}{4}$

$\sqrt{\left(t-\frac{1}{2}\right)^2}=\pm\sqrt{\frac{5}{4}}$

$t-\frac{1}{2}=\pm\frac{\sqrt{5}}{2}$

$t-\frac{1}{2}=\frac{\sqrt{5}}{2}$ $\quad t-\frac{1}{2}=-\frac{\sqrt{5}}{2}$

$t=\frac{1}{2}+\frac{\sqrt{5}}{2}$ $\quad t=\frac{1}{2}-\frac{\sqrt{5}}{2}$

The solutions are $\frac{1+\sqrt{5}}{2}$ and $\frac{1-\sqrt{5}}{2}$.

23. $y^2-6y=4$

Complete the square.

$y^2-6y+9=4+9$

$(y-3)^2=13$

$\sqrt{(y-3)^2}=\pm\sqrt{13}$

$y-3=\pm\sqrt{13}$

$y-3=\sqrt{13}$ $\quad y-3=-\sqrt{13}$

$y=3+\sqrt{13}$ $\quad y=3-\sqrt{13}$

The solutions are $3+\sqrt{13}$ and $3-\sqrt{13}$.

25. $x^2=8x-15$

$x^2-8x=-15$

Complete the square.

$x^2-8x+16=-15+16$

$(x-4)^2=1$

$\sqrt{(x-4)^2}=\pm\sqrt{1}$

$x-4=\pm1$

$x-4=1$ $\quad x-4=-1$

$x=5$ $\quad x=3$

The solutions are 5 and 3.

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27. $v^2 = 4v - 13$

$v^2 - 4v = -13$

Complete the square.

$v^2 - 4v + 4 = -13 + 4$

$(v-2)^2 = -9$

$\sqrt{(v-2)^2} = \pm\sqrt{-9}$

$v - 2 = \pm 3i$

$v - 2 = 3i \qquad v - 2 = -3i$

$v = 2 + 3i \qquad v = 2 - 3i$

The solutions are $2 + 3i$ and $2 - 3i$.

29. $p^2 + 6p = -13$

Complete the square.

$p^2 + 6p + 9 = -13 + 9$

$(p+3)^2 = -4$

$\sqrt{(p+3)^2} = \pm\sqrt{-4}$

$p + 3 = \pm 2i$

$p + 3 = 2i \qquad p + 3 = -2i$

$p = -3 + 2i \qquad p = -3 - 2i$

The solutions are $-3 + 2i$ and $-3 - 2i$.

31. $y^2 - 2y = 17$

Complete the square.

$y^2 - 2y + 1 = 17 + 1$

$(y-1)^2 = 18$

$\sqrt{(y-1)^2} = \pm\sqrt{18}$

$y - 1 = \pm 3\sqrt{2}$

$y - 1 = 3\sqrt{2} \qquad y - 1 = -3\sqrt{2}$

$y = 1 + 3\sqrt{2} \qquad y = 1 - 3\sqrt{2}$

The solutions are $1 + 3\sqrt{2}$ and $1 - 3\sqrt{2}$.

33. $z^2 = z + 4$

$z^2 - z = 4$

Complete the square.

$z^2 - z + \frac{1}{4} = 4 + \frac{1}{4}$

$\left(z - \frac{1}{2}\right)^2 = \frac{17}{4}$

$\sqrt{\left(z - \frac{1}{2}\right)^2} = \pm\sqrt{\frac{17}{4}}$

$z - \frac{1}{2} = \pm\frac{\sqrt{17}}{2}$

$z - \frac{1}{2} = \frac{\sqrt{17}}{2} \qquad z - \frac{1}{2} = -\frac{\sqrt{17}}{2}$

$z = \frac{1}{2} + \frac{\sqrt{17}}{2} \qquad z = \frac{1}{2} - \frac{\sqrt{17}}{2}$

The solutions are $\frac{1+\sqrt{17}}{2}$ and $\frac{1-\sqrt{17}}{2}$.

35. $x^2 + 13 = 2x$

$x^2 - 2x = -13$

Complete the square.

$x^2 - 2x + 1 = -13 + 1$

$(x-1)^2 = -12$

$\sqrt{(x-1)^2} = \pm\sqrt{-12}$

$x - 1 = \pm 2i\sqrt{3}$

$x - 1 = 2i\sqrt{3} \qquad x - 1 = -2i\sqrt{3}$

$x = 1 + 2i\sqrt{3} \qquad x = 1 - 2i\sqrt{3}$

The solutions are $1 + 2i\sqrt{3}$ and $1 - 2i\sqrt{3}$.

37. $4x^2 - 4x + 5 = 0$

$4x^2 - 4x = -5$

$\frac{1}{4}(4x^2 - 4x) = \frac{1}{4}(-5)$

$x^2 - x = -\frac{5}{4}$

Complete the square.

$x^2 - x + \frac{1}{4} = -\frac{5}{4} + \frac{1}{4}$

$\left(x - \frac{1}{2}\right)^2 = -1$

$\sqrt{\left(x - \frac{1}{2}\right)^2} = \pm\sqrt{-1}$

$x - \frac{1}{2} = \pm i$

$x - \frac{1}{2} = i \qquad x - \frac{1}{2} = -i$

$x = \frac{1}{2} + i \qquad x = \frac{1}{2} - i$

The solutions are $\frac{1}{2} + i$ and $\frac{1}{2} - i$.

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39. $9x^2 - 6x + 2 = 0$

$9x^2 - 6x = -2$

$\frac{1}{9}(9x^2 - 6x) = \frac{1}{9}(-2)$

$x^2 - \frac{2}{3}x = -\frac{2}{9}$

Complete the square.

$x^2 - \frac{2}{3}x + \frac{1}{9} = -\frac{2}{9} + \frac{1}{9}$

$\left(x - \frac{1}{3}\right)^2 = -\frac{1}{9}$

$\sqrt{\left(x - \frac{1}{3}\right)^2} = \pm\sqrt{-\frac{1}{9}}$

$x - \frac{1}{3} = \pm\frac{1}{3}i$

$x - \frac{1}{3} = \frac{1}{3}i \qquad x - \frac{1}{3} = -\frac{1}{3}i$

$x = \frac{1}{3} + \frac{1}{3}i \qquad x = \frac{1}{3} - \frac{1}{3}i$

The solutions are $\frac{1}{3} + \frac{1}{3}i$ and $\frac{1}{3} - \frac{1}{3}i$.

41. $2s^2 = 4s + 5$

$2s^2 - 4s = 5$

$\frac{1}{2}(2s^2 - 4s) = \frac{1}{2}(5)$

$s^2 - 2s = \frac{5}{2}$

Complete the square.

$s^2 - 2s + 1 = \frac{5}{2} + 1$

$(s-1)^2 = \frac{7}{2}$

$\sqrt{(s-1)^2} = \pm\sqrt{\frac{7}{2}}$

$s - 1 = \pm\sqrt{\frac{7}{2}} = \pm\frac{\sqrt{14}}{2}$

$s - 1 = \frac{\sqrt{14}}{2} \qquad s - 1 = -\frac{\sqrt{14}}{2}$

$s = \frac{2}{2} + \frac{\sqrt{14}}{2} \qquad s = \frac{2}{2} - \frac{\sqrt{14}}{2}$

The solutions are $\frac{2+\sqrt{14}}{2}$ and $\frac{2-\sqrt{14}}{2}$.

43. $2r^2 = 3 - r$

$2r^2 + r = 3$

$\frac{1}{2}(2r^2 + r) = \frac{1}{2}(3)$

$r^2 + \frac{1}{2}r = \frac{3}{2}$

Complete the square.

$r^2 + \frac{1}{2}r + \frac{1}{16} = \frac{3}{2} + \frac{1}{16}$

$\left(r + \frac{1}{4}\right)^2 = \frac{25}{16}$

$\sqrt{\left(r + \frac{1}{4}\right)^2} = \pm\sqrt{\frac{25}{16}}$

$r + \frac{1}{4} = \pm\frac{5}{4}$

$r + \frac{1}{4} = \frac{5}{4} \qquad r + \frac{1}{4} = -\frac{5}{4}$

$r = \frac{4}{4} = 1 \qquad r = -\frac{6}{4} = -\frac{3}{2}$

The solutions are 1 and $-\frac{3}{2}$.

45. $y - 2 = (y-3)(y+2)$

$y - 2 = y^2 - y - 6$

$y^2 - 2y = 4$

Complete the square.

$y^2 - 2y + 1 = 4 + 1$

$(y-1)^2 = 5$

$\sqrt{(y-1)^2} = \pm\sqrt{5}$

$y - 1 = \pm\sqrt{5}$

$y - 1 = \sqrt{5} \qquad y - 1 = -\sqrt{5}$

$y = 1 + \sqrt{5} \qquad y = 1 - \sqrt{5}$

The solutions are $1 + \sqrt{5}$ and $1 - \sqrt{5}$.

47. $6t - 2 = (2t-3)(t-1)$

$6t - 2 = 2t^2 - 5t + 3$

$2t^2 - 11t = -5$

$\frac{1}{2}(2t^2 - 11t) = \frac{1}{2}(-5)$

$t^2 - \frac{11}{2}t = -\frac{5}{2}$

Complete the square.

$t^2 - \frac{11}{2}t + \frac{121}{16} = -\frac{5}{2} + \frac{121}{16}$

$\left(t - \frac{11}{4}\right)^2 = \frac{81}{16}$

$\sqrt{\left(t - \frac{11}{4}\right)^2} = \pm\sqrt{\frac{81}{16}}$

$t - \frac{11}{4} = \pm\frac{9}{4}$

$t - \frac{11}{4} = \frac{9}{4} \qquad t - \frac{11}{4} = -\frac{9}{4}$

$t = \frac{20}{4} = 5 \qquad t = \frac{2}{4} = \frac{1}{2}$

The solutions are 5 and $\frac{1}{2}$.

49. $(x-4)(x+1) = x - 3$

$x^2 - 3x - 4 = x - 3$

$x^2 - 4x = 1$

Complete the square.

$x^2 - 4x + 4 = 1 + 4$

$(x-2)^2 = 5$

$\sqrt{(x-2)^2} = \pm\sqrt{5}$

$x - 2 = \pm\sqrt{5}$

$x - 2 = \sqrt{5} \qquad x - 2 = -\sqrt{5}$

$x = 2 + \sqrt{5} \qquad x = 2 - \sqrt{5}$

The solutions are $2 + \sqrt{5}$ and $2 - \sqrt{5}$.

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Applying the Concepts

51.
$$z^2+2z=4$$
$$z^2+2z+1=4+1$$
$$(z+1)^2=5$$
$$\sqrt{(z+1)^2}=\pm\sqrt{5}$$
$$z+1=\pm\sqrt{5}$$
$$z+1=\sqrt{5} \qquad z+1=-\sqrt{5}$$
$$z=-1+\sqrt{5} \qquad z=-1-\sqrt{5}$$
$$z=1.236 \qquad z=-3.236$$

53.
$$2x^2=4x-1$$
$$2x^2-4x=-1$$
$$x^2-2x=-\frac{1}{2}$$
$$x^2-2x+1=-\frac{1}{2}+1$$
$$(x-1)^2=\frac{1}{2}$$
$$\sqrt{(x-1)^2}=\pm\sqrt{\frac{1}{2}}$$
$$x-1=\pm\sqrt{\frac{1}{2}}$$
$$x-1=\sqrt{\frac{1}{2}} \qquad x-1=-\sqrt{\frac{1}{2}}$$
$$x=1+\sqrt{\frac{1}{2}} \qquad x=1-\sqrt{\frac{1}{2}}$$
$$x=1.707 \qquad x=0.293$$

55.
$$4z^2+2z=1$$
$$z^2+\frac{1}{2}z=\frac{1}{4}$$
$$z^2+\frac{1}{2}z+\frac{1}{16}=\frac{1}{4}+\frac{1}{16}$$
$$\left(z+\frac{1}{4}\right)^2=\frac{5}{16}$$
$$\sqrt{\left(z+\frac{1}{4}\right)^2}=\pm\sqrt{\frac{5}{16}}$$
$$z+\frac{1}{4}=\pm\frac{\sqrt{5}}{4}$$
$$z+\frac{1}{4}=\frac{\sqrt{5}}{4} \qquad z+\frac{1}{4}=-\frac{\sqrt{5}}{4}$$
$$z=-\frac{1}{4}+\frac{\sqrt{5}}{4} \qquad z=-\frac{1}{4}-\frac{\sqrt{5}}{4}$$
$$z=0.309 \qquad z=-0.809$$

57.
$$x^2-ax-2a^2=0$$
$$x^2-ax=2a^2$$
Complete the square.
$$x^2-ax+\frac{1}{4}a^2=2a^2+\frac{1}{4}a^2$$
$$\left(x-\frac{1}{2}a\right)^2=\frac{9}{4}a^2$$
$$\sqrt{\left(x-\frac{1}{2}a\right)^2}=\pm\sqrt{\frac{9}{4}a^2}$$
$$x-\frac{1}{2}a=\pm\sqrt{\frac{9}{4}a^2}=\pm\frac{3}{2}a$$
$$x-\frac{1}{2}a=\frac{3}{2}a \qquad x-\frac{1}{2}a=-\frac{3}{2}a$$
$$x=2a \qquad x=-a$$
The solutions are $2a$ and $-a$.

59.
$$x^2+3ax-10a^2=0$$
$$x^2+3ax=10a^2$$
Complete the square.
$$x^2+3ax+\frac{9}{4}a^2=10a^2+\frac{9}{4}a^2$$
$$\left(x+\frac{3}{2}a\right)^2=\frac{49}{4}a^2$$
$$\sqrt{\left(x+\frac{3}{2}a\right)^2}=\pm\sqrt{\frac{49}{4}a^2}$$
$$x+\frac{3}{2}a=\pm\frac{7}{2}a$$
$$x+\frac{3}{2}a=\frac{7}{2}a \qquad x+\frac{3}{2}a=-\frac{7}{2}a$$
$$x=2a \qquad x=-5a$$
The solutions are $2a$ and $-5a$.

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Section 10.3

Objective A Exercises

3. $x^2 - 3x - 10 = 0$

$a = 1,\ b = -3,\ c = -10$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-10)}}{2(1)}$$

$$= \frac{3 \pm \sqrt{9 + 40}}{2} = \frac{3 \pm \sqrt{49}}{2} = \frac{3 + 7}{2}$$

$$x = \frac{3+7}{2} \quad x = \frac{3-7}{2}$$

$$= \frac{10}{2} = 5 \quad = \frac{-4}{2} = -2$$

The solutions are 5 and –2.

5. $y^2 + 5y - 36 = 0$

$a = 1,\ b = 5,\ c = -36$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-5 \pm \sqrt{(5)^2 - 4(1)(-36)}}{2(1)}$$

$$= \frac{-5 \pm \sqrt{25 + 144}}{2} = \frac{-5 \pm \sqrt{169}}{2}$$

$$= \frac{-5 \pm 13}{2}$$

$$y = \frac{-5+13}{2} \quad y = \frac{-5-13}{2}$$

$$= \frac{8}{2} = 4 \quad = \frac{-18}{2} = -9$$

The solutions are 4 and –9.

7. $w^2 = 8w + 72$

$w^2 - 8w - 72 = 0$

$a = 1,\ b = -8,\ c = -72$

$$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(-72)}}{2(1)}$$

$$= \frac{8 \pm \sqrt{64 + 288}}{2} = \frac{8 \pm \sqrt{352}}{2}$$

$$= \frac{8 \pm 4\sqrt{22}}{2} = 4 \pm 2\sqrt{22}$$

The solutions are $4 + 2\sqrt{22}$ and $4 - 2\sqrt{22}$.

9. $v^2 = 24 - 5v$

$v^2 + 5v - 24 = 0$

$a = 1,\ b = 5,\ c = -24$

$$v = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-5 \pm \sqrt{(5)^2 - 4(1)(-24)}}{2(1)}$$

$$= \frac{-5 \pm \sqrt{25 + 96}}{2} = \frac{-5 \pm \sqrt{121}}{2}$$

$$= \frac{-5 \pm 11}{2}$$

$$v = \frac{-5+11}{2} \quad v = \frac{-5-11}{2}$$

$$= \frac{6}{2} = 3 \quad = \frac{-16}{2} = -8$$

The solutions are 3 and –8.

11. $2y^2 + 5y - 1 = 0$

$a = 2,\ b = 5,\ c = -1$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-5 \pm \sqrt{(5)^2 - 4(2)(-1)}}{2(2)}$$

$$= \frac{-5 \pm \sqrt{25 + 8}}{4} = \frac{-5 \pm \sqrt{33}}{4}$$

The solutions are $\frac{-5 + \sqrt{33}}{4}$ and $\frac{-5 - \sqrt{33}}{4}$.

13. $8s^2 = 10s + 3$

$8s^2 - 10s - 3 = 0$

$a = 4,\ b = -10,\ c = 3$

$$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-10) \pm \sqrt{(-10)^2 - 4(8)(-3)}}{2(8)}$$

$$= \frac{10 \pm \sqrt{100 + 96}}{16} = \frac{10 \pm \sqrt{196}}{16}$$

$$= \frac{10 \pm 14}{16}$$

$$s = \frac{10+14}{16} \quad s = \frac{10-14}{16}$$

$$= \frac{24}{16} = \frac{3}{2} \quad = \frac{-4}{16} = -\frac{1}{4}$$

The solutions are $\frac{3}{2}$ and $-\frac{1}{4}$.

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15. $x^2 = 14x - 4$

$x^2 - 14x + 4 = 0$

$a = 1,\ b = -14,\ c = 4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-14) \pm \sqrt{(-14)^2 - 4(1)(4)}}{2(1)}$$

$$= \frac{14 \pm \sqrt{196 - 16}}{2} = \frac{14 \pm \sqrt{180}}{2}$$

$$= \frac{14 \pm 6\sqrt{5}}{2} = 7 \pm 3\sqrt{5}$$

The solutions are $7 + 3\sqrt{5}$ and $7 - 3\sqrt{5}$.

17. $2z^2 - 2z - 1 = 0$

$a = 2,\ b = -2,\ c = -1$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(-1)}}{2(2)}$$

$$= \frac{2 \pm \sqrt{4 + 8}}{4} = \frac{2 \pm \sqrt{12}}{4}$$

$$= \frac{2 \pm 2\sqrt{3}}{4} = \frac{1 \pm \sqrt{3}}{2}$$

The solutions are $\frac{1 + \sqrt{3}}{2}$ and $\frac{1 - \sqrt{3}}{2}$.

19. $z^2 + 2z + 2 = 0$

$a = 1,\ b = 2,\ c = 2$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{(2)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{4 - 8}}{2} = \frac{-2 \pm \sqrt{-4}}{2}$$

$$= \frac{-2 \pm 2i}{2} = -1 \pm i$$

The solutions are $-1 + i$ and $-1 - i$.

21. $y^2 - 2y + 5 = 0$

$a = 1,\ b = -2,\ c = 5$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(5)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4 - 20}}{2} = \frac{2 \pm \sqrt{-16}}{2}$$

$$= \frac{2 \pm 4i}{2} = 1 \pm 2i$$

The solutions are $1 + 2i$ and $1 - 2i$.

23. $s^2 - 4s + 13 = 0$

$a = 1,\ b = -4,\ c = 13$

$$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 - 52}}{2} = \frac{4 \pm \sqrt{-36}}{2}$$

$$= \frac{4 \pm 6i}{2} = 2 \pm 3i$$

The solutions are $2 + 3i$ and $2 - 3i$.

25. $2w^2 - 2w - 5 = 0$

$a = 2,\ b = -2,\ c = -5$

$$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(-5)}}{2(2)}$$

$$= \frac{2 \pm \sqrt{4 + 40}}{4} = \frac{2 \pm \sqrt{44}}{4}$$

$$= \frac{2 \pm 2\sqrt{11}}{4} = \frac{1 \pm \sqrt{11}}{2}$$

The solutions are $\frac{1 + \sqrt{11}}{2}$ and $\frac{1 - \sqrt{11}}{2}$.

27. $2x^2 + 6x + 5 = 0$

$a = 2,\ b = 6,\ c = 5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-6 \pm \sqrt{(6)^2 - 4(2)(5)}}{2(2)}$$

$$= \frac{-6 \pm \sqrt{36 - 40}}{4} = \frac{-6 \pm \sqrt{-4}}{4}$$

$$= \frac{-6 \pm 2i}{4} = \frac{-3 \pm i}{2}$$

The solutions are $-\frac{3}{2} + \frac{1}{2}i$ and $-\frac{3}{2} - \frac{1}{2}i$.

29. $4t^2 - 6t + 9 = 0$

$a = 4,\ b = -6,\ c = 9$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(4)(9)}}{2(4)}$$

$$= \frac{6 \pm \sqrt{36 - 144}}{8} = \frac{6 \pm \sqrt{-108}}{8}$$

$$= \frac{6 \pm 6i\sqrt{3}}{8} = \frac{3 \pm 3i\sqrt{3}}{4}$$

The solutions are $\frac{3}{4} + \frac{3\sqrt{3}}{4}i$ and $\frac{3}{4} - \frac{3\sqrt{3}}{4}i$.

31. $p^2 - 8p + 3 = 0$

$a = 1,\ b = -8,\ c = 3$

$$p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(3)}}{2(1)}$$

$$= \frac{8 \pm \sqrt{64 - 12}}{2}$$

$$= \frac{8 \pm \sqrt{52}}{2}$$

$$= \frac{8 \pm 2\sqrt{13}}{2}$$

$$= 4 \pm \sqrt{13}$$

$$\approx 4 \pm 3.606$$

$p \approx 4 + 3.606$ $\quad$ $p \approx 4 - 3.606$

≈ 7.606 $\quad$ ≈ 0.394

The solutions are approximately 7.606 and 0.394.

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33. $w^2+4w-1=0$

$a=1,\ b=4,\ c=-1$

$$w=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-4\pm\sqrt{(4)^2-4(1)(-1)}}{2(1)}$$

$$=\frac{-4\pm\sqrt{16+4}}{2}$$

$$=\frac{-4\pm\sqrt{20}}{2}$$

$$=\frac{-4\pm2\sqrt{5}}{2}$$

$$=-2\pm\sqrt{5}$$

$$\approx-2\pm2.236$$

$w\approx-2+2.236$ ≈0.236 $\qquad$ $w\approx-2-2.236$ ≈-4.236

The solutions are approximately 0.236 and –4.236.

35. $2y^2=y+5$

$2y^2-y-5=0$

$a=2,\ b=-1,\ c=-5$

$$y=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-(-1)\pm\sqrt{(-1)^2-4(2)(-5)}}{2(2)}$$

$$=\frac{1\pm\sqrt{1+40}}{4}$$

$$=\frac{1\pm\sqrt{41}}{4}$$

$$\approx\frac{1\pm6.403}{4}$$

$y\approx\dfrac{1+6.403}{4}\approx1.851$ $\qquad$ $y\approx\dfrac{1-6.403}{4}\approx-1.351$

The solutions are approximately 1.851 and –1.351.

37. $3y^2+y+1=0$

$a=3,\ b=1,\ c=1$

b^2-4ac

$1^2-4(3)(1)=1-12=-11$

$-11<0$

Since the discriminant is less than zero, the equation has two complex number solutions.

39. $4x^2+20x+25=0$

$a=4,\ b=20,\ c=25$

b^2-4ac

$20^2-4(4)(25)=400-400=0$

Since the discriminant is equal to zero, the equation has one real number solution, a double root.

41. $3w^2+3w-2=0$

$a=3,\ b=3,\ c=-2$

b^2-4ac

$3^2-4(3)(-2)=9+24=33$

$33>0$

Since the discriminant is greater than zero, the equation has two real number solutions that are not equal.

Applying the Concepts

43. $x^2-6x+p=0$

$x^2-6x=-p$

Complete the square.

$x^2-6x+9=-p+9$

$(x-3)^2=-p+9$

$\sqrt{(x-3)^2}=\sqrt{9-p}$

$x-3=\pm\sqrt{9-p}$

$x=3\pm\sqrt{9-p}$

x will be real, and there will be two nonequal solutions if $9-p>0$. Solving the inequality gives $p<9$. The values of p are $\{p|p<9\}$.

45. $x^2-2x+p=0$

$a=1,\ b=-2,\ c=p$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-(-2)\pm\sqrt{(-2)^2-4(p)(1)}}{2(1)}$$

$$=\frac{2\pm\sqrt{4-4p}}{2}=1\pm\sqrt{1-p}$$

x will be complex, and there will be two solutions if $1-p<0$. Solving the inequality gives $p>1$. The values of p are $\{p|p>1\}$.

47. $x^2+ix+2=0$

$a=1,\ b=i,\ c=2$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-i\pm\sqrt{i^2-4(1)(2)}}{2(1)}$$

$$=\frac{-i\pm\sqrt{-1-8}}{2}=\frac{-i\pm\sqrt{-9}}{2}$$

$$=\frac{-i\pm3i}{2}$$

$x=\dfrac{-i+3i}{2}=\dfrac{2i}{2}=i$ $\qquad$ $x=\dfrac{-i-3i}{2}=\dfrac{-4i}{2}=-2i$

The possible solutions are i and $-2i$.

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Section 10.4

Objective A Exercises

1. $x^4 - 13x^2 + 36 = 0$

$(x^2)^2 - 13(x^2) + 36 = 0$

$u^2 - 13u + 36 = 0$

$(u - 4)(u - 9) = 0$

$u - 4 = 0 \quad u - 9 = 0$

$u = 4 \qquad u = 9$

Replace u by x^2.

$x^2 = 4 \qquad x^2 = 9$

$\sqrt{x^2} = \pm\sqrt{4} \quad \sqrt{x^2} = \pm\sqrt{9}$

$x = \pm 2 \qquad x = \pm 3$

The solutions are 2, –2, 3, and –3.

3. $z^4 - 6z^2 + 8 = 0$

$(z^2)^2 - 6(z^2) + 8 = 0$

$u^2 - 6u + 8 = 0$

$(u - 4)(u - 2) = 0$

$u - 4 = 0 \quad u - 2 = 0$

$u = 4 \qquad u = 2$

Replace u by z^2.

$z^2 = 4 \qquad z^2 = 2$

$\sqrt{z^2} = \pm\sqrt{4} \quad \sqrt{z^2} = \pm\sqrt{2}$

$z = \pm 2 \qquad z = \pm 2$

The solutions are 2, –2, $\sqrt{2}$, and $-\sqrt{2}$.

5. $p - 3p^{1/2} + 2 = 0$

$(p^{1/2})^2 - 3(p^{1/2}) + 2 = 0$

$u^2 - 3u + 2 = 0$

$(u - 1)(u - 2) = 0$

$u - 1 = 0 \quad u - 2 = 0$

$u = 1 \qquad u = 2$

Replace u by $p^{1/2}$.

$p^{1/2} = 1 \qquad p^{1/2} = 2$

$(p^{1/2})^2 = 1^2 \quad (p^{1/2})^2 = 2^2$

$p = 1 \qquad p = 4$

The solutions are 1 and 4.

7. $x - x^{1/2} - 12 = 0$

$(x^{1/2})^2 - (x^{1/2}) - 12 = 0$

$u^2 - u - 12 = 0$

$(u + 3)(u - 4) = 0$

$u + 3 = 0 \quad u - 4 = 0$

$u = -3 \qquad u = 4$

Replace u by $x^{1/2}$.

$x^{1/2} = -3 \qquad x^{1/2} = 4$

$(x^{1/2})^2 = (-3)^2 \quad (x^{1/2})^2 = 4^2$

$x = 9 \qquad x = 16$

16 checks as a solution. 9 does not check as a solution. The solution is 16.

9. $z^4 + 3z^2 - 4 = 0$

$(z^2)^2 + 3(z^2) - 4 = 0$

$u^2 + 3u - 4 = 0$

$(u + 4)(u - 1) = 0$

$u + 4 = 0 \quad u - 1 = 0$

$u = -4 \qquad u = 1$

Replace u by z^2.

$z^2 = -4 \qquad z^2 = 1$

$\sqrt{z^2} = \pm\sqrt{-4} \quad \sqrt{z^2} = \pm\sqrt{1}$

$z = \pm 2i \qquad z = \pm 1$

The solutions are $2i$, $-2i$, 1, and –1.

11. $x^4 + 12x^2 - 64 = 0$

$(x^2)^2 + 12(x^2) - 64 = 0$

$u^2 + 12u - 64 = 0$

$(u + 16)(u - 4) = 0$

$u + 16 = 0 \quad u - 4 = 0$

$u = -16 \qquad u = 4$

Replace u by x^2.

$x^2 = -16 \qquad x^2 = 4$

$\sqrt{x^2} = \pm\sqrt{-16} \quad \sqrt{x^2} = \pm\sqrt{4}$

$x = \pm 4i \qquad x = \pm 2$

The solutions are $4i$, $-4i$, 2, and –2.

13. $p + 2p^{1/2} - 24 = 0$

$(p^{1/2})^2 + 2(p^{1/2}) - 24 = 0$

$u^2 + 2u - 24 = 0$

$(u + 6)(u - 4) = 0$

$u + 6 = 0 \quad u - 4 = 0$

$u = -6 \qquad u = 4$

Replace u by $p^{1/2}$.

$p^{1/2} = -6 \qquad p^{1/2} = 4$

$(p^{1/2})^2 = (-6)^2 \quad (p^{1/2})^2 = 4^2$

$p = 36 \qquad p = 16$

16 checks as a solution. 36 does not check as a solution. The solution is 16.

15. $y^{2/3} - 9y^{1/3} + 8 = 0$

$(y^{1/3})^2 - 9(y^{1/3}) + 8 = 0$

$u^2 - 9u + 8 = 0$

$(u - 1)(u - 8) = 0$

$u - 1 = 0 \quad u - 8 = 0$

$u = 1 \qquad u = 8$

Replace u by $y^{1/3}$.

$y^{1/3} = 1 \qquad y^{1/3} = 8$

$(y^{1/3})^3 = 1^3 \quad (y^{1/3})^3 = 8^3$

$y = 1 \qquad y = 512$

The solutions are 1 and 512.

17. $9w^4 - 13w^2 + 4 = 0$

$9(w^2)^2 - 13(w^2) + 4 = 0$

$9u^2 - 13u + 4 = 0$

$(9u - 4)(u - 1) = 0$

$9u - 4 = 0 \quad u - 1 = 0$

$9u = 4 \qquad u = 1$

$u = \frac{4}{9}$

Replace u by w^2.

$w^2 = \frac{4}{9} \qquad w^2 = 1$

$\sqrt{w^2} = \pm\sqrt{\frac{4}{9}} \quad \sqrt{w^2} = \pm\sqrt{1}$

$w = \pm\frac{2}{3} \qquad w = \pm 1$

The solutions are $\frac{2}{3}$, $-\frac{2}{3}$, 1, and –1.

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Objective B Exercises

19. $\sqrt{x+1}+x=5$

$\sqrt{x+1}=5-x$

$\left(\sqrt{x+1}\right)^2=(5-x)^2$

$x+1=25-10x+x^2$

$0=24-11x+x^2$

$0=(3-x)(8-x)$

$3-x=0 \quad 8-x=0$

$3=x \qquad 8=x$

3 checks as a solution. 8 does not check as a solution. The solution is 3.

21. $x=\sqrt{x}+6$

$x-6=\sqrt{x}$

$(x-6)^2=\left(\sqrt{x}\right)^2$

$x^2-12x+36=x$

$x^2-13x+36=0$

$(x-4)(x-9)=0$

$x-4=0 \quad x-9=0$

$x=4 \qquad x=9$

9 checks as a solution. 4 does not check as a solution. The solution is 9.

23. $\sqrt{3w+3}=w+1$

$\left(\sqrt{3w+3}\right)^2=(w+1)^2$

$3w+3=w^2+2w+1$

$0=w^2-w-2$

$0=(w-2)(w+1)$

$w-2=0 \quad w+1=0$

$w=2 \qquad w=-1$

2 and –1 check as solutions. The solutions are 2 and –1.

25. $\sqrt{4y+1}-y=1$

$\sqrt{4y+1}=y+1$

$\left(\sqrt{4y+1}\right)^2=(y+1)^2$

$4y+1=y^2+2y+1$

$0=y^2-2y$

$0=y(y-2)$

$y=0 \quad y-2=0$

$y=2$

0 and 2 check as solutions. The solutions are 0 and 2.

27. $\sqrt{10x+5}-2x=1$

$\sqrt{10x+5}=2x+1$

$\left(\sqrt{10x+5}\right)^2=(2x+1)^2$

$10x+5=4x^2+4x+1$

$0=4x^2-6x-4$

$0=2(2x^2-3x-2)$

$0=2(2x+1)(x-2)$

$2x+1=0 \qquad x-2=0$

$2x=-1 \qquad x=2$

$x=-\frac{1}{2}$

$-\frac{1}{2}$ and 2 check as solutions. The solutions are $-\frac{1}{2}$ and 2.

29. $\sqrt{p+11}=1-p$

$\left(\sqrt{p+11}\right)^2=(1-p)^2$

$p+11=1-2p+p^2$

$0=-10-3p+p^2$

$0=p^2-3p-10$

$0=(p-5)(p+2)$

$p-5=0 \quad p+2=0$

$p=5 \qquad p=-2$

–2 checks as a solution. 5 does not check as a solution. The solution is –2.

31. $\sqrt{x-1}-\sqrt{x}=-1$

$\sqrt{x-1}=\sqrt{x}-1$

$\left(\sqrt{x-1}\right)^2=\left(\sqrt{x}-1\right)^2$

$x-1=x-2\sqrt{x}+1$

$2\sqrt{x}=2$

$\sqrt{x}=1$

$\left(\sqrt{x}\right)^2=1^2$

$x=1$

1 checks as a solution. The solution is 1.

33. $\sqrt{2x-1}=1-\sqrt{x-1}$

$\left(\sqrt{2x-1}\right)^2=\left(1-\sqrt{x-1}\right)^2$

$2x-1=1-2\sqrt{x-1}+x-1$

$2\sqrt{x-1}=-x+1$

$\left(2\sqrt{x-1}\right)^2=(-x+1)^2$

$4(x-1)=x^2-2x+1$

$4x-4=x^2-2x+1$

$0=x^2-6x+5$

$0=(x-5)(x-1)$

$x-5=0 \quad x-1=0$

$x=5 \qquad x=1$

1 checks as a solution. 5 does not check as a solution. The solution is 1.

35. $\sqrt{t+3}+\sqrt{2t+7}=1$

$\sqrt{2t+7}=1-\sqrt{t+3}$

$\left(\sqrt{2t+7}\right)^2=\left(1-\sqrt{t+3}\right)^2$

$2t+7=1-2\sqrt{t+3}+t+3$

$t+3=-2\sqrt{t+3}$

$(t+3)^2=\left(-2\sqrt{t+3}\right)^2$

$t^2+6t+9=4(t+3)$

$t^2+6t+9=4t+12$

$t^2+2t-3=0$

$(t+3)(t-1)=0$

$t+3=0 \quad t-1=0$

$t=-3 \qquad t=1$

–3 checks as a solution. 1 does not check as a solution. The solution is –3.

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Objective C Exercises

37.

$$x = \frac{10}{x-9}$$
$$(x-9)x = (x-9)\frac{10}{x-9}$$
$$x^2 - 9x = 10$$
$$x^2 - 9x - 10 = 0$$
$$(x-10)(x+1) = 0$$
$$x-10 = 0 \quad x+1 = 0$$
$$x = 10 \quad x = -1$$

The solutions are 10 and –1.

39.

$$\frac{t}{t+1} = \frac{-2}{t-1}$$
$$(t-1)(t+1)\frac{t}{t+1} = (t-1)(t+1)\frac{-2}{t-1}$$
$$(t-1)t = (t+1)(-2)$$
$$t^2 - t = -2t - 2$$
$$t^2 + t + 2 = 0$$
$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-1 \pm \sqrt{1^2 - 4(1)(2)}}{2(1)}$$
$$= \frac{-1 \pm \sqrt{1-8}}{2} = \frac{-1 \pm \sqrt{-7}}{2}$$
$$= \frac{-1 \pm i\sqrt{7}}{2}$$

The solutions are $-\frac{1}{2} + \frac{\sqrt{7}}{2}i$ and $-\frac{1}{2} - \frac{\sqrt{7}}{2}i$.

41.

$$\frac{y-1}{y+2} + y = 1$$
$$(y+2)\left(\frac{y-1}{y+2} + y\right) = (y+2)1$$
$$(y+2)\frac{y-1}{y+2} + (y+2)y = y+2$$
$$y - 1 + y^2 + 2y = y + 2$$
$$y^2 + 3y - 1 = y + 2$$
$$y^2 + 2y - 3 = 0$$
$$(y+3)(y-1) = 0$$
$$y+3 = 0 \quad y-1 = 0$$
$$y = -3 \quad y = 1$$

The solutions are –3 and 1.

43.

$$\frac{3r+2}{r+2} - 2r = 1$$
$$(r+2)\left(\frac{3r+2}{r+2} - 2r\right) = (r+2)1$$
$$(r+2)\frac{3r+2}{r+2} - (r+2)2r = r+2$$
$$3r + 2 - 2r^2 - 4r = r + 2$$
$$-2r^2 - r + 2 = r + 2$$
$$-2r^2 - 2r = 0$$
$$-2r(r+1) = 0$$
$$-2r = 0 \quad r+1 = 0$$
$$r = 0 \quad r = -1$$

The solutions are 0 and –1.

45.

$$\frac{2}{2x+1} + \frac{1}{x} = 3$$
$$x(2x+1)\left(\frac{2}{2x+1} + \frac{1}{x}\right) = x(2x+1)3$$
$$x(2x+1)\frac{2}{2x+1} + x(2x+1)\frac{1}{x} = 3x(2x+1)$$
$$2x + 2x + 1 = 6x^2 + 3x$$
$$4x + 1 = 6x^2 + 3x$$
$$0 = 6x^2 - x - 1$$
$$0 = (2x-1)(3x+1)$$
$$2x - 1 = 0 \quad 3x + 1 = 0$$
$$2x = 1 \quad 3x = -1$$
$$x = \frac{1}{2} \quad x = -\frac{1}{3}$$

The solutions are $\frac{1}{2}$ and $-\frac{1}{3}$.

47.

$$\frac{16}{z-2} + \frac{16}{z+2} = 6$$
$$(z-2)(z+2)\left(\frac{16}{z-2} + \frac{16}{z+2}\right) = (z-2)(z+2)6$$
$$(z-2)(z+2)\frac{16}{z-2} + (z-2)(z+2)\frac{16}{z+2} = (z^2-4)6$$
$$(z+2)16 + (z-2)16 = 6z^2 - 24$$
$$16z + 32 + 16z - 32 = 6z^2 - 24$$
$$32z = 6z^2 - 24$$
$$0 = 6z^2 - 32z - 24$$
$$0 = 2(3z^2 - 16z - 12)$$
$$0 = 2(3z+2)(z-6)$$
$$3z + 2 = 0 \quad z - 6 = 0$$
$$3z = -2 \quad z = 6$$
$$z = -\frac{2}{3}$$

The solutions are $-\frac{2}{3}$ and 6.

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49.

$$\frac{t}{t-2}+\frac{2}{t-1}=4$$

$$(t-2)(t-1)\left(\frac{t}{t-2}+\frac{t}{t-1}\right)=(t-2)(t-1)4$$

$$(t-2)(t-1)\frac{t}{t-2}+(t-2)(t-1)\frac{2}{t-1}=\left(t^2-3t+2\right)4$$

$$(t-1)t+(t-2)2=4t^2-12t+8$$

$$t^2-t+2t-4=4t^2-12t+8$$

$$t^2+t-4=4t^2-12t+8$$

$$0=3t^2-13t+12$$

$$0=(3t-4)(t-3)$$

$3t-4=0 \quad t-3=0$

$3t=4 \quad t=3$

$t=\frac{4}{3}$

The solutions are $\frac{4}{3}$ and 3.

51.

$$\frac{5}{2p-1}+\frac{4}{p+1}=2$$

$$(2p-1)(p+1)\left(\frac{5}{2p-1}+\frac{4}{p+1}\right)=(2p-1)(p+1)2$$

$$(2p-1)(p+1)\frac{5}{2p-1}+(2p-1)(p+1)\frac{4}{p+1}=\left(2p^2+p-1\right)2$$

$$(p+1)5+(2p-1)4=4p^2+2p-2$$

$$5p+5+8p-4=4p^2+2p-2$$

$$13p+1=4p^2+2p-2$$

$$0=4p^2-11p-3$$

$$0=(4p+1)(p-3)$$

$4p+1=0 \quad p-3=0$

$4p=-1 \quad p=3$

$p=-\frac{1}{4}$

The solutions are $-\frac{1}{4}$ and 3.

Applying the Concepts

53. $\left(\sqrt{x}+3\right)^2-4\sqrt{x}-17=0$.

Let $u=\sqrt{x}+3$.

$u^2-4(u-3)-17=0$

$u^2-4u+12-17=0$

$u^2-4u-5=0$

$(u-5)(u+1)=0$

$u-5=0 \quad u+1=0$

$u=5 \quad u=-1$

Replace u by $\sqrt{x}+3$.

$\sqrt{x}+3=5 \quad \sqrt{x}+3=-1$

$\sqrt{x}=2 \quad \sqrt{x}=-4$

$\left(\sqrt{x}\right)^2=(2)^2 \quad \left(\sqrt{x}\right)^2=(-4)^2$

$x=4 \quad x=16$

4 checks as a solution. 16 does not check as a solution. The solution is 4.

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Section 10.5

Objective A Application Problems

1. Strategy

- This is a geometry problem.
- The height of the triangle: h
 The base of the triangle: $5h - 1$
- The area of the triangle is 21 cm^2. Use the equation for the area of a triangle $\left(A = \frac{1}{2}bh\right)$.

Solution

$$A = \frac{1}{2}bh$$
$$21 = \frac{1}{2}(5h-1)h$$
$$42 = 5h^2 - h$$
$$0 = 5h^2 - h - 42$$
$$0 = (5h+14)(h-3)$$
$$5h + 14 = 0 \qquad h - 3 = 0$$
$$h = -\frac{14}{5} \qquad h = 3$$

Since the height cannot be negative, $-\frac{14}{5}$ cannot be a solution. $5(3) - 1 = 15 - 1 = 14$. The height is 3 cm. The base is 14 cm.

3. Strategy

- This is a geometry problem.
- The width of the rectangle: x
 The length of the rectangle: $x + 111$
- The area of the rectangle is 104,000 mi^2. Use the equation for the area of a rectangle $(A = L \cdot W)$.

Solution

$$A = L \cdot W$$
$$104{,}000 = (x + 111)x$$
$$104{,}000 = x^2 + 111x$$
$$0 = x^2 + 111x - 104{,}000$$
$$a = 1,\ b = 111,\ c = -104{,}000$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-111 \pm \sqrt{111^2 - 4(1)(-104{,}000)}}{2(1)}$$
$$= \frac{-111 \pm 654.46}{2}$$
$$x = 272,\ -383$$

Since the width of Colorado cannot be negative, –383 cannot be a solution. $x + 111 = 383$. The width of Colorado is 272 mi. The length of Colorado is 383 mi.

5. Strategy

To find the maximum safe speed if the car must stop within a given distance, substitute 60 ft for d and solve for v.

Solution

$$d = 0.04v^2 + 0.5v$$
$$60 = 0.04v^2 + 0.5v$$
$$0 = 0.04v^2 + 0.5v - 60$$
$$0 = v^2 + 12.5v - 1500$$
$$a = 1,\ b = 12.5,\ c = -1500$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-12.5 \pm \sqrt{(12.5)^2 - 4(1)(-1500)}}{2(1)}$$
$$= \frac{-12.5 \pm 78.46}{2}$$
$$x = 33,\ -45$$

Since the car cannot travel at a negative speed, –45 cannot be a solution. The maximum safe speed the car can travel is 33 mph.

7. Strategy

To find the time for a projectile to return to Earth, substitute for height ($s = 0$) and initial velocity ($v_0 = 200$ ft/s) and solve for t.

Solution

$$s = v_0 t - 16t^2$$
$$0 = 200t - 16t^2$$
$$0 = 8t(25 - 2t)$$
$$8t = 0 \qquad 25 - 2t = 0$$
$$t = 0 \qquad t = 12.5$$

The solution $t = 0$ is not possible because the projectile has not yet left Earth. The rocket takes 12.5 s to return to Earth.

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9. Strategy

To find the maximum speed a driver can be going and still be able to stop within 150 m, substitute 150 in for d in the equation $d = 0.019v^2 + 0.69v$ and solve for v.

Solution

$$d = 0.019v^2 + 0.69v$$
$$150 = 0.019v^2 + 0.69v$$
$$0 = 0.019v^2 + 0.69v - 150$$
$$a = 0.019, b = 0.69, c = -150$$
$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-0.69 \pm \sqrt{(0.69)^2 - 4(0.019)(-150)}}{2(0.019)}$$
$$= \frac{-0.69 \pm \sqrt{0.4761 + 11.4}}{0.038} = \frac{-0.69 \pm \sqrt{00.8761}}{0.038}$$
$$t \approx -108.8 \quad \text{or} \quad 72.5$$

The speed cannot be a negative number. A driver can be going approximately 72.5 km/h and still be able to stop within 150 m.

11. Strategy

- This is a work problem.
- Time for the smaller pipe to fill the tank: t
 Time for the larger pipe to fill the tank: $t - 6$

	Rate	Time	Part
Smaller pipe	$\frac{1}{t}$	4	$\frac{4}{t}$
Larger pipe	$\frac{1}{t-6}$	4	$\frac{4}{t-6}$

- The sum of the parts of the task completed must equal 1.

Solution

$$\frac{4}{t} + \frac{4}{t-6} = 1$$
$$t(t-6)\left(\frac{4}{t} + \frac{4}{t-6}\right) = t(t-6)1$$
$$(t-6)4 + 4t = t^2 - 6t$$
$$4t - 24 + 4t = t^2 - 6t$$
$$8t - 24 = t^2 - 6t$$
$$0 = t^2 - 14t + 24$$
$$0 = (t-12)(t-2)$$
$$t - 12 = 0 \qquad t - 2 = 0$$
$$t = 12 \qquad t = 2$$
$$t - 6 = 12 - 6 = 6 \qquad t - 6 = 2 - 6 = -4$$

The solution –4 is not possible, since time cannot be a negative number. It would take the larger pipe 6 min to fill the tank. It would take the smaller pipe 12 min to fill the tank.

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13. Strategy

- This is a distance-rate problem.
- Rate of the wind: w

	Distance	Rate	Time
With wind	4000	$1320+w$	$\frac{4000}{1320+w}$
Against wind	4000	$1320-w$	$\frac{4000}{1320-w}$

- The time with the wind is 0.5 h less than the time against the wind.

Solution

$$\frac{4000}{1320+w}=\frac{4000}{1320-w}-0.5$$

$$(1320+w)(1320-w)\frac{4000}{1320+w}=\left(\frac{4000}{1320-w}-0.5\right)(1320+w)(1320-w)$$

$$4000(1320-w)=4000(1320+w)-0.5(1320+w)(1320-w)$$

$$5{,}280{,}000-4000w=5{,}280{,}000+4000w-871{,}200+0.5w^2$$

$$0=0.5w^2+8000w-871{,}200$$

$$0=w^2+16{,}000w-1{,}742{,}400$$

$a=1,\ b=16{,}000,\ c=-1{,}742{,}400$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-16{,}000\pm\sqrt{(16{,}000)^2-4(1)(-1{,}742{,}400)}}{2(1)}$$

$$=\frac{-16{,}000\pm16{,}216.3}{2}$$

$x=108,\ -16{,}108.15$

Since the wind cannot travel at a negative speed, –16,108 is not a solution. The wind is traveling at a rate of 108 mph.

15. Strategy

- This is a uniform motion problem.
- Rate of crew in calm water: x

	Distance	Rate	Time
With current	5	$r+4$	$\frac{5}{r+4}$
Against current	5	$r-4$	$\frac{5}{r-4}$

- The total trip took 3 h.

Solution

$$\frac{5}{r+4}+\frac{5}{r-4}=3$$

$$(r+4)(r-4)\left(\frac{5}{r+4}+\frac{5}{r-4}\right)=3(r+4)(r-4)$$

$$5(r-4)+5(r+4)=3(r^2-16)$$

$$5r-20+5r+20=3r^2-48$$

$$10r=3r^2-48$$

$$0=3r^2-10r-48$$

$$0=(3r+8)(r-6)$$

$3r+8=0 \qquad r-6=0$

$r=-\frac{8}{3} \qquad r=6$

The rate cannot be a negative number. The rowing rate of the guide is 6 mph.

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Applying the Concepts

17. Strategy To find the radius of the cone, substitute 11.25π in for A and 6 in for s in the equation $A = \pi r^2 + \pi rs$ and solve for r..

Solution

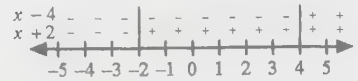
$$A = \pi r^2 + \pi rs$$
$$11.25\pi = \pi r^2 + \pi r(6)$$
$$0 = \pi r^2 + 6\pi r - 11.25\pi$$
$$a = \pi,\ b = 6\pi,\ c = -11.25\pi$$
$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-6\pi \pm \sqrt{(6\pi)^2 - 4(\pi)(-11.25\pi)}}{2(\pi)}$$
$$= \frac{-6\pi \pm \sqrt{36\pi^2 + 45\pi^2}}{2\pi} = \frac{-6\pi \pm \sqrt{81\pi^2}}{2\pi}$$
$$= \frac{-6\pi \pm 9\pi}{2\pi} = \frac{-6 \pm 9}{2}$$
$$t = 1.5 \text{ or } -7.5$$

The radius cannot be a negative number. The radius of the cone is 1.5 in.

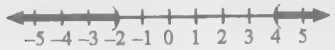
Section 10.6

Objective A Exercises

3. $(x-4)(x+2) > 0$

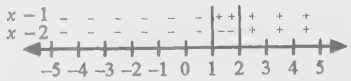


$\{x \mid x < -2 \text{ or } x > 4\}$

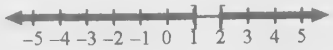


5. $x^2 - 3x + 2 \geq 0$

$(x-1)(x-2) \geq 0$

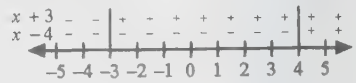


$\{x \mid x \leq 1 \text{ or } x \geq 2\}$



7. $x^2 - x - 12 < 0$

$(x+3)(x-4) < 0$



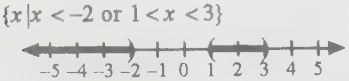
$\{x \mid -3 < x < 4\}$



9. $(x-1)(x+2)(x-3) < 0$



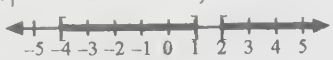
$\{x \mid x < -2 \text{ or } 1 < x < 3\}$



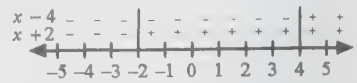
11. $(x+4)(x-2)(x-1) \geq 0$



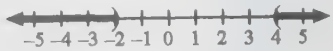
$\{x \mid -4 \leq x \leq 1 \text{ or } x \geq 2\}$



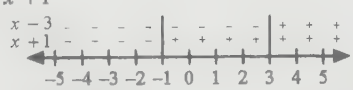
13. $\dfrac{x-4}{x+2} > 0$



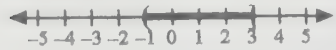
$\{x \mid x < -2 \text{ or } x > 4\}$



15. $\dfrac{x-3}{x+1} \leq 0$



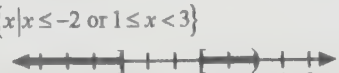
$\{x \mid -1 < x \leq 3\}$



17. $\dfrac{(x-1)(x+2)}{x-3} \leq 0$

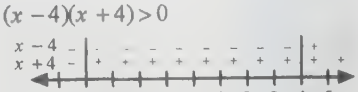


$\{x \mid x \leq -2 \text{ or } 1 \leq x < 3\}$

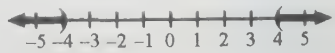


19. $x^2 - 16 > 0$

$(x-4)(x+4) > 0$



$\{x \mid x > 4 \text{ or } x < -4\}$

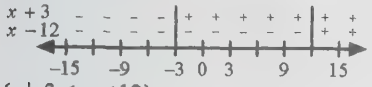


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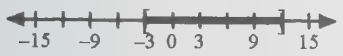
21. $x^2 - 9x \le 36$

$x^2 - 9x - 36 \le 0$

$(x+3)(x-12) \le 0$



$\{x \mid -3 \le x \le 12\}$

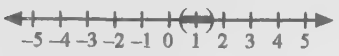


23. $4x^2 - 8x + 3 < 0$

$(2x-1)(2x-3) < 0$



$\left\{x \mid \frac{1}{2} < x < \frac{3}{2}\right\}$

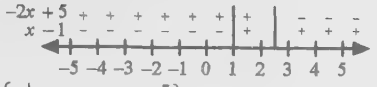


25. $\frac{3}{x-1} < 2$

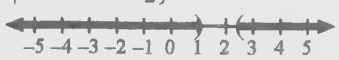
$\frac{3}{x-1} - 2 < 0$

$\frac{3}{x-1} - \frac{2x-2}{x-1} < 0$

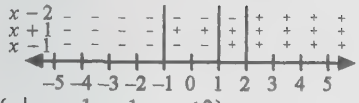
$\frac{-2x+5}{x-1} < 0$



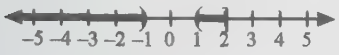
$\left\{x \mid x < 1 \text{ or } x > \frac{5}{2}\right\}$



27. $\frac{x-2}{(x+1)(x-1)} \le 0$



$\{x \mid x < -1 \text{ or } 1 < x \le 2\}$



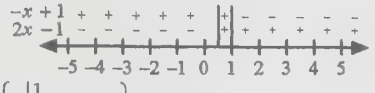
29. $\frac{x}{2x-1} \ge 1$

$\frac{x}{2x-1} - 1 \ge 0$

$\frac{x}{2x-1} - \frac{2x-1}{2x-1} \ge 0$

$\frac{x-(2x-1)}{2x-1} \ge 0$

$\frac{-x+1}{2x-1} \ge 0$



$\left\{x \mid \frac{1}{2} < x \le 1\right\}$



31. $\frac{x}{2-x} \le -3$

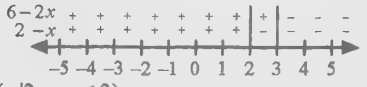
$\frac{x}{2-x} + 3 \le 0$

$\frac{x}{2-x} - \frac{3(2-x)}{2-x} \le 0$

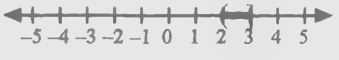
$\frac{x+3(2-x)}{2-x} \le 0$

$\frac{x+6-3x}{2-x} \le 0$

$\frac{6-2x}{2-x} \le 0$



$\{x \mid 2 < x \le 3\}$



33. $\frac{3}{x-5} > \frac{1}{x+1}$

$\frac{3}{x-5} - \frac{1}{x+1} > 0$

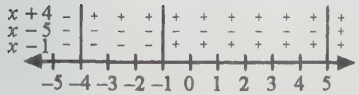
$\frac{3(x+1)}{(x-5)(x+1)} - \frac{1(x-5)}{(x-5)(x+1)} > 0$

$\frac{3(x+1)-1(x-5)}{(x-5)(x+1)} > 0$

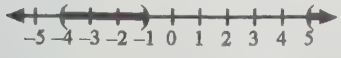
$\frac{3x+3-x+5}{(x-5)(x+1)} > 0$

$\frac{2x+8}{(x-5)(x+1)} > 0$

$\frac{2(x+4)}{(x-5)(x+1)} > 0$



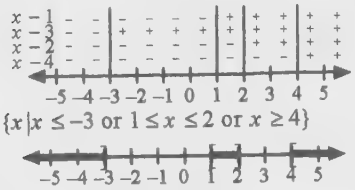
$\{x \mid x > 5 \text{ or } -4 < x < -1\}$



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Applying the Concepts

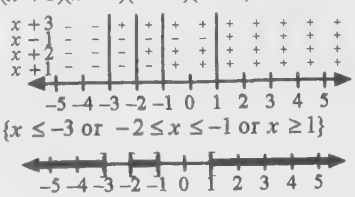
35. $(x-1)(x+3)(x-2)(x-4) \geq 0$



$\{x \mid x \leq -3 \text{ or } 1 \leq x \leq 2 \text{ or } x \geq 4\}$

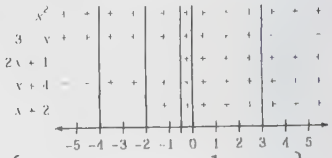
37. $(x^2+2x-3)(x^2+3x+2) \geq 0$

$(x+3)(x-1)(x+2)(x+1) \geq 0$

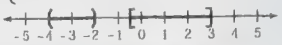


$\{x \leq -3 \text{ or } -2 \leq x \leq -1 \text{ or } x \geq 1\}$

39. $\dfrac{x^2(3-x)(2x+1)}{(x+4)(x+2)} \geq 0$



$\left\{x \mid -4 < x < -2 \text{ or } -\frac{1}{2} \leq x \leq 3\right\}$



Chapter Review

1.
$$2x^3 - 3x = 0$$
$$2x(2x-3) = 0$$
$$x = 0$$
$$2x - 3 = 0$$
$$x = \frac{3}{2}$$

2.
$$6x^2 + 9cx = 6c^2$$
$$6x^2 + 9cx - 6c^2 = 0$$
$$3(2x^2 + 3cx - 2c^2) = 0$$
$$(2x - 1c)(x + 2c) = 0$$
$$2x - 1c = 0$$
$$2x = 1c$$
$$x = \frac{c}{2}$$
$$x + 2c = 0$$
$$x = -2c$$

3.
$$x^2 = 48$$
$$\sqrt{x^2} = \sqrt{48}$$
$$x = 4\sqrt{3}$$
$$x = -4\sqrt{3}$$

4.
$$\left(x + \frac{1}{2}\right)^2 + 4 = 0$$
$$\left(x + \frac{1}{2}\right)^2 = -4$$
$$\sqrt{\left(x + \frac{1}{2}\right)^2} = \sqrt{-4}$$
$$x + \frac{1}{2} = \pm\sqrt{-4}$$
$$x + \frac{1}{2} = -\sqrt{-4} \qquad x + \frac{1}{2} = \sqrt{-4}$$
$$x + \frac{1}{2} = -2i \qquad x + \frac{1}{2} = 2i$$
$$x = -\frac{1}{2} - 2i \qquad x = -\frac{1}{2} + 2i$$

5.
$$x^2 + 4x + 3 = 0$$
$$x^2 + 4x + 4 = 4 - 3$$
$$(x+2)^2 = 1$$
$$\sqrt{(x+2)^2} = \sqrt{1}$$
$$x + 2 = \pm 1$$
$$x + 2 = -1 \qquad x + 2 = 1$$
$$x = -3 \qquad x = -1$$

6.
$$7x^2 - 14x + 3 = 0$$
$$7x^2 - 14x = -3$$
$$x^2 - 2x = -\frac{3}{7}$$
$$x^2 - 2x + 1 = -\frac{3}{7} + 1$$
$$(x-1)^2 = \frac{4}{7}$$
$$\sqrt{(x-1)^2} = \sqrt{\frac{4}{7}}$$
$$x - 1 = \pm\frac{2}{\sqrt{7}}$$
$$x - 1 = \frac{2}{\sqrt{7}} \qquad x - 1 = \frac{-2}{\sqrt{7}}$$
$$x = \frac{2}{\sqrt{7}} + 1 \qquad x = \frac{-2}{\sqrt{7}} + 1$$
$$x = \frac{2+\sqrt{7}}{\sqrt{7}} \qquad x = \frac{-2+\sqrt{7}}{\sqrt{7}}$$
$$x = \frac{2\sqrt{7}+7}{\sqrt{7}} \qquad x = \frac{-2\sqrt{7}+7}{\sqrt{7}}$$

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7. $12x^2 - 25x + 12 = 0$

$b = -25,\ a = 12,\ c = 12$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{25 \pm \sqrt{(-25)^2 - 4(12)(12)}}{2(12)}$$

$$= \frac{25 \pm \sqrt{49}}{24}$$

$$x = \frac{25+7}{24} \quad x = \frac{25-7}{24}$$

$$x = \frac{32}{24} \quad x = \frac{18}{24}$$

$$x = \frac{4}{3} \quad x = \frac{3}{4}$$

8. $x^2 - x + 8 = 0$

$a = 1,\ b = -1,\ c = 8$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{1 \pm \sqrt{(-1)^2 - 4(1)(8)}}{2(1)} = \frac{1 \pm \sqrt{-31}}{2}$$

$$x = \frac{1+\sqrt{31}}{2} \quad x = \frac{1-\sqrt{31}}{2}$$

$$x = \frac{1}{2} + \frac{\sqrt{31}}{2} \quad x = \frac{1}{2} - \frac{\sqrt{31}}{2}$$

9. $(x - r_1)(x - r_2) = 0$

$(x - 0)(x - (-3)) = 0$

$x(x + 3) = 0$

$x^2 + 3x = 0$

10. $(x - r_1)(x - r_2) = 0$

$$\left(x - \frac{3}{4}\right)\left(x - \left(-\frac{2}{3}\right)\right) = 0$$

$$\left(x - \frac{3}{4}\right)\left(x + \frac{2}{3}\right) = 0$$

$$x^2 - \frac{3}{4}x + \frac{2}{3}x - \frac{6}{12} = 0$$

$$x^2 - \frac{1}{12}x - \frac{1}{2} = 0$$

$$12\left(x^2 - \frac{1}{12}x - \frac{1}{2}\right) = 0$$

$$12x^2 - x - 6 = 0$$

11. $x^2 - 2x + 8 = 0$

$x^2 - 2x + 1 = -8 + 1$

$(x - 1)^2 = -7$

$\sqrt{(x-1)^2} = \sqrt{-7}$

$x - 1 = \pm\sqrt{7}$

$x - 1 = \sqrt{-7} \quad x - 1 = -\sqrt{-7}$

$x - 1 = i\sqrt{7} \quad x - 1 = -i\sqrt{7}$

$x = 1 + i\sqrt{7} \quad x = 1 - i\sqrt{7}$

12. $(x - 2)(x + 3) = x - 10$

$x^2 + x - 6 = x - 10$

$x^2 = -4$

$\sqrt{x^2} = \sqrt{-4}$

$x = \pm i\sqrt{4}$

13. $3x(x - 3) = 2x - 4$

$3x^2 - 9x = 2x - 4$

$3x^2 - 11x + 4 = 0$

$a = 3,\ b = -11,\ c = 4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{11 \pm \sqrt{(-11)^2 - 4(3)(4)}}{2(3)}$$

$$= \frac{11 \pm \sqrt{73}}{6}$$

14. $3x^2 - 5x + 1 = 0$

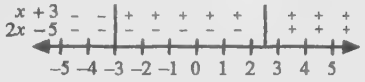
$a = 3,\ b = -5,\ c = 1$

$\sqrt{b^2 - 4ac} = \sqrt{(-5)^2 - 4(3)(1)}$

$\sqrt{13} > 0$

Two real number solutions.

15. $(x + 3)(2x - 5) < 0$

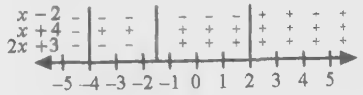


$x > -3$ or $x < 2\frac{1}{2}$

$\left\{x \middle| -3 < x < 2\frac{1}{2}\right\}$

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16. $(x-2)(x+4)(2x+3) \le 0$



$-\frac{3}{2} \le x \le 2$ or $x \le -4$

$\left\{x \middle| x \le -4 \text{ or } -\frac{3}{2} \le x \le 2\right\}$

17. $x^{2/3} + x^{1/3} - 12 = 0$. Let $u = x^{1/3}$.

$u^2 + u - 12 = 0$

$(u+4)(u-3) = 0$

$u + 4 = 0 \quad u - 3 = 0$

$u = -4 \quad u = 3$

$x^{1/3} = -4 \quad x^{1/3} = 3$

$\left(x^{1/3}\right)^3 = (-4)^3 \quad \left(x^{1/3}\right)^3 = (3)^3$

$x = -64 \quad x = 27$

Both –64 and 27 check as solutions.

18. $2(x-1) + 3\sqrt{x-1} - 2 = 0$. Let $u = \sqrt{x-1}$.

$2u^2 + 3u - 2 = 0$

$(2u-1)(u+2) = 0$

$2u - 1 = 0 \quad u + 2 = 0$

$u = \frac{1}{2} \quad u = -2$

$\sqrt{x-1} = \frac{1}{2} \quad \sqrt{x-1} = -2$

$\left(\sqrt{x-1}\right)^2 = \left(\frac{1}{2}\right)^2 \quad \left(\sqrt{x-1}\right)^2 = (-2)^2$

$x - 1 = \frac{1}{4} \quad x - 1 = 4$

$x = \frac{5}{4} \quad x = 5$

$\frac{5}{4}$ checks as a solution. 5 does not check as a solution.

19. $3x = \frac{9}{x-2}$

$3x(x-2) = 9$

$3x^2 - 6x = 9$

$3x^2 - 6x - 9 = 0$

$3(x^2 - 2x - 3) = 0$

$(x-3)(x+1) = 0$

$x - 3 = 0 \quad x + 1 = 0$

$x = 3 \quad x = -1$

Both 3 and –1 check as solutions.

20. $\frac{3x+7}{x+2} + x = 3$

$\frac{3x+7}{x+2} = -x + 3$

$3x + 7 = (-x+3)(x+2)$

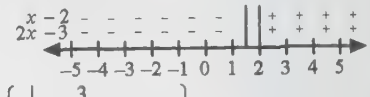
$3x + 7 = -x^2 + x + 6$

$x^2 + 2x + 1 = 0$

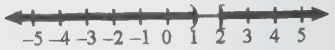
$(x+1)^2 = 0$

$x = -1$

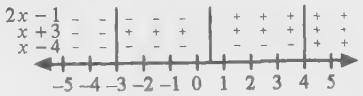
21. $\frac{x-2}{2x-3} \ge 0$



$\left\{x \middle| x < \frac{3}{2} \text{ or } x \ge 2\right\}$



22. $\frac{(2x-1)(x+3)}{x-4} \le 0$



$\frac{1}{2} \le x < 4$ or $x < -3$



$\left\{x \middle| x \le -3 \text{ or } \frac{1}{2} \le x < 4\right\}$

23. $x = \sqrt{x} + 2$

$x - \sqrt{x} - 2 = 0$

Let $u = \sqrt{x}$.

$u^2 - u - 2 = 0$

$(u-2)(u+1) = 0$

$u = 2 \quad u = -1$

$\sqrt{x} = 2 \quad \sqrt{x} = -1$

$\left(\sqrt{x}\right)^2 = 2^2 \quad \left(\sqrt{x}\right)^2 = (-1)^2$

$x = 4 \quad x = 1$

4 checks as a solution. 1 does not check as a solution.

24. $2x = \sqrt{5x+24} + 3$

$2x - 3 = \sqrt{5x+24}$

$(2x-3)^2 = \left(\sqrt{5x+24}\right)^2$

$4x^2 - 12x + 9 = 5x + 24$

$4x^2 - 17x - 15 = 0$

$(4x+3)(x-5) = 0$

$4x + 3 = 0 \quad x - 5 = 0$

$x = -\frac{3}{4} \quad x = 5$

$-\frac{3}{4}$ does not check as a solution. 5 checks as a solution.

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25.

$$\frac{x-2}{2x+3}-\frac{x-4}{x}=2$$

$$x(2x+3)\left[\left(\frac{x-2}{2x+3}\right)-\left(\frac{x-4}{x}\right)\right]=2x(2x+3)$$

$$x^2-2x-2x^2+8x-3x+12=4x^2+6x$$

$$5x^2+3x-12=0$$

$a=5,\ b=3,\ c=-12$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-3\pm\sqrt{3^2-4(5)(-12)}}{2(5)}$$

$$=\frac{-3\pm\sqrt{249}}{10}$$

26.

$$1-\frac{x+4}{2-x}=\frac{x-3}{x+2}$$

$$(x+2)(2-x)\left(1-\frac{x+4}{2-x}\right)=(x+2)(2-x)\frac{x-3}{x+2}$$

$$(x+2)(2-x)-(x+2)(x+4)=(2-x)(x-3)$$

$$2x-x^2+4-2x-\left(x^2+6x+8\right)=2x-6-x^2+3x$$

$$2x-x^2+4-2x-x^2-6x-8=-x^2+5x-6$$

$$-2x^2-6x-4=-x^2+5x-6$$

$$x^2+11x-2=0$$

$a=1,\ b=11,\ c=-2$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-11\pm\sqrt{(11)^2-4(1)(-2)}}{2(1)}$$

$$=\frac{-11\pm\sqrt{129}}{2}$$

27. Strategy

- The width: x
 The length: $2x+2$

Solution

The area is 60 cm^2

$$A=L\cdot W$$

$$60=(2x+2)(x)$$

$$60=2x^2+2x$$

$$2x^2+2x-60=0$$

$$(2x+12)(x-5)=0$$

$2x+12=0 \qquad x-5=0$

$x=-\frac{12}{2} \qquad x=5$

The distance can not be a negative number. $2x+2=12$. The length is 12 cm and the width of the rectangle is 5 cm.

28. Strategy

- First Integer: x
- Second Integer: $x+2$
- Third Integer: $x+4$

Solution

The sum of three consecutive even integers equals 56.

$$x^2+(x+2)^2+(x+4)^2=56$$

$$x^2+x^2+4x+4+x^2+8x+16=56$$

$$3x^2+12x+20=56$$

$$3x^2+12x-36=0$$

$$3\left(x^2+4x-12\right)=0$$

$$(x+6)(x-2)=0$$

$x+6=0 \qquad x-2=0$

$x=-6 \qquad x=2$

$x+2=-4 \qquad x+2=4$

$x+4=-2 \qquad x+4=6$

The three integers are 2, 4, and 6 or –6, –4, and –2.

29. Strategy

- Time of the new computer: t

	Rate	Time	Part
New computer	$\frac{1}{t}$	8	$\frac{8}{t}$
Old computer	$\frac{1}{t+12}$	8	$\frac{8}{t+12}$

Solution

The sum of the parts equals 1.

$$\frac{8}{t}+\frac{8}{t+12}=1$$

$$t(t+12)\left[\frac{8}{t}+\frac{8}{t+12}\right]=t(t+12)$$

$$8(t+12)+8t=t^2+12t$$

$$8t+96+8t=t^2+12t$$

$$t^2-4t-96=0$$

$$(t-12)(t+8)=0$$

$t-12=0 \qquad t+8=0$

$t=12 \qquad t=-8$

The time cannot be a negative number. The new computer takes 12 min to print the payroll.

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30. Strategy
- Time for the first car: t
- Time for the second car: $t-1$

	Distance	Time	Rate
First Car	200	t	$\frac{200}{t}$
Second Car	200	$t-1$	$\frac{200}{t-1}$

Solution The rate of the second car is 10 mph faster than the first.

$$\frac{200}{t-1}=\frac{200}{t}+10$$
$$t(t-1)\left(\frac{200}{t-1}\right)=t(t-1)\left(\frac{200}{t}+10\right)$$
$$200t=200t-200+10t^2-10t$$
$$10t^2-10t-200=0$$
$$t^2-t-20=0$$
$$(t-5)(t+4)=0$$
$$t-5=0 \quad t+4=0$$
$$t=5 \qquad t=-4$$

$\frac{200}{t}=40$ the time cannot be negative.

$\frac{200}{t-1}=50$

The first car traveled at 40 mph and the second car traveled at 50 mph.

Chapter Test

1.
$$3x^2+10x=8$$
$$3x^2+10x-8=0$$
$$(3x-2)(x+4)=0$$
$$3x-2=0 \quad x+4=0$$
$$3x=2 \qquad x=-4$$
$$x=\frac{2}{3}$$

The solutions are $\frac{2}{3}$ and -4.

2.
$$6x^2-5x-6=0$$
$$(2x-3)(3x+2)=0$$
$$2x-3=0 \quad 3x+2=0$$
$$2x=3 \qquad 3x=-2$$
$$x=\frac{3}{2} \qquad x=-\frac{2}{3}$$

The solutions are $\frac{3}{2}$ and $-\frac{2}{3}$.

3.
$$(x-r_1)(x-r_2)=0$$
$$(x-3)(x-(-3))=0$$
$$(x-3)(x+3)=0$$
$$x^2-9=0$$

4.
$$(x-r_1)(x-r_2)=0$$
$$\left(x-\frac{1}{2}\right)(x-(-4))=0$$
$$\left(x-\frac{1}{2}\right)(x+4)=0$$
$$x^2+\frac{7}{2}x-2=0$$
$$2\left(x^2+\frac{7}{2}x-2\right)=2\cdot 0$$
$$2x^2+7x-4=0$$

5.
$$3(x-2)^2-24=0$$
$$3(x-2)^2=24$$
$$(x-2)^2=8$$
$$\sqrt{(x-2)^2}=\pm\sqrt{8}$$
$$x-2=\pm 2\sqrt{2}$$
$$x-2=2\sqrt{2} \qquad x-2=-2\sqrt{2}$$
$$x=2+2\sqrt{2} \qquad x=2-2\sqrt{2}$$

The solutions are $2+2\sqrt{2}$ and $2-2\sqrt{2}$.

6.
$$x^2-6x-2=0$$
$$x^2-6x=2$$

Complete the square.

$$x^2-6x+9=2+9$$
$$(x-3)^2=11$$
$$\sqrt{(x-3)^2}=\pm\sqrt{11}$$
$$x-3=\pm\sqrt{11}$$
$$x-3=\sqrt{11} \qquad x-3=-\sqrt{11}$$
$$x=3+\sqrt{11} \qquad x=3-\sqrt{11}$$

The solutions are $3+\sqrt{11}$ and $3-\sqrt{11}$.

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7. $3x^2 - 6x = 2$

$\frac{1}{3}(3x^2 - 6x) = \frac{1}{3}(2)$

$x^2 - 2x = \frac{2}{3}$

Complete the square.

$x^2 - 2x + 1 = \frac{2}{3} + 1$

$(x-1)^2 = \frac{5}{3}$

$\sqrt{(x-1)^2} = \pm\sqrt{\frac{5}{3}}$

$x - 1 = \pm\frac{\sqrt{15}}{3}$

$x - 1 = \frac{\sqrt{15}}{3}$ $\quad$ $x - 1 = \frac{-\sqrt{15}}{3}$

$x = 1 + \frac{\sqrt{15}}{3}$ $\quad$ $x = 1 - \frac{\sqrt{15}}{3}$

$= \frac{3+\sqrt{15}}{3}$ $\quad$ $= \frac{3-\sqrt{15}}{3}$

The solutions are $\frac{3+\sqrt{15}}{3}$ and $\frac{3-\sqrt{15}}{3}$.

8. $2x^2 - 2x = 1$

$2x^2 - 2x - 1 = 0$

$a = 2,\ b = -2,\ c = -1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(-1)}}{2(2)}$

$= \frac{2 \pm \sqrt{4+8}}{4} = \frac{2 \pm \sqrt{12}}{4}$

$= \frac{2 \pm 2\sqrt{3}}{4} = \frac{1 \pm \sqrt{3}}{2}$

The solutions are $\frac{1+\sqrt{3}}{2}$ and $\frac{1-\sqrt{3}}{2}$.

9. $x^2 + 4x + 12 = 0$

$a = 1,\ b = 4,\ c = 12$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-4 \pm \sqrt{(4)^2 - 4(1)(12)}}{2(1)}$

$= \frac{-4 \pm \sqrt{16-48}}{2} = \frac{-4 \pm \sqrt{-32}}{2}$

$= \frac{-4 \pm 4i\sqrt{2}}{2} = -2 \pm 2i\sqrt{2}$

The solutions are $-2 + 2i\sqrt{2}$ and $-2 - 2i\sqrt{2}$.

10. $3x^2 - 4x = 1$

$4x^2 - 4x - 1 = 0$

$a = 3,\ b = -4,\ c = -1$

$b^2 - 4ac = (-4)^2 - 4(3)(-1) = 16 + 12$

$= 28$

$28 > 0$

Since the discriminant is greater than zero, the equation has two real number solutions.

11. $x^2 - 6x = -15$

$x^2 - 6x + 15 = 0$

$a = 1,\ b = -6,\ c = 15$

$b^2 - 4ac = (-6)^2 - 4(1)(15)$

$36 - 60 = -24$

$-24 < 0$

Since the discriminant is less than zero, the equation has two complex number solutions.

12. $2x + 7x^{1/2} - 4 = 0$

$2(x^{1/2})^2 + 7x^{1/2} - 4 = 0$

$2u^2 + 7u - 4 = 0$

$(2u - 1)(u + 4) = 0$

$2u - 1 = 0$ $\quad$ $u + 4 = 0$

$2u = 1$ $\quad$ $u = -4$

$u = \frac{1}{2}$

Replace u by $x^{1/2}$.

$x^{1/2} = \frac{1}{2}$ $\quad$ $x^{1/2} = -4$

$(x^{1/2})^2 = \left(\frac{1}{2}\right)^2$ $\quad$ $(x^{1/2})^2 = (-4)^2$

$x = \frac{1}{4}$ $\quad$ $x = 16$

16 does not check as a solution. $\frac{1}{4}$ does check as a solution. The solution is $\frac{1}{4}$.

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13. $x^4 - 4x^2 + 3 = 0$

$(x^2)^2 - 4x^2 + 3 = 0$

$u^2 - 4u + 3 = 0$

$(u-3)(u-1) = 0$

$u - 3 = 0 \qquad u - 1 = 0$

$u = 3 \qquad u = 1$

Replace u by x^2.

$x^2 = 3 \qquad u - 1 = 0$

$\sqrt{x^2} = \pm\sqrt{3} \qquad \sqrt{x^2} = \pm\sqrt{1}$

$x = \pm\sqrt{3} \qquad x = \pm 1$

The solutions are $\sqrt{3}$, $-\sqrt{3}$, 1, and -1.

14. $\sqrt{2x+1} + 5 = 2x$

$\sqrt{2x+1} = 2x - 5$

$\left(\sqrt{2x+1}\right)^2 = (2x-5)^2$

$2x + 1 = 4x^2 - 20x + 25$

$0 = 4x^2 - 22x + 24$

$0 = 2(2x^2 - 11x + 12)$

$0 = 2(2x-3)(x-4)$

$2x - 3 = 0 \qquad x - 4 = 0$

$2x = 3 \qquad x = 4$

$x = \frac{3}{2}$

$\frac{3}{2}$ does not check as a solution. 4 does check as a solution. The solution is 4.

15. $\sqrt{x-2} = \sqrt{x} - 2$

$\left(\sqrt{x-2}\right)^2 = \left(\sqrt{x} - 2\right)^2$

$x - 2 = x - 4\sqrt{x} + 4$

$-6 = -4\sqrt{x}$

$-\frac{1}{4}(-6) = \left(-4\sqrt{x}\right)\left(-\frac{1}{4}\right)$

$\frac{3}{2} = \sqrt{x}$

$\left(\frac{3}{2}\right)^2 = \left(\sqrt{x}\right)^2$

$\frac{9}{4} = x$

$\frac{9}{4}$ does not check as a solution. The equation has no solution.

16. $\frac{2x}{x-3} + \frac{5}{x-1} = 1$

$(x-3)(x-1)\left(\frac{2x}{x-3} + \frac{5}{x-1}\right) = (x-3)(x-1)1$

$2x(x-1) + 5(x-3) = (x-3)(x-1)$

$2x^2 - 2x + 5x - 15 = x^2 - 4x + 3$

$2x^2 + 3x - 15 = x^2 - 4x + 3$

$x^2 + 7x - 18 = 0$

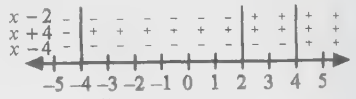
$(x+9)(x-2) = 0$

$x + 9 = 0 \qquad x - 2 = 0$

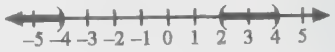
$x = -9 \qquad x = 2$

The solutions are -9 and 2.

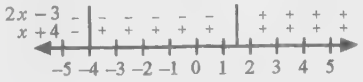
17. $(x-2)(x+4)(x-4) < 0$



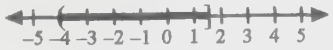
$\{x \mid x < -4 \text{ or } 2 < x < 4\}$



18. $\frac{2x-3}{x+4} \le 0$



$\left\{x \mid -4 < x \le \frac{3}{2}\right\}$



19. Strategy

- This is a geometry problem.
- The height of the triangle: x
 The base of the triangle: $3x + 3$
- The area of the triangle is 30 ft^2.
 Use the formula for the area of the triangle $\left(A = \frac{1}{2}bh\right)$.

Solution

$A = \frac{1}{2}bh$

$30 = \frac{1}{2}(3x+3)x$

$60 = 3x^2 + 3x$

$0 = 3x^2 + 3x - 60$

$0 = 3(x^2 + x - 20)$

$0 = 3(x+5)(x-4)$

$x + 5 = 0 \qquad x - 4 = 0$

$x = -5 \qquad x = 4$

The solution -5 is not possible, since height cannot be a negative number.

$x = 4$, $3x + 3 = 3(4) + 3 = 15$

The height of the triangle is 4 ft. The base of the triangle is 15 ft.

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20. Strategy
- This is a distance-rate problem.
- The rate of the canoe in calm water: x

	Distance	Rate	Time
With current	6	$x+2$	$\frac{6}{x+2}$
Against current	6	$x-2$	$\frac{6}{x-2}$

- The total traveling time is 4 h.

Solution

$$\frac{6}{x+2}+\frac{6}{x-2}=4$$

$$(x+2)(x-2)\left(\frac{6}{x+2}+\frac{6}{x-2}\right)=(4)(x+2)(x-2)$$

$$6(x-2)+6(x+2)=4(x^2-4)$$

$$6x-12+6x+12=4x^2-16$$

$$12x=4x^2-16$$

$$0=4x^2-12x-16$$

$$0=4(x^2-3x-4)$$

$$0=4(x-4)(x+1)$$

$x-4=0 \qquad x+1=0$

$x=4 \qquad x=-1$

The solution –1 is not possible, since rate cannot be a negative number. The rate of the canoe in calm water is 4 mph.

Cumulative Review

1. $2a^2-b^2+c^2=2(3)^2-(-4)^2+(-2)^2$

$=2(9)-16+4$

$=18-16+4$

$=18-4=14$

2. $|3x-2|<8$

$-8<3x-2<8$

$-8+2<3x-2+2<8+2$

$-6<3x<10$

$\frac{1}{3}\cdot(-6)<\frac{1}{3}\cdot(3x)<\frac{1}{3}\cdot 10$

$-2<x<\frac{10}{3}$

$\left\{x\,\middle|\,-2<x<\frac{10}{3}\right\}$

3. $V=\pi r^2 h=\pi(3)^2(6)=\pi(9)(6)$

$=54\pi$

The volume is $54\pi\ \text{m}^3$.

4. $f(x)=\frac{2x-3}{x^2-1}$

$f(-2)=\frac{2(-2)-3}{(-2)^2-1}$

$f(-2)=-\frac{7}{3}$

5. $P_1(3,\ -4),\ P_2(-1,\ 2)$

$m=\frac{y_2-y_1}{x_2-x_1}=\frac{2-(-4)}{-1-3}=\frac{2+4}{-4}$

$=\frac{6}{-4}=-\frac{3}{2}$

6. $6x-5y=15$

$6x-5(0)=15$

$6x=15$

$x=\frac{15}{6}=\frac{5}{2}$

The x-intercept is $\left(\frac{5}{2},\ 0\right)$.

$6(0)-5y=15$

$-5y=15$

$y=-3$

The y-intercept is $(0,\ -3)$.

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7. $x - y = 1$

$-y = -x + 1$

$y = x - 1$

$m = 1 \quad (x_1, y_1) = (1, 2)$

$y - y_1 = m(x - x_1)$

$y - 2 = 1(x - 1)$

$y - 2 = x - 1$

$y = x + 1$

8. $x + y + z = 2$

$-x + 2y - 3z = -9$

$x - 2y - 2z = -1$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ -1 & 2 & -3 \\ 1 & -2 & -2 \end{vmatrix} = 1\begin{vmatrix} 2 & -3 \\ -2 & -2 \end{vmatrix} - 1\begin{vmatrix} -1 & -3 \\ 1 & -2 \end{vmatrix} + 1\begin{vmatrix} -1 & 2 \\ 1 & -2 \end{vmatrix} = -15$$

$$D_x = \begin{vmatrix} 2 & 1 & 1 \\ -9 & 2 & -3 \\ -1 & -2 & -2 \end{vmatrix} = 2\begin{vmatrix} 2 & -3 \\ -2 & -2 \end{vmatrix} - 1\begin{vmatrix} -9 & -3 \\ -1 & -2 \end{vmatrix} + 1\begin{vmatrix} -9 & 2 \\ -1 & -2 \end{vmatrix} = -15$$

$$D_y = \begin{vmatrix} 1 & 2 & 1 \\ -1 & -9 & -3 \\ 1 & -1 & -2 \end{vmatrix} = 1\begin{vmatrix} -9 & -3 \\ -1 & -2 \end{vmatrix} - 2\begin{vmatrix} -1 & -3 \\ 1 & -2 \end{vmatrix} + 1\begin{vmatrix} -1 & -9 \\ 1 & -1 \end{vmatrix} = -15$$

$$D_z = \begin{vmatrix} 1 & 1 & 2 \\ -1 & 2 & -9 \\ 1 & -2 & -1 \end{vmatrix} = 1\begin{vmatrix} 2 & -9 \\ -2 & -1 \end{vmatrix} - 1\begin{vmatrix} -1 & -9 \\ 1 & -1 \end{vmatrix} + 2\begin{vmatrix} -1 & 2 \\ 1 & -2 \end{vmatrix} = -30$$

$$x = \frac{D_x}{D} = \frac{-15}{-15} = 1$$

$$y = \frac{D_y}{D} = \frac{15}{-15} = -1$$

$$z = \frac{D_z}{D} = \frac{-30}{-15} = 2$$

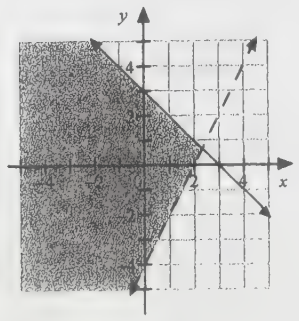
The solution is $(1, -1, 2)$.

9. Solve each inequality.

$x + y \le 3 \qquad 2x - y < 4$

$y \le 3 - x \qquad -y < 4 - 2x$

$y > -4 + 2x$



10. $\dfrac{BC}{EF} = \dfrac{\text{height of triangle } ABC}{\text{height of triangle } DEF}$

$\dfrac{12}{24} = \dfrac{8}{h}$

$12h = 24(8)$

$12h = 192$

$h = 16$

The height of triangle DEF is 16 cm.

11.

$$3x - 4 \overline{)\, 3x^3 - 13x^2 + 0 + 10}$$

$$\underline{3x^3 - 4x^2}$$

$$-9x^2 + 0$$

$$\underline{-9x^2 + 12x}$$

$$-12x + 10$$

$$\underline{-12x + 16}$$

$$-6$$

$$(3x^3 - 13x^2 + 10) \div (3x - 4)$$

$$= x^2 - 3x - 4 - \frac{6}{3x - 4}$$

12. $-3x^3y + 6x^2y^2 - 9xy^3 = -3xy(x^2 - 2xy + 3y^2)$

13. $6x^2 - 7x - 20 = (2x - 5)(3x + 4)$

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14. $\dfrac{x^2+2x+1}{8x^2+8x}\cdot\dfrac{4x^3-4x^2}{x^2-1}=\dfrac{(x+1)(x+1)}{8x(x+1)}\cdot\dfrac{4x^2(x-1)}{(x+1)(x-1)}$

$=\dfrac{\cancel{(x+1)}^{1}\cancel{(x+1)}^{1}4x^2\cancel{(x-1)}^{1}}{8x\cancel{(x+1)}^{1}\cancel{(x+1)}^{1}\cancel{(x-1)}^{1}}=\dfrac{x}{2}$

15. $\dfrac{x}{x+2}-\dfrac{4x}{x+3}=1$

$(x+2)(x+3)\left[\dfrac{x}{x+2}-\dfrac{4x}{x+3}\right]=(x+2)(x+3)1$

$x(x+3)-4x(x+2)=(x+2)(x+3)$

$x^2+3x-4x^2-8x=x^2+5x+6$

$-3x^2-5x=x^2+5x+6$

$0=4x^2+10x+6$

$0=2(2x^2+5x+3)$

$0=2(2x+3)(x+1)$

$2x+3=0 \qquad x+1=0$

$2x=-3 \qquad x=-1$

$x=-\dfrac{3}{2}$

The solutions are $-\dfrac{3}{2}$ and -1.

16. $S=\dfrac{n}{2}(a+b)$

$2S=n(a+b)$

$2S=an+bn$

$2S-an=bn$

$\dfrac{2S-an}{n}=\dfrac{bn}{n}$

$\dfrac{2S-an}{n}=b$

17. $a^{-1/2}\left(a^{1/2}-a^{3/2}\right)=a^0-a^1=1-a$

18. $-2i(7-4i)=-14i+8i^2=-14i-8$

$=-8-14i$

19. $\sqrt{3x+1}-1=x$

$\sqrt{3x+1}=x+1$

$\left(\sqrt{3x+1}\right)^2=(x+1)^2$

$3x+1=x^2+2x+1$

$0=x^2-x$

$0=x(x-1)$

$x=0 \qquad x-1=0$

$x=1$

0 and 1 both check as solutions. The solutions are 0 and 1.

20. $x^4-6x^2+8=0$

$\left(x^2\right)^2-6x^2+8=0$

$u^2-6u+8=0$

$(u-4)(u-2)=0$

$u-4=0 \qquad u-2=0$

$u=4 \qquad u=2$

Replace u by x^2.

$x^2=4 \qquad x^2=2$

$\sqrt{x^2}=\pm\sqrt{4} \qquad \sqrt{x^2}=\pm\sqrt{2}$

$x=\pm2 \qquad x=\pm\sqrt{2}$

The solutions are 2, –2, $\sqrt{2}$, and $-\sqrt{2}$.

21. Strategy

Let p represent the length of the piston rod, T the tolerance, and m the given length.
Solve the absolute value inequality $|m-p|\le T$ for m.

Solution

$|m-p|\le T$

$\left|m-9\dfrac{3}{8}\right|\le\dfrac{1}{64}$

$-\dfrac{1}{64}\le\left|m-9\dfrac{3}{8}\right|\le\dfrac{1}{64}$

$-\dfrac{1}{64}+9\dfrac{3}{8}\le m\le\dfrac{1}{64}+9\dfrac{3}{8}$

$9\dfrac{23}{64}\le m\le 9\dfrac{25}{64}$

The lower limit is $9\dfrac{23}{64}$ in. The upper limit is $9\dfrac{25}{64}$ in.

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22. $A = \frac{1}{2}b \cdot h = \frac{1}{2}(x+8)(2x-4) = \frac{1}{2}(2x^2+12x-32)$

$= (x^2+6x-16)$

The area is $(x^2+6x-16)\ \text{ft}^2$.

23. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-250{,}000}{30-0} = \frac{-250{,}000}{30} = -\frac{25{,}000}{3}$

The slope represents the amount in dollars that the building depreciates each year $\left(-\frac{25{,}000}{3} \approx \$8333\right)$.

24. Strategy To find the distance, use the Pythagorean Theorem. The hypotenuse is the length of the ladder. One leg is the distance from the bottom of the ladder to the base of the building. The distance along the building from the ground to the top of the ladder is the unknown leg.

Solution

$a^2 + b^2 = c^2$

$8^2 + b^2 = 17^2$

$64 + b^2 = 289$

$b^2 = 255$

$\sqrt{b^2} = \pm\sqrt{255}$

$b = \pm 15$

The solution $b = -15$ is not possible, since the distance cannot be a negative number. The distance is 15 ft.

25. $2x^2 + 4x + 3 = 0$

$a = 2,\ b = 4,\ c = 3$

$b^2 - 4ac = 4^2 - 4(2)(3)$

$16 - 24 = -8$

$-8 < 0$

Since the discriminant is less than zero, the equation has two complex number solutions.

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Basic Algebra II

Worksheets



Northern Essex Community College

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Multiplication of Polynomials

Multiplication of Monomials by Monomials - Three steps: a) multiply the signs, b) multiply the numerical coefficients, and c) add the exponents of the same bases.

Examples: **a)** $(2x^3)(+4x^2y^4) = +8x^5y^4$ **b.** $(-2a^5b^3)(-10a^2b^3)(3b^2) = -60a^7b^8$

Multiply:

1) $x^3 \bullet x^3$

2) $b^8 \bullet b^3$

3) $y^8 \bullet y^2$

4) $4x^4(10x^2)$

5) $(5y^8)(5y^3)$

6) $(-2a^3)(7a^4)$

7) $(7y^8)(-5y^9)$

8) $-8x^6(3x^9)$

9) $(3b^4c^4)(-2b^2c^5)$

10) $(-9m)(+2m)$

11) $(+8x^3)(4x^8)$

12) $(2x^2y^5)(-14xy)$

13) $(-8x^3)(-6x^7)$

14) $(4x^9)(\frac{1}{4}x^2)$

15) $(-2y^2z^2)(-7y^3z^3)$

16) $(-5x^7y)(3xy^3)$

17) $(-4x^3)(4x^8)(4x^6)$

18) $(2x^5)(-2x^5)(-11x^3)$

19) $(-10c)(3c^3)(-2c^5)$

20) $(-10b^4)(2b^5)(-\frac{1}{5}b^3)$

21) $a^9b^9 \bullet a^2b^6$

22) $(-x^2y^2)(-x^5y^5)$

23) $(3a)(-7a^5)(\frac{1}{3}a^5)$

24) $(-10x^2)(-7x^6)(\frac{1}{5}x^5)$

25) $(a^2b^3)(-2bc)(-2a^5c^5)$

26) $(14mn)(2mn)(2mn)$

27) $(-2a^5b)(6b^2)(5a^2)$

28) $(6x^5)(-8xy^2)$

29) $(-3x^5y)(-2x^2y^4)$

30) $(-y^5)(+3y^5)$

31) $(3x^7)(-6x^3y^5z)(-5x^2z^4)$

32) $(2z^7)(10z^8)(5z^9)$

33) $(x^3)(3x^8yz^4)(-2x^4z)$

Multiplication of Monomials (Exponents outside Parentheses)

Multiplication of Monomials with Exponents outside of Parentheses - The exponent outside a parentheses indicates the power to which the parentheses must be raised.

Examples **a.** $(2a^2)^4$
$= (2a^2)\ (2a^2)\ (2a^2)\ (2a^2)$
$= 16a^8$

b. If there is no numerical coefficient, multiply the exponents inside the parentheses by the exponent that is outside the parentheses. $(a^3b^5)^6 = a^{18}y^{30}$

Multiply:

1) $(3r^2s)^2$ **2)** $(5x^3y^3)^3$ **3)** $(-4x^3y^2)^3$ **4)** $(8x^3y^4)^2$

5) $(6a^3b^5)^2$ **6)** $(-2x^4y^4)^4$ **7)** $(10x)^3$ **8)** $(3a^2)^4$

9) $(2y^5z)^3$ **10)** $(-4ab^2)^3$ **11)** $(a^3b^3)^3$ **12)** $(-5xy^2)^3$

13) $(x^5y^2)^6$ **14)** $(-2ab^8)^2$ **15)** $(3a^3)^2$ **16)** $(x^5)^5$

17) $(z^7)^3$ **18)** $(a^2)^3$ **19)** $(m^3n^4)^5$ **20)** $(xy^2)^2$

21) $(-p^3q^3)^7$ **22)** $(-a^5b^6)^5$ **23)** $(-v^6)^6$ **24)** $(b^3c^3)^3$

25) $(abc^3)^5$ **26)** $(x^5y^2)^8$ **27)** $(-a^4b^4)^5$ **28)** $(-a^4c^4)^7$

29) $(x^3y^3z^2)^4$ **30)** $(d^2)^{15}$ **31)** $(4x^3)^4$ **32)** $(f^7)^6$

33) $(7x^2y^3)^3$ **34)** $(x^2y^2)^5$ **35)** $(-5a^5b^4)^3$ **36.** $(-m^5n^2)^7$

Multiplication of Monomials and Monomials (Exponents outside the Parentheses)

Examples

a. $(3x^2y^2)^2(2xy)^3$
$= (3x^2y^2)\ (3x^2y^2)\ (2xy)\ (2xy)\ (2xy)$
$= 72x^7y^7$

b. $(-2a^5)^3(a^2b^5)^4$
$= (-2a^5)\ (-2a^5)\ (-2a^5)\ (a^2b^5)\ (a^2b^5)\ (a^2b^5)\ (a^2b^5)$
$= -8a^{23}y^{20}$

Multiply:

1) $(4xy)^3(x^3)^2$

2) $(6x^2y^3)^3(x^2)^4$

3) $(-2a^2)^2(a)^3$

4) $(-3x^2y)\ (xy^3)^3$

5) $(12x^3)^2(-2x^2)^3$

6) $(2x)^4(-2x)$

7) $(4b^2)^2(2a^2b^3)^2$

8) $(2x^2)^5(2x^3y^2)$

9) $(-8x)^3(x^4y^3)^2$

10) $(mn^2)^4(-2)^2$

11) $(b^5)^3(-5b^3)^2$

12) $(x^2y)^2(xy^3)^4$

13) $(x^2y)^4(-xy^2)^4$

14) $(xy^3)(-3x^3y^2)$

15) $(4b^3)^3(-a^3b^2)$

16) $(2x^5y^7)(5xy^4)^2$

17) $(4x^2y)^3(xy^2)$

18) $(2a^3b^4)^2(8ab)^2$

19) $(-3xy)^3(xy^7)^4$

20) $(-4x^2)^2(x^2)^9$

21) $(-2mn)(-m^4)^4$

22) $(2pq)^3(-4p^2q^2)^2$

23) $(-3y^4)^3(x^5y^6)^7$

24) $(r^5s^4)^3(r^5s^4)^3$

More Division of Monomials

Division of Monomials - If the largest exponent is in the numerator, the variable remains in the numerator, but if the largest exponent is in the denominator, then the variable stays in the denominator.

Examples

a. $\frac{x^5}{x^3} = \frac{x^2}{1} = x^2$ **b.** $\frac{x^3y^3}{x^2y^5} = \frac{x}{y^2}$ **c.** $\frac{5m^3}{10} = \frac{m^3}{2}$ **d.** $\frac{8a^6b^2}{6a^2b^4} = \frac{4a^4}{3b^2}$

Simplify:

1) $\frac{x^5}{x^4}$ **2)** $\frac{a^8}{a^7}$ **3)** $\frac{c^7}{-c^4}$

4) $\frac{-r}{r^2}$ **5)** $\frac{z^4}{z^7}$ **6)** $\frac{-3b^3}{-3b^5}$

7) $\frac{a^2b^7}{a^4b^5}$ **8)** $\frac{r^{10}s^7}{r^7s^{10}}$ **9)** $\frac{-ab^3}{a^4b}$

10) $\frac{x^3y^3}{x^2y^9}$ **11)** $\frac{5b^3}{15}$ **12)** $\frac{7}{21y^3}$

13) $\frac{8x^2}{16y^2}$ **14)** $\frac{4a^4}{14c^4}$ **15)** $\frac{3x^{12}}{9x^{10}}$

16) $\frac{-3a^6b^2}{15a^5b^4}$ **17)** $\frac{-4a^5z^8}{12a^7z^4}$ **18)** $\frac{3a^9b^7}{6a^7b^8}$

19) $\frac{-x^3}{-x^7}$ **20)** $\frac{5a^7b^3}{25a^5b^5}$ **21)** $\frac{11x^{12}}{33}$

22) $\frac{-12y^3z^3}{8yz^5}$ **23)** $\frac{-a^{14}b^8}{a^{12}b^3}$ **24)** $\frac{c^{15}}{-c^3}$

Negative Exponents

Negative Exponents - To change a negative exponent to a positive exponent, move the exponent and its base from the numerator to the denominator. If the exponent is in the denominator, move it to the numerator.

Examples

a. $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

b. $\frac{5}{x^{-2}} = \frac{5x^2}{1} = 5x^2$

c. $\frac{y^6}{y^{-4}} = \frac{y^6 \bullet y^4}{1} = y^{10}$

Change all negative exponents to positive exponents and simplify.

1) 2^{-4}

2) 7^{-2}

3) 5^{-2}

4) a^{-3}

5) $\frac{5}{y^{-3}}$

6) $\frac{8}{x^{-4}}$

7) $\frac{1}{z^{-3}}$

8) $\frac{x^4}{x^{-3}}$

9) $\frac{m^3}{m^{-3}}$

10) $\frac{4x^2}{x^{-2}}$

11) $\frac{12z^4}{z^{-2}}$

12) $\frac{a^3b^3}{a^{-4}b^{-4}}$

13) $\frac{a^5b^5}{a^{-5}b^{-3}}$

14) $\frac{x^2y^7}{x^{-1}y^{-3}}$

15) $\frac{r^6s^2}{r^{-3}s^{-1}}$

16) $\frac{a^7}{a^{-2}}$

17) $\frac{x^{-2}y^{-4}}{x^4y^2}$

18) $\frac{m^6}{m^{-3}n^{-3}}$

19) $\frac{x^9}{y^{-7}}$

20) $p^{-3}q^7$

21) $\frac{a^7}{a^{-7}b^{-2}}$

22) $\frac{13a^3}{b^{-5}}$

23) $\frac{mn}{m^{-1}n^{-1}}$

24) $x^{-3}y^{-3}$

25) $\frac{a^5b^5}{a^{-4}}$

26) $y^{-3}z^{-2}$

27) $\frac{x^3y^2}{y^{-5}}$

28) $\frac{8a}{b^{-4}}$

Negative Exponents (Cont.)

Write with a positive exponent. Then evaluate.

1) 3^{-2}

2) 2^{-3}

3) 4^{-1}

4) 10^{-1}

5) 6^{-2}

6) $\frac{2^{-2}}{2}$

7) $\frac{4^{-2}}{4}$

8) $\frac{3^{-2}}{3^3}$

9) $\frac{2^{-4}}{2^2}$

10) $\frac{3^{-3}}{3}$

11) $\frac{2^{-2}}{2^2}$

12) $\frac{5^{-2}}{5}$

Simplify.

13) x^{-1}

14) x^{-9}

15) a^{-5}

16) b^{-3}

17) $a^{-1}b$

18) xy^{-2}

19) x^3x^{-3}

20) $a^{-3}b^2$

21) $x^{-2}y^{-1}$

22) $\frac{x^{-1}}{x^3}$

23) $\frac{x^{-2}}{x}$

24) $\frac{a^{-2}}{a^4}$

25) $\frac{a^{-7}}{a^7}$

26) $\frac{x^{-1}}{xy^{-2}}$

27) $\frac{x^3y^{-2}}{x}$

28) $(x^2)^{-4}$

29) $(y^{-2})^{-3}$

30) $(a^{-4})^{-2}$

31) $(a^{-3})^0$

32) $(a^{-1}b^2)^0$

Negative Exponents Outside Parentheses

Negative Exponent, with Exponents Outside Parentheses - 1) Multiply each exponent inside the parentheses by the exponent outside the parentheses, 2) To change a negative exponent to a positive exponent, move the exponent and its base from the numerator to the denominator. If the negative exponent is in the denominator, move it to the numerator. 3) Divide and/or multiply the like variables.

Examples

a. $\left(\frac{y^3}{y^4}\right)^2 = \frac{y^6}{y^8} = \frac{1}{y^2}$

b. $\left(\frac{2}{a^5}\right)^{-2} = \frac{2^{-2}}{a^{-10}} = \frac{a^{10}}{2^2} = \frac{a^{10}}{4}$

c. $\left(\frac{m^{-3}n}{m^2n^{-3}}\right)^{-4} = \left(\frac{m^{12}n^{-4}}{m^{-8}n^{12}}\right) = \left(\frac{m^{12}m^8}{n^{12}n^4}\right) = \frac{m20}{n^{16}}$

Remove the parentheses and simplify.

1) $\left(\frac{y^3}{y^4}\right)^2$

2) $\left(\frac{a^2}{a^7}\right)^3$

3) $\left(\frac{x^5}{x^3}\right)^{-2}$

4) $\left(\frac{v}{w^4}\right)^{-3}$

5) $\left(\frac{2}{z^5}\right)^{-4}$

6) $\left(\frac{x^{-2}y^{-2}}{x^3y^3}\right)^{-5}$

7) $\left(\frac{x^{-2}y^{-3}}{x^{-2}y^5}\right)^{-2}$

8) $\left(\frac{m^{-2}n^2}{m^3n^{-5}}\right)^{-2}$

9) $\left(\frac{c^{10}}{s^{-3}}\right)^{-5}$

10) $\left(\frac{m^{-4}n^{-5}}{m^{-2}n^{-3}}\right)^{-1}$

Zero Exponents

Zero exponents - any number, variable, or entire term raised to the zero power is equal to "1". The only exception to this rule is "0" to the "0" power.

Examples:

a. $x^0 = 1$ **b.** $x^0y = 1y = y$ **c.** $\frac{(ab)^0}{b} = \frac{1}{b}$ **d.** $a^3b^0c = a^3c$

Simplify:

1) $a^0 =$

2) $y^0 =$

3) $r^0 =$

4) $(xz)^0 =$

5) $(ax)^0 =$

6) $(xyz)^0 =$

7) $x^0b =$

8) $r^0c =$

9) $xc^0 =$

10) $a^2b^0 =$

11) $\frac{a^0}{c} =$

12) $\frac{x^0b}{5} =$

13) $a^2b^0x^2 =$

14) $3xy^0 =$

15) $-xy^0 =$

16) $(3a^2)^0 =$

17) $3(ab)^0 =$

18) $-3(c^2d)^0 =$

Addition and Subtraction of Polynomials

Combining Polynomials - To add or subtract polynomials, combine the numerical coefficients of the like terms. (Like terms are terms that have the **same** variables with the **same** exponents.)

Examples:

a. $(4x^2 + 3x - 2) + (2x^2 - 5x - 6)$

$(4x^2 + 2x^2) + (3x - 5x) + (-2 - 6)$

$6x^2 - 2x - 8$

b. $(3a^2 - 5a + 2) - (4a^2 + a + 2)$

$3a^2 - 5a + 2 - 4a = -a - 2$

$(3a^2 - 4a^2) + (-5a - a) + (2 - 2)$

$-a^2 - 6a$

Simplify.

1) $(x^2 + 5x) + (-2x^2 - 3x)$

2) $(y^2 + 3y) + (-2y - 5)$

3) $(x^2 + 4x + 9) + (x^2 - x - 6)$

4) $(x^3 - 5x + 6) + (3x^2 - x - 6)$

5) $(4y^3 + 2y^2 - 2) + (-3y^3 - 2y^2 - 1)$

6) $(x^3 - 3x) - (x^2 - 7x)$

7) $(x^2 - 3x + 2) - (x^2 + 6x + 7)$

8) $(3x^3 + 6x + 3) - (-2x^2 + 3x + 2)$

9). $(5y^3 + 4y - 1) - (y^3 + y^2 + 6)$

10) $(2x^3 - 5x + 6) - (x^3 - x + 7)$

11) $(y^3 - 6xy + 2) + (y^3 - 6xy - 7)$

12) $(x^2 - 2xy) - (-2x^2 + 3xy)$

13) $(2x^2 + x - 1) - (x^2 + 6x - 3)$

14) $(3x^2 + 2x - 2) + (x^2 + 5x - 6)$

15) $(3x^3 - 2x - 6) - (2x^2 + 6x - 1)$

Distributive Property

Simplify.

1) $x(x + 1)$

2) $y(2 - y)$

3) $-x(x + 2)$

4) $-y(8 - y)$

5) $2a(a - 1)$

6) $3b(b + 5)$

7) $-2x^2(x - 1)$

8) $-4y^2(y + 6)$

9) $-6y^2(y^2 - y)$

10) $-x^2(2x^2 - 3)$

11) $2x(5x^2 - 2x)$

12) $3y(2y - y^2)$

13) $(2x - 3)4x$

14) $(2y - 1)y$

15) $(2x - 3)x$

16) $(2x - 1)3x$

17) $-x^2y(x - y^2)$

18) $-xy^2(2x - y)$

19) $x(x^2 - 2x + 1)$

20) $x(x^2 - 3x - 2)$

21) $y(-y^2 + 4y - 3)$

22) $-y(y^2 - 5y - 6)$

23) $-a(a^2 - 6a - 1)$

24) $-b(2b^2 + 3b - 6)$

25) $x^2(2x^2 - 3x - 2)$

26) $y^2(-3y^2 - 5y - 3)$

27) $x^3(-x^2 - 5x - 6)$

28) $y^3(-2y^2 - 3y - 4)$

29) $2y^2(-2y^2 - 5y + 8)$

30) $3x^2(4x^2 - 2x + 7)$

31) $4x^2(5x^2 - x - 9)$

32) $5y^2(-y^2 + 3y - 6)$

33) $xy(x^2 - xy + y^2)$

34) $ab(a^2 - 3ab - 4b^2)$

35) $xy(x^2 - 2xy + 2y^2)$

36) $ab(a^2 + 5ab - 7b^2)$

Multiplying Binomials

Simplify.

1) $(x + 1)(x + 4)$

2) $(y + 2)(y + 3)$

3) $(a - 2)(a + 5)$

4) $(b - 5)(b + 4)$

5) $(y + 2)(y - 7)$

6) $(x + 9)(x - 4)$

7) $(y - 6)(y - 2)$

8) $(a - 7)(a - 8)$

9. $(a - 2)(a - 8)$

10) $(x + 11)(x - 3)$

11) $(2x + 1)(x + 6)$

12) $(y + 1)(3y + 2)$

13) $(2x - 3)(x + 3)$

14) $(5x - 2)(x + 3)$

15) $(3x - 2)(x - 5)$

16) $(2x - 1)(3x - 5)$

17) $(2y - 9)(y + 1)$

18) $(4y - 7)(y + 2)$

19) $(3x + 4)(3x + 7)$

20) $(5a + 2)(6a + 1)$

21) $(6a - 13)(2a - 5)$

22. $(5a - 9)(2a - 7)$

23) $(3b + 11)(5b - 4)$

24) $(3a + 10)(4a - 3)$

25) $(x + y)(x + 2y)$

26) $(2a + b)(a + 2b)$

27) $(2x - 3y)(x - y)$

28) $(a - 3b)(2a + 3b)$

29) $(4a - b)(2a + 5b)$

30) $(2x - y)(x + y)$

31) $(3x - 5y)(3x + 2y)$

32) $(5x + 2y)(6x + y)$

Multiplying Polynomials (Cont.)

Simplify.

1) $(x^2 + 2x + 1)(x + 2)$

2) $(x^2 - 3x + 5)(x - 1)$

3) $(a^2 - a + 2)(a - 2)$

4) $(x^2 - 4x - 5)(x - 3)$

5) $(a^2 - 5a + 3)(a - 1)$

6) $(x^2 - 5x - 4)(3x - 1)$

7) $(-2b^2 - b + 2)(b - 4)$

8) $(-a^2 + 4a - 3)(2a + 3)$

9) $(-x^2 + 6x - 3)(2x - 3)$

10) $(- a^2 - a + 2)(3a + 1)$

11) $(x^2 - 4)(x + 1)$

12) $(y^2 + 3y)(y - 3)$

13) $(x^2 + 3)(x - 5)$

14) $(y^2 - y)(2y + 4)$

15) $(x^3 - 2x + 1)(x - 3)$

16) $(y^3 + 2y^2 - 5)(2y - 3)$

17) $(4y^2 + 7y - 1)(2y - 7)$

18) $(3y^2 + y - 3)(3y - 2)$

19) $(3a^3 - 2a + 3)(a - 2)$

20) $(2y^2 + 4y - 2)(2y - 1)$

21) $(4a^3 - 4a + 1)(a - 3)$

22) $(2b^3 - 3b^2 + 6)(3b - 1)$

23) $(y^3 + y^2 - 2y + 2)(y + 1)$

24) $(a^3 - 2a^2 + 3a - 1)(3a - 2)$

Special Products

Simplify.

1) $(x + 1)(x - 1)$

2) $(x - 3)(x + 3)$

3) $(x + 5)(x - 5)$

4) $(x - 7)(x + 7)$

5) $(2x - 1)(2x + 1)$

6) $(3x - 1)(3x + 1)$

7) $(4x - 3)(4x + 3)$

8) $(x + 5)^2$

9) $(y - 4)^2$

10) $(3y - 1)^2$

11) $(x - 1)^2$

12) $(x - 3)^2$

13) $(x + 7)^2$

14) $(x + 9)^2$

15) $(x - y)^2$

16) $(2a - 5)^2$

17) $(5x - 4)^2$

18) $(3x - 7)^2$

19) $(3a - 5)(3a + 5)$

20) $(6x + 5)(6x - 5)$

21) $(2x + 5)^2$

22) $(9x - 2)^2$

23) $(a - 2b)^2$

24) $(x + 2y)^2$

25) $(5x - 6)(5x + 6)$

26) $(b - 6a)(b + 6a)$

27) $(x + 5y)^2$

28) $(2 - 7y)^2$

29) $(3 - 5y)^2$

30) $(3 - 5y)(3 + 5y)$

31) $(4x - 1)(4x + 1)$

32) $(2a + 3b)^2$

33) $(x + 6y)^2$

Applications with Polynomials

Solve.

1. The length of a rectangle is $3x$. The width is $3x - 1$. Find the area of the rectangle in terms of the variable x.

2. The width of a rectangle is $x - 2$. The length is $3x + 2$. Find the area of the rectangle in terms of the variable x.

3. The length of a rectangle is $3x + 1$. The width is $2x - 1$. Find the area of the rectangle in terms of the variable x.

4. The width of a rectangle is $x + 7$. The length is $4x + 3$. Find the area of the rectangle in terms of the variable x.

5. The length of a side of a square is $x + 3$. Use the equation $A = s^2$ where s is the length of a side of a square, to find the area of the square in terms of the variable x.

6. The length of a side of a square is $x - 8$. Use the equation $A = s^2$. where s is the length of the side of a square, to find the area of the square in terms of the variable x.

7. The length of a side of a square is $2x + 1$. Find the area of the square m terms of the variable x.

8. The length of a side of a square is $3x - 4$. Find the area of the square in terms of the variable x

9. The radius of a circle is $x + 4$. Use the equation $A = \pi r^2$ where r is the radius, to find the area of the circle in terms of the variable x. Use 3.14 for π.

10. The radius of a circle is $x - 3$. Use the equation $A = \pi r^2$, where r is the radius, to find the area of the circle in terms of the variable x. Use 3.14 for π.

11. The radius of a circle is $x + 6$. Find the area of the circle in terms of the variable x. Use 3.14 for π.

12. The radius of a circle is $2x + 1$. Find the area of the circle in terms of the variable x. Use 3.14 for π.

Dividing a Polynomial by a Monomial

Simplify.

1) $\frac{4x+4}{4}$

2) $\frac{7y+7}{7}$

3) $\frac{8a-12}{4}$

4) $\frac{15a-35}{5}$

5) $\frac{20a-30}{10}$

6) $\frac{12b-30}{6}$

7) $\frac{2a^2+3a}{a}$

8) $\frac{5y^2+3y}{y}$

9) $\frac{3b^3-2b}{b}$

10) $\frac{20y^2-10y}{10y}$

11) $\frac{x^3+2x^2-4x}{x}$

12) $\frac{a^3-4a^2+5a}{a}$

13) $\frac{x^4-2x^3-x^2}{x^2}$

14) $\frac{a^6-6a^5-4a^4}{a^3}$

15) $\frac{4x^2y^2-12xy}{4xy}$

16) $\frac{6x^2y^2+30xy}{6xy}$

17) $\frac{10y^6-25y^4}{-5y^4}$

18) $\frac{8x^4-6x^2}{-2x^2}$

19) $\frac{3y^4-12y^3}{-3y^3}$

20) $\frac{6x^2-9x^4}{-3x^2}$

21) $\frac{-7x^2-4x-6}{x}$

22. $\frac{4x^2+x-5}{x}$

23) $\frac{6y^2+4y-5}{y}$

24) $\frac{-2x^2+5x-4}{x}$

25) $\frac{8a^2b-10ab+12ab^2}{2ab}$

26) $\frac{24x^2y^2-12xy-36xy^2}{12xy}$

27) $\frac{12a^2b-18ab+30ab^2}{6ab}$

28) $\frac{20a^2b+10ab-40ab^2}{10ab}$

29) $\frac{5x^2y+6xy-2xy^2}{xy}$

30) $\frac{6ab^2-24ab+18a^2b^2}{6ab}$

Dividing a Polynomial by a Polynomial

Simplify.

1) $(x^2 + 6x + 9) \div (x + 3)$

2) $(x^2 + 8x + 16) \div (x + 4)$

3) $(x^2 - 12x + 36) \div (x - 6)$

4) $(a^2 - 14a + 49) \div (a - 7)$

5) $(x^2 - 2x - 8) \div (x + 2)$

6) $(y^2 - y - 30) \div (y - 6)$

7) $(x^2 - 3x - 18) \div (x - 6)$

8) $(y^2 + 5y - 14) \div (y+7)$

9) $(2x^2 + 5x + 3) \div (x + 1)$

10) $(2y^2 - 11y + 14) \div (y - 2)$

11) $(2y^2 + 7y + 3) \div (y + 3)$

12) $(2y^2 + 11y - 21) \div (2y - 3)$

13) $(3x^2 - 27) \div (3x - 9)$

14) $(2y^2 + y - 15) \div (y + 3)$

15) $(x^2 - 1) \div (x + 1)$

16) $(x^2 + 9) \div (x + 3)$

17) $(4x^2 - 10x) \div (2x - 3)$

18) $(4y^2 + 2y) \div (2y - 1)$

19) $(a^2 + 3a + 10) \div (a + 5)$

20) $(b^2 - 7b - 8) \div (b - 4)$

21) $(10y^2 - 29y - 21) \div (2y - 7)$

22) $(12a^2 - 6a - 6) \div (2a + 3)$

23) $(6a^2 + 11a - 10) \div (3a - 2)$

24) $(15x^2 + 14x - 8) \div (5x - 2)$

Removing a Common Factor

Factor.

1) $4a + 4$

2) $6c - 6$

3) $8 - 4a^2$

4) $9 + 21x^2$

5) $3x + 9$

6. $10a^2 + 15$

7) $24a - 8$

8) $24x + 12$

9) $9x - 6$

10) $16a^2 - 8a$

11) $12xy - 16y$

12) $6b^2 - 5b^3$

13) $20x^3 - 24x^2$

14) $12a^5 - 36a^2$

15) $24a^3b^4 - 18a^2b^2$

16) $4a^5b + 6ab^4$

17) $a^3b^2 + a^4b^3$

18) $25x^2y^2 - 15x^3y$

19) $3x^2y - 5xy^2$

20) $8a^3b - 12a^2b^3$

21) $x^3y^2 - x^2y^4$

22) $x^3 - 5x^2 + 7x$

23) $y^3 - 6y^2 - 8y$

24) $4x^2 - 16x + 20$

25) $6y^2 - 9y + 12$

26) $3X^2 - 9x + 18$

27) $b^4 - 3b^3 + 7b^2$

28) $4x^2 - 8x^3 + 12x^4$

29) $12y^2 - 16y + 48$

30) $5y^4 + 10y^3 - 35y$

31) $4x^4 + 12X^3 - 28X^2$

32) $45a^4b^2 - 75a^3b + 30ab^4$

33) $32x^4y^2 - 96x^2y^4 - 48x^6y^2$

Factoring by Grouping

Factor:

1) x(a+ b) + 3(a + b)

2) a(x - y) + 5(x - y)

3) x(b -1) - y(b - 1)

4) a(c - d) + b(c - d)

5) y(a - 1) - (a - 1)

6) a(y + 3) - (y + 3)

7) x(y - 2) - (y - 2)

8) 3x(y - 7) - (y - 7)

9) 2x(x - 5) - (x - 5)

10) 5y(x - 3) - (3 - x)

11) x(a - 2b) + y(a - 2b)

12) 4a(a + 1) - (a + 1)

13) a(x - 9) - (x - 9)

14) b(a - 5) -2(a - 5)

15) c (x - 3y) + d(x - 3y)

16) 3x(a + 1) + 4(a + 1)

17) a(x - 1) + 3(1 - x)

18) x(a - 4) + y(4 – a)

19) x(a - 3b) + y(3b - a)

20) x(a - 9) - (9 - a)

21) a(x - 7) + 3(7 - x)

22) 2x(x - 6) + (6 – x)

23) d(e - 5) + (5 - e)

24) 2x(a - 4) - y(4 - a)

25) m(a - 9) - n(9 - a)

26) 3m(n - 2) - (2 - n)

27) 2a(b - 1) - c(b - 1)

28) x(a - 2) - 3y(a - 2)

29) x(y - 5) + (5 - y)

30) 2(a - b) + c(b - a)

31) m(n - 1)+ 2(1 - n)

32) x(c - 7) - y(7 - c)

33) a(b - c) - 3(c - b)

34) 5(x - y) - z(y - x)

Factoring by Grouping (Cont.)

Factor by grouping:

1) $2xy - 6x + 3y - 9$

2) $x^2 + xy + 2x + 2y$

3) $ax + 5x + 6a + 30$

4) $2x^2 - 2xy + x - y$

5) $6x^2 + 2x + 6xy + 2y$

6) $4a^2 + 4ab + 3a + 3b$

7) $ax - 7a - 2x + 14$

8) $2x^2 + 2xy - 5x - 5y$

9) $ax + a - 2x - 2$

10) $3xy + y - 9x - 3$

11) $3a^2 - a - 6ab + 2b$

12) $2ax + x - 6a - 3$

13) $2ax - 3x - 4a + 6$

14) $5a^2 - 15a - 3ax + 9x$

15) $xy - 3x - y^2 + 3y$

16) $7a^2 - ay - 7ab + by$

17) $6x^2 - 4x - 3xy + 2y$

18) $4a^2 + 12a - ab - 3b$

19) $2a^2 + 3a - 8ax - 12x$

20) $3x^2 - 6x - xy + 2y$

21) $8ax - 2a + 4xy - y$

Factoring Trinomials with Coefficients of 1

Factor.

1) $a^2 - 2a - 35$

2) $a^2 - 4a + 3$

3) $a^2 + 3a - 10$

4) $a^2 - 5a + 6$

5) $b^2 - 7b + 10$

6) $b^2 + 8b + 15$

7) $b^2 + 6b - 16$

8) $y^2 - y - 30$

9) $y^2 + 3y - 40$

10) $y^2 + 5y - 66$

11) $x^2 - 4x - 60$

12) $y^2 - 7y + 10$

13) $y^2 - 9y + 18$

14) $x^2 - 12x + 36$

15) $x^2 - 4x - 96$

16) $a^2 + 3a - 28$

17) $x^2 + 10x + 16$

18) $b^2 - 11b - 180$

19) $x^2 + 10x + 25$

20) $x^2 - 14x + 49$

21) $b^2 + 7b + 12$

22) $b^2 + 10b + 16$

23) $x^2 - 9x - 36$

24) $x^2 - 7x - 60$

25) $x^2 + 10x - 56$

26) $x^2 - 8x - 128$

27) $x^2 - 4x - 77$

28) $b^2 - 20b + 84$

29) $b^2 - 21b + 108$

30) $b^2 - 27b + 180$

31) $a^2 + 16a + 63$

32) $x^2 - 19x + 60$

33) $x^2 - 25x + 84$

Factoring Trinomials with Coefficients Greater than 1

Factor.

1) $2x^2 - 5x + 2$

2) $3x^2 - 2x - 1$

3) $2a^2 + 7a + 3$

4) $3x^2 + x - 2$

5) $2b^2 - 13b + 6$

6) $3a^2 - 7a + 2$

7) $3x^2 - 13x + 4$

8) $4x^2 + 4x - 3$

9) $5a^2 + 2a - 3$

10) $5a^2 + 13a - 6$

11) $6y^2 + 5y - 6$

12) $6x^2 + x - 5$

13) $5x^2 - 3x - 2$

14) $7x^2 - 15x + 2$

15) $7y^2 + 8y + 1$

16) $14x^2 - 9x + 1$

17) $7y^2 + 18y + 8$

18) $9a^2 - 3a - 2$

19) $8x^2 - 26x - 7$

20) $3a^2 - 5a - 12$

21) $3x^2 - 10x - 8$

22) $6x^2 - 5x - 6$

23) $4y^2 + 25y + 25$

24) $7x^2 + 20x - 3$

25) $5x^2 + 2x - 7$

26) $10x^2 - 11x - 6$

27) $15x^2 + 14x - 8$

28) $8x^2 - 26x + 15$

29) $12x^2 - 7x - 10$

30) $9x^2 - 12x - 5$

31) $8x^2 - 2x - 15$

32) $10x^2 - 21x - 10$

33) $15x^2 - 26x + 8$

34) $12y^2 - 16y - 3$

Factoring Completely

Factor.

1) $2x^2 + 8x + 6$

2) $3x^2 + 15x + 12$

3) $3a^2 + 3a - 36$

4) $4x^2 - 8x - 12$

5) $ab^2 - ab - 2a$

6) $ab^2 + 5ab - 14a$

7) $xy^2 - 7xy + 10x$

8) $xy^2 + 11xy + 28x$

9) $x^3 - 8x^2 + 12x$

10) $2a^3 - 10a^2 + 8a$

11) $3y^3 - 18y^2 + 15y$

12) $5y^3 + 10y^2 - 40y$

13) $4x^2 + 28x - 32$

14) $2x^2 - 2x - 12$

15) $3z^2 - 21z - 24$

16) $z^3 + 14z^2 + 45z$

17) $a^2 - 2ab - 48b^2$

18) $c^3 + 17c^2 + 30c$

19) $x^2 - 9xy + 14y^2$

20) $4y^2 - 16y - 128$

21) $3x^2 + 3x - 18$

22) $2x^2 - 14x - 60$

23) $4a^2 + 28a - 32$

24) $3y^2 - 24y + 45$

25) $5y^2 - 50y + 45$

26) $2x^3 + 2x^2 - 12x$

27) $y^2 + 7xy - 8x^2$

28) $a^2 - 12ab + 35b^2$

29) $x^3 + 3x^2 - 40x$

30) $10x^3 + 60x^2 + 80x$

31) $2x^2y - 34xy - 36y$

32) $x^4 + 2x^2 - 63$

33) $2x^2 + 14x - 240$

34) $a^4 + a^3 - 72a^2$

Factoring Completely (Cont.)

Factor.

1) $3x^2 - 6x + 3$

2) $8x^2 + 22x - 6$

3) $6y^2 - 20y + 14$

4) $3x^3 - 8x^2 + 4x$

5) $15y^2 - 5y - 10$

6) $2a^2b + 8ab - 24b$

7) $4x^2 - 25xy + 6y^2$

8) $3y^2 + 8yz + 5z^2$

9) $x^2 - 11xy + 30y^2$

10) $2x^3 - 7x^2 - 4x$

11) $15 + 8x + x^2$

12) $21 + 4x - x^2$

13) $32 + 4x - x^2$

14) $4y^2 - 4y - 3$

15) $12y^2 + 12y - 9$

16) $12x^2 - 19x + 4$

17) $27 - 6x - x^2$

18) $5x^2 - 35x - 40$

19) $8y^2 - 8y - 6$

20) $5x^3 - 6x^2 + x$

21) $3x^2 - 11x - 20$

22) $12y^2 - 12y - 9$

23) $8x^2 + 8x - 6$

24) $6x^2 + 22x - 40$

25) $35a^2 - 16ab - 3b^2$

26) $10a^3 - 15a^2 - 25a$

27) $6x^3 + 17x^2 + 5x$

28) $6x^3 - 26x^2 + 8x$

29) $10b^2 + 14b - 12$

30) $22z - 9z^2 - z^3$

31) $10a^2 - 31ab + 15b^2$

32) $6x^3 - 32x^2 + 10x$

33) $19 + 18x - x^2$

34) $13x^2 + 49x - 12$

Difference of Perfect Squares

Factor.

1) $x^2 - 16$

2) $x^2 - 25$

3) $x^2 - 64$

4) $9x^2 - 1$

5) $16x^2 - 25$

6) $9x^2 - 49$

7) $x^4 - 4$

8) $x^8 - 100$

9) $36x^2 - 1$

10) $81x^2 - 1$

11) $1 - 100x^2$

12) $1 - 81x^2$

13) $y^4 - 121$

14) $1 - 144x^2$

15) $x^2 + 25$

16) $x^2 + 81$

17) $x^2 - y^6$

18) $x^4 - y^8$

19) $1 - 25x^2$

20) $1 - 36x^2$

21) $4 - 9x^2$

22) $16 - 49x^2$

23) $b^2 - 144c^2$

24) $a^2 - 49b^2$

25) $x^2y^2 - 100$

26) $x^6 - 81$

27) $9x^2 - 16y^2$

28) $25x^2 - 144$

29) $x^2y^2 - 1$

30) $x^2 - 400$

31) $36a^2 - 1$

32) $49x^2 - 4$

33) $x^4 - 4$

Sum and Difference of Cubes

Factor Completely.

1) $x^3 + y^3$

2) $x^3 + 8$

3) $x^3 - y^3$

4) $x^3 - 8$

5) $x^3 - 1$

6) $a^3 - 1$

7) $27a^3 + 1$

8) $8x^3 + 1$

9) $8x^3 - y^3$

10) $27a^3 - b^3$

11) $27a^3 + 8b^3$

12) $8x^3 + 125y^3$

13) $125a^3 - 27b^3$

14) $64x^3 - 27y^3$

15) $x^3 - y^6$

16) $8a^3 - 125$

17) $8a^6 + b^3$

18) $64x^6 + 1$

19) $a^9 - 1$

20) $a^9 + 1$

Factoring Completely

Factor Completely.

1) $3x^2 - 12$

2) $2x^2 - 50$

3) $x^3 + 2x^2 + x$

4) $y^3 - 8y^2 + 16y$

5) $x^4 + x^3 - 6x^2$

6) $a^4 - 3a^3 - 40a^2$

7) $3b^2 + 30b + 63$

8) $5a^2 + 7a - 6$

9) $4y^2 - 32y + 28$

10) $2a^2 - 18a - 44$

11) $x^3 - 8x^2 - 20x$

12) $b^3 - 5b^2 - 6b$

13) $a^4 - 12a^3 - 28a^2$

14) $5a^3 - 30a^2 + 45a$

15) $4x^2 - 6x + 2$

16) $2x^4 - 11x^3 + 5x^2$

17) $x^4 - 16x^2$

18) $a^4 - 81$

19) $15x^3 - 18x^2 + 3x$

20) $9a^3 + 39a^2 + 12a$

21) $3xy^2 + 11xy - 20x$

22) $24 + 6x - 3x^2$

23) $a^2b^2 + 7ab^2 - 8b^2$

24) $4x^2y + 12xy + 8y$

25) $72 + 2a^2$

26) $18a^3 - 54a^2 + 36a$

27) $x^2y^2 - y^2$

28) $5x^2 - 45y^2$

29) $x^4 - 9x^2$

30) $a^4 - 1$

31) $a(3x - 3) + b(3x - 3)$

32) $y^2(a + b) - 4(a + b)$

Solving Quadratic Equations by Factoring

Solve.

1) $(y+1)(y+2) = 0$

2) $(y - 4)(y - 6) = 0$

3) $(z - 6)(z - 1) = 0$

4) $(x + 7)(x - 5) = 0$

5) $x(x - 8) = 0$

6) $x(x + 1) = 0$

7) $a(a - 4) = 0$

8) $a(a + 7) = 0$

9) $y(3y + 2) = 0$

10) $t(2t - 5) = 0$

11) $3a(2a - 1) = 0$

12) $2b(4b + 3) = 0$

13) $(b - 1)(b - 4) = 0$

14) $(b - 7)(b + 4) = 0$

15) $x^2 - 16 = 0$

16) $x^2 - 4x - 21 = 0$

17) $x^2 + 6x - 16 = 0$

18) $x^2 - 5x = 6$

19) $x^2 - 7x = 18$

20) $x^2 - 8x = 9$

21) $x^2 - 5x = 14$

22) $2a^2 - a = 3$

23) $4t^2 - 13t = -3$

24) $5a^2 + 13a = 6$

25) $2x^2 + 5x = -2$

26) $x(x+10) = 11$

27) $y(y - 9) = -18$

28) $x(x+ 5) = 50$

29) $x(x - 11) = -30$

30) $(2x + 3)(x - 1) = 25$

31) $(z + 1)(z - 9) = 39$

32) $(y + 2)(y - 3) = 6$

33) $(y + 3)(2y - 1) = 9$

Applications of Quadratic Equations

Solve.

1) The square of a positive number is four more than three times the positive number. Find the number.

2) The square of a negative number is forty-eight more than twice the negative number. Find the number.

3) The sum of two numbers is seven. The sum of the squares of the two numbers is twenty-nine. Find the two numbers.

4) The sum of two numbers is ten. The sum of the squares of the two numbers is fifty-two Find the two numbers.

5) The sum of two numbers is twelve. The product of the two numbers is twenty-seven. Find the two numbers.

6) The square of the sum of a number and six is eighty-one. Find the number.

7) The sum of the squares of two consecutive positive integers is sixty-one. Find the two integers.

8) The product of two consecutive even positive integers is one hundred-twenty. Find the integers.

9) The length of the base of a triangle is three times the height. The area of the triangle is 96 ft. Find the base and height of the triangle.

10) The length of a rectangle is four times the width. The area is 144 in^2. Find the length and width.

11) The width of a rectangle is 7 ft. less than the length. The area of the rectangle is 120 $ft.^2$. Find the length and width of the rectangle.

12) The length of each side of a square is extended by 3 in. The area of the resulting square is 121 in^2. Find the length of a side of the original square.

Simplifying Algebraic Fractions

Simplify.

1) $\frac{8x^2}{12x^4}$

2) $\frac{24x^2y}{32xy^3}$

3) $\frac{(x+1)^3}{(x+1)^5}$

4) $\frac{(3x-2)^6}{(3x-2)^4}$

5) $\frac{2n-3}{3-2n}$

6) $\frac{15y^2(y+3)}{20y^3(y+3)}$

7) $\frac{15y(y+1)}{45y^2(y+1)}$

8) $\frac{24x^3(4-x)}{18x(4-x)}$

9) $\frac{6x^3(2-x)}{12x(2-x)}$

10) $\frac{12x^3(5-2x)}{18x(2x-5)}$

11) $\frac{a^2+6a}{ac+6c}$

12) $\frac{x^2+4x}{3x+12}$

13) $\frac{6-8x}{4x^2-3x}$

14) $\frac{x^2+2x-15}{x^2-10x+21}$

15) $\frac{x^2-2x-3}{x^2-5x-6}$

16) $\frac{x^2-7x+12}{x^2-x-12}$

17) $\frac{x^2-3x-10}{x^2-6x+5}$

18) $\frac{x^2-8x+16}{x^2-9x+20}$

19) $\frac{x^2-7x+10}{x^2-4x-5}$

20) $\frac{x^2+6x+9}{x^2+2x-3}$

21) $\frac{x^2-2x-24}{16-x^2}$

22) $\frac{9-y^2}{y^2+2y-15}$

23) $\frac{3x^3+18x^2-21x}{x^3+5x^2-14x}$

24) $\frac{2x^2-18x}{4x^3-32x^2-36x}$

25) $\frac{5x^2-11x+2}{5x^2-7x-6}$

26) $\frac{2x^2-7x+3}{2x^2+9x-5}$

27) $\frac{x^2+2x-24}{12-4x-x^2}$

28) $\frac{2x^2-8}{x^2+4x-12}$

29) $\frac{2x^2-9x+7}{x^2+2x-3}$

30) $\frac{8x^2-14x+6}{4x-3}$

Multiplying Algebraic Fractions

Simplify.

1) $\frac{9xy}{35a^3b^2} \cdot \frac{14a^2b^3}{45x^3y^2}$

2) $\frac{8xy^3}{75a^2b^2} \cdot \frac{15a^3b^2}{32x^2y^2}$

3) $\frac{9x-18}{4x+16} \cdot \frac{20x+80}{12x-24}$

4) $\frac{6x-9}{12x+6} \cdot \frac{36x+18}{24x-36}$

5) $\frac{x^2-16}{x^2-4x} \cdot \frac{x^2+x}{x^2+8x+16}$

6) $\frac{x^2-2x+1}{x^2+x-2} \cdot \frac{x^2+8x}{x^2+7x-8}$

7) $\frac{x^2-3x-10}{x^2-5x} \cdot \frac{x+2}{x^2-4}$

8) $\frac{x^2-x-20}{x^2y^3} \cdot \frac{x^3y^2}{x^2-10x+25}$

9) $\frac{3x^2-7x}{3xy+y} \cdot \frac{3xy^2+y^2}{6x^2-14x}$

10) $\frac{xy^2}{x^2+12x+32} \cdot \frac{x^2+3x-4}{x^3y^2}$

11) $\frac{x^2+5x-14}{x^2+6x-16} \cdot \frac{x^2-64}{x^2-x-56}$

12) $\frac{x^2-3x-10}{x^2+6x+8} \cdot \frac{x^2-x-20}{x^2-25}$

13) $\frac{x^2-7x-18}{x^2-81} \cdot \frac{x^2-11x+18}{x^2-4}$

14) $\frac{x^2+2x-15}{x^2+12x+35} \cdot \frac{x^2+16x+63}{x^2-x-6}$

15) $\frac{x^2+5x-14}{x^2-x-6} \cdot \frac{x^2+x-2}{x^2-3x+2}$

16) $\frac{x^2-3x-54}{x^2-8x-20} \cdot \frac{x^2-9x-10}{x^2-8x-9}$

17) $\frac{x^2-4x-32}{x^2-8x} \cdot \frac{x^3+2x^2+x}{x^2+5x+4}$

18) $\frac{2x^2+3x-20}{2x^2+7x-30} \cdot \frac{4x^2-12x+9}{2x^2+5x-12}$

19) $\frac{3x^2-10x+3}{2x^2-5x-3} \cdot \frac{2x^2-3x-2}{2x^2-x-6}$

20) $\frac{x^2+4x+3}{6x^2+17x+12} \cdot \frac{3x^2+x-4}{x^2-1}$

Dividing Algebraic Fractions

Simplify.

1) $\frac{6x^2y^3}{25a^2b^2} \div \frac{8xy}{15ab}$

2) $\frac{18a^2b^4}{25a^3b^2} \div \frac{27a^3b^3}{100a^4b^2}$

3) $\frac{5x-10}{6x-24} \div \frac{15x-30}{9x-36}$

4) $\frac{18x-9}{24x-36} \div \frac{12x+36}{20x-30}$

5) $\frac{x^2+2x}{x^2-4} \div \frac{x+2}{x-2}$

6) $\frac{x^2-x-6}{x^2+2x+1} \div \frac{x+2}{x+1}$

7) $\frac{x^2+3x+2}{xy^2} \div \frac{x^2+4x+4}{x^2y^2}$

8) $\frac{x^2-4}{x+2} \div \frac{x-2}{x+3}$

9) $\frac{x^2-25}{x^3y^4} \div \frac{x^2-10x+25}{x^4y^2}$

10) $\frac{6ax-9a}{y^2} \div \frac{2x-3}{y^4}$

11) $\frac{x^2-5x-14}{x^2-9x+14} \div \frac{x^2+6x+8}{x^2+2x-8}$

12) $\frac{x^2+x-6}{x^2-9} \div \frac{x^2-9x+14}{x^2-8x+15}$

13) $\frac{x^2-3x+2}{x^2-7x+10} \div \frac{x^2-1}{x^2-4x-5}$

14) $\frac{x^2-x-30}{x^2+6x+5} \div \frac{x^2-11x+30}{x^2-6x+5}$

15) $\frac{x^2-2x-63}{x^2-4x-45} \div \frac{x^2+3x-28}{x^2+9x+20}$

16) $\frac{x^2-11x+28}{x^2-13x+42} \div \frac{x^2-9x+20}{x^2+x-30}$

17) $\frac{x^2-6x-40}{x^2-7x-30} \div \frac{2x^2+7x+3}{2x^2+5x+2}$

18) $\frac{3x^2+17x+10}{3x^2-22x-16} \div \frac{x^2-8x-48}{x^2-4x-32}$

19) $\frac{2x^2+5x-3}{x^2-2x-15} \div \frac{2x^2-5x+2}{x-5}$

20) $\frac{3y^2-14y+8}{2y^2-7y-4} \div \frac{3y^2-16y+5}{2y^2-9y-5}$

Addition and Subtraction of Fractions with Like Denominators

Simplify:

1) $\frac{2}{x^2}+\frac{5}{x^2}$

2) $\frac{9}{xy}-\frac{3}{xy}$

3) $\frac{2}{x+3}-\frac{7}{x+3}$

4) $\frac{x}{x-5}-\frac{3}{x-5}$

5) $\frac{4x}{3x+2}+\frac{6x}{3x+2}$

6) $\frac{5y}{5y-1}-\frac{8y}{5y-1}$

7) $\frac{3x+2}{x-2}+\frac{x+4}{x-2}$

8) $\frac{5x+4}{2x-1}+\frac{3x-9}{2x-1}$

9) $\frac{4x-3}{x+8}-\frac{2x-19}{x+8}$

10) $\frac{7x+4}{x-9}-\frac{4x+31}{x-9}$

11) $\frac{x-6}{3x+5}-\frac{5x-4}{3x+5}$

12) $\frac{3n}{2n+9}-\frac{6n-5}{2n+9}$

13) $\frac{x}{x^2-3x-10}+\frac{2}{x^2-3x-10}$

14) $\frac{2x}{x^2-x-12}-\frac{8}{x^2-x-12}$

15) $\frac{3x+5}{x^2-6x+8}-\frac{2x+9}{x^2-6x+8}$

16) $\frac{2x-7}{x^2+4x-12}-\frac{x-13}{x^2+4x-12}$

17) $\frac{3y-5}{2y^2+9y-5}-\frac{y-4}{2y^2+9y-5}$

18) $\frac{4x-3}{2x^2+x-3}+\frac{x-2}{2x^2+x-3}$

19) $\frac{3x^2+x}{x^2-6x+5}+\frac{2x^2+3x}{x^2-6x+5}-\frac{5x^2+4}{x^2-6x+5}$

20) $\frac{2x^2-7x}{x^2-x-56}-\frac{x^2+2x+10}{x^2-x-56}-\frac{x-26}{x^2-x-56}$

Addition and Subtraction of Fractions with Unlike Denominators

Simplify:

1) $\frac{8}{x} - \frac{4}{3x}$

2) $\frac{3}{2a} - \frac{2}{3a}$

3) $\frac{2}{3x} + \frac{4}{9x} - \frac{1}{6x}$

4) $\frac{5}{y^2} + \frac{2}{3y} - \frac{3}{5y}$

5) $\frac{2x-1}{2x} - \frac{x+1}{3x}$

6) $\frac{3x-2}{8x} + \frac{x+4}{12x}$

7) $\frac{4x-7}{9x} - \frac{x-3}{4x}$

8) $\frac{x+1}{xy^2} - \frac{3x-3}{xy}$

9) $\frac{x-9}{6x^2} + \frac{x-2}{2x}$

10) $\frac{x+4}{9x} - \frac{x-4}{12x^2}$

11) $\frac{4}{x-1} - \frac{3}{x+2}$

12) $\frac{5}{y+6} - \frac{2}{y-3}$

13) $\frac{2x}{x-8} + \frac{8}{x-2}$

14) $\frac{3x}{2x-1} - \frac{6}{x-3}$

15) $\frac{x}{x^2-4} - \frac{2}{x-2}$

16) $\frac{4x}{x^2-2x-8} + \frac{3}{x-4}$

17) $\frac{x+2}{x^2-x-6} - \frac{3}{3-x}$

18) $\frac{2x+1}{x-2} - \frac{x-3}{x-1} + \frac{x+2}{x^2-3x+2}$

Complex Fractions

Simplify:

1) $\dfrac{1-\dfrac{5}{x}}{1-\dfrac{25}{x^2}}$

2) $\dfrac{2+\dfrac{6}{x}}{1-\dfrac{36}{x^2}}$

3) $\dfrac{1-\dfrac{5}{x+3}}{2-\dfrac{10}{x+3}}$

4) $\dfrac{6-\dfrac{18}{x-5}}{2-\dfrac{6}{x-5}}$

5) $\dfrac{5-\dfrac{10}{2x+1}}{3-\dfrac{18}{2x+1}}$

6) $\dfrac{2-\dfrac{4}{x+6}}{3-\dfrac{6}{x+6}}$

7) $\dfrac{1-\dfrac{12}{x}+\dfrac{27}{x^2}}{1-\dfrac{9}{x^2}}$

8) $\dfrac{1+\dfrac{9}{x}+\dfrac{14}{x^2}}{1-\dfrac{49}{x^2}}$

9) $\dfrac{1-\dfrac{11}{x}+\dfrac{30}{x^2}}{1-\dfrac{2}{x}-\dfrac{15}{x^2}}$

10) $\dfrac{1-\dfrac{3}{x}-\dfrac{10}{x^2}}{1-\dfrac{8}{x}+\dfrac{15}{x^2}}$

11) $\dfrac{x-\dfrac{3x}{x+3}}{x+\dfrac{3x}{x+3}}$

12) $\dfrac{x+\dfrac{1}{x-1}}{1+\dfrac{1}{x-1}}$

13) $\dfrac{x+3+\dfrac{12}{x+2}}{x+2-\dfrac{3}{x+2}}$

14) $\dfrac{x+2-\dfrac{7}{x-4}}{x+3-\dfrac{8}{x-4}}$

15) $\dfrac{\dfrac{2}{x}-\dfrac{1}{x+1}}{\dfrac{6}{x}+\dfrac{1}{x+1}}$

16) $\dfrac{\dfrac{4}{2x+1}-\dfrac{2}{x}}{\dfrac{6}{x}-\dfrac{3}{2x+1}}$

Fractional Equations

Solve.

1) $\frac{2x}{5}-\frac{4}{3}=-\frac{8}{15}$

2) $\frac{x}{2}-\frac{2}{3}=\frac{5}{6}$

3) $\frac{3x-4}{6}+\frac{1}{2}=\frac{x}{4}+\frac{7}{12}$

4) $\frac{2x+3}{10}-\frac{1}{2}=\frac{4x+2}{10}-\frac{3}{5}$

5) $\frac{5}{2a-1}=3$

6) $\frac{20}{2x-3}=4$

7) $3+\frac{9}{x}=6$

8) $2+\frac{7}{n}=3$

9) $\frac{2}{x-1}=\frac{3}{x}$

10) $\frac{4}{x+2}=\frac{3}{x+1}$

11) $\frac{3}{2x-7}=\frac{2}{3x+2}$

12) $\frac{4}{4x-1}=\frac{-2}{1-3x}$

13) $\frac{x}{x-2}=3-\frac{2}{x-2}$

14) $\frac{6}{y+3}=\frac{3}{y-1}$

15) $\frac{3x}{x-2}-3=\frac{x}{x-2}$

16) $\frac{n}{n+8}=\frac{1}{n+3}$

17) $\frac{x}{x-1}=\frac{6}{x+1}$

18) $\frac{x}{x+12}=\frac{2}{x+4}$

19) $\frac{6}{y}=\frac{1}{y-1}+2$

20) $\frac{2x}{3x-4}=\frac{x-3}{x}$

Solving Proportions

Solve.

1) $\frac{x}{18} = \frac{5}{6}$

2) $\frac{12}{x} = \frac{2}{5}$

3) $\frac{5}{8} = \frac{x}{40}$

4) $\frac{x}{20} = \frac{4}{5}$

5) $\frac{8}{x} = \frac{2}{3}$

6) $\frac{6}{7} = \frac{x}{28}$

7) $\frac{15}{8} = \frac{75}{x}$

8) $\frac{x+2}{10} = \frac{4}{5}$

9) $\frac{7}{8} = \frac{x-3}{16}$

10) $\frac{12}{7} = \frac{48}{x}$

11) $\frac{x+1}{16} = \frac{3}{4}$

12) $\frac{5}{6} = \frac{x-2}{18}$

13) $\frac{15}{x+6} = \frac{5}{8}$

14) $\frac{3}{10} = \frac{18}{x-4}$

15) $\frac{3}{x} = \frac{12}{x+3}$

16) $\frac{16}{x-3} = \frac{2}{3}$

17) $\frac{4}{9} = \frac{16}{x+2}$

18) $\frac{2}{x} = \frac{6}{x-1}$

19) $\frac{12}{x-1} = \frac{6}{x}$

20) $\frac{x+2}{5} = \frac{x}{20}$

21) $\frac{x-5}{2} = \frac{x}{7}$

22) $\frac{20}{x-3} = \frac{5}{x}$

23) $\frac{x+1}{3} = \frac{x}{9}$

24) $\frac{x-7}{4} = \frac{x}{6}$

25) $\frac{3}{x-2} = \frac{12}{2x-1}$

26) $\frac{15}{x+3} = \frac{3}{x-1}$

27) $\frac{3x}{5} = \frac{x-3}{10}$

28) $\frac{4}{x-3} = \frac{16}{3x+1}$

29) $\frac{8}{x+4} = \frac{2}{x-1}$

30) $\frac{5x}{6} = \frac{x-1}{12}$

Applications with Proportions

Solve.

1) The monthly loan payment for a car is $31.05 for each $1000 borrowed. At this rate, find the monthly payment for a $7000 car loan.

2) An investment of $1000 earns $130 each year. At the same rate, how much *additional* money must be invested to earn $325 each year?

3) Eighteen ceramic tiles are required to tile a 12-square-foot area. At this rate, how many feet can be tiled using 324 ceramic tiles?

4) A pre-election survey showed that 4 out of every 5 voters would vote for a new library. At this rate, how many people would be expected to vote for the library if 20,000 votes were cast?

5) A quality control inspector found 3 defective electric clocks in a shipment of 100 clocks. At this rate, how many clocks would be defective in a shipment of 3000?

6) A license fee for a car which cost $6000 was $72. At the same rate, what is the license fee for a car which costs $8000?

7) In a wildlife preserve, 20 deer are captured, tagged, and then released. Later, 50 deer are captured. Four of the 50 are found to have tags Estimate the number of deer in the preserve.

8) A stock investment of 100 shares paid a dividend of $140. At this rate, how many *additional* shares are required to earn a dividend of $210?

9) Two ounces of a medication are required for a 120-pound adult. At the same rate, how many *additional* ounces of medication are required for a 180-pound adult?

10) A caterer estimates that 2 gallons of fruit punch will serve 30 people. How much *additional* punch is necessary to serve 75 people?

Literal Equations

Rewrite in terms of y.

1) $2x + y = 8$

2) $5x - y = 2$

3) $2x + 5y = 20$

4) $5x - 2y = 10$

5) $3x + 6y = 18$

6) $x - 2y = 6$

7) $x + 4y = 8$

8) $2x - 3y - 18 = 0$

9) $3x - y + 9 = 0$

Rewrite in terms of x.

10) $x + 4y = 6$

11) $x - 5y = 7$

12) $2x - y = 4$

13) $3x + y = 12$

14) $2x + 3y = 8$

15) $2x + 5y = 10$

16) $3x - 5y - 30 = 0$

17) $2x - 7y - 14 = 0$

18) $5x - 4y + 5 = 0$

Rewrite the formula in terms of the variable given.

19) $a = \frac{1}{2}bh$; b

20) $P = a + b + c$; a

21) $d = rt$; r

22) $PV = nRT$; P

23) $P = 2l + 2w$; w

24) $R = \frac{C - S}{t}$; C

25) $I = \frac{E}{R + r}$; r

26) $R = \frac{MB - T}{K}$; M

27) $K = mr - mv$; m

28) $S = \pi r(r - l)$: l

29) $A = P + Prt$; r

30) $A = \frac{P}{1 - c}$; c

Work Problems

Solve.

1) Two farmers are planting peanuts. One farmer, using an old planter and working alone, required 6 hours to plant the crop. A second farmer, using a modern planter, can plant the crop in 2 hours. How long would it take to plant the peanut crop with both planters working together?

2) A high speed copier can completely copy a report in 10 hours. An older copier requires 15 hours to do the same task. How long would it take to copy the report with both copiers operating?

3) One member of a golf course greens crew can mow a fairway in 40 minutes, while the second member of the crew requires 24 minutes to mow the same fairway. How long would it take to mow the fairway when they work together?

4) A small heating unit will heat a room 10° in 60 minutes. A larger heater will heat the room 10° in 40 minutes. How long would it take to heat the room 10° with both heaters operating?

5) With cold and hot water running, a bathtub can be filled in 8 minutes. The hot water faucet working alone requires 24 minutes to fill the tub. How long would it take the cold water faucet working alone to fill the tub?

6) An experienced bricklayer can construct a wall twice as fast as an apprentice bricklayer. Working together, the bricklayers require 8 hours to build the wall. How long would it take the apprentice working alone to build the wall?

7) A large and small inlet pipe work together to fill a storage tank. The larger pipe requires 8 hours working alone to fill the tank. After both pipes have been operating for 3 hours, the larger pipe is turned off. The small pipe requires 10 more hours to fill the storage tank. How long would it take the smaller pipe working alone to fill the tank?

8) One welder requires 20 hours to make the welds in a steel frame, while a second welder requires 35 hours to do the same job. The first welder worked for 8 hours, and then quit. How long would it take the second welder to complete the welds?

9) Two computer printers that work at the same rate are working together to print the payroll checks for a large corporation. After working together for 3 hours, one of the printers quits. The second requires 2 more hours to complete the payroll checks. Find the time it would take one printer alone to print the payroll checks.

10) Two roofers who work at the same rate are roofing a section of an apartment. After working together for 8 hours, one roofer quits. The second roofer required 15 more hours to finish the job. How long would it take one of the roofers working alone to roof the apartment?

Distance Problems

Solve.

1) An express train travels 440 miles in the same amount of time that a freight train travels 280 miles. The rate of the express train is 20 mph faster than the freight train. Find the rate of each train.

2) A commuter plane flew 720 miles in the same amount of time it took a jet to fly 1520 mi. The rate of the jet was 200 mph greater than the rate of the commuter plane. Find the rate of each plane.

3) A sales accountant traveled 75 miles by car and 1800 miles by jet plane. The rate of the jet is eight times the rate of the car. The entire trip took 6 hours. Find the rate of each.

4) A single-engine plane and a corporate jet leave an airport at noon and head for another airport 1040 miles away. The rate of the corporate jet is four times the rate of the single-engine plane The single-engine plane arrives 8 hours after the corporate jet. Find the rate of each plane.

5) The speed of a boat in still water is 28 mph. The boat traveled 70 miles down a river in the same amount of time it traveled 42 miles up the river. Find the rate of the river's current.

6) A jet can fly 550 mph in calm air. Traveling with the wind, the plane can fly 2400 miles in the same amount of time as it flies 2000 miles against the wind. Find the rate of the wind.

7) A motorist drove 270 miles on a four-lane highway before driving 54 miles on a two-lane mountain road The rate of speed on the four-lane highway was two times the rate on the mountain road. The time spent traveling the 324 miles was 7 hours. Find the rate of the motorist on the mountain road.

8) A plane can fly at a rate of 220 mph in calm air. Traveling with the wind, the plane flew 1000 miles in the same amount of time as it flew 760 miles against the wind. Find the rate of the wind.

9) A canoeist can paddle at the rate of 8 mph in calm water. Traveling with the current, the canoe traveled 30 miles in the same amount of time as it traveled 18 miles against the current. Find the rate of the current.

10) A corporate jet can fly at 280 mph in calm air. Flying with the wind the plane can fly 960 miles in the same amount of time as it can fly 720 miles against the wind. Find the rate of the wind.

Applications Of Variation

1) Salary. Robin earns \$12 an hour (h). How much does she earn in 8 h?

2) Salary. Kwang earns \$9.50 per hour. How much does he earn in 40 h?

3) Distance traveled. Lee is traveling at a constant rate of 55 miles per hour (mi/h). How far does she travel in 8 h?

4) Distance traveled. An airplane is traveling at a constant rate of 450 mi/h. How far does it travel in 7 h?

Translate each to an algebraic equation using *k* as the constant of the variation.

5) s varies directly as x

6) r is inversely proportional to s

7) V is directly proportional to t

8) m varies inversely as n

9) y varies inversely as the square of x

10) V varies directly as the cube of r

Find *k*, the constant of the variation.

11) y varies directly with x, $y = 54$ when $x = 6$

12) m varies directly with n, $m = 144$ when $n = 8$

13) V is directly proportional to h, $V = 189$ when $h = 9$

14) d directly proportional to t, d =750 when $t = 15$

15) m varies inversely as p, m = 20 when $p = 25$

16) v varies inversely as x, $v = 12$ when $x = \frac{1}{2}$

17) w is inversely proportional to t, $w = 3$ when $t = 12$

18) r is inversely proportional to t; $r = \frac{1}{2}$ when $t = 14$

$\sqrt{\text{Simplifying radicals}}$

a) Perfect Squares

These numbers have a set of "twins" as factors:

$\sqrt{16} = \sqrt{4 \cdot 4}$ (notice the "twins" as factors) = 4

$\sqrt{9} = \sqrt{3 \cdot 3} = 3$

$\sqrt{4} = 2$

$\sqrt{1} = 1$

$\sqrt{144} = 12$

Try these:

1) $\sqrt{121}$ _____ **2)** $\sqrt{25}$ _____ **3)** $\sqrt{49}$ _____ **4)** $\sqrt{100}$ _____

5) $\sqrt{36}$ _____ **6)** $\sqrt{9}$ _____ **7)** $\sqrt{64}$ _____ **8)** $\sqrt{81}$ _____

b) <u>NOT so perfect squares:</u> Choose a set of factors, where one is a perfect square. Look for the largest perfect square that you can find.

$\sqrt{18} = \sqrt{9 \cdot 2} = \sqrt{9} \cdot \sqrt{2} = 3\sqrt{2}$

$\sqrt{32} = \sqrt{16 \cdot 2} = \sqrt{16} \cdot \sqrt{2} = 4\sqrt{2}$

$\sqrt{75} = \sqrt{25 \cdot 3} = \sqrt{25} \cdot \sqrt{3} = 5\sqrt{3}$

$\sqrt{200} = \sqrt{100 \cdot 2} = \sqrt{100} \cdot \sqrt{2} = 10\sqrt{2}$

Try these:

1) $\sqrt{72}$ _____ **2)** $\sqrt{12}$ _____ **3)** $\sqrt{50}$ _____ **4)** $\sqrt{20}$ _____

5) $\sqrt{27}$ _____ **6)** $\sqrt{8}$ _____ **7)** $\sqrt{48}$ _____ **8)** $\sqrt{45}$ _____

Simplifying Radicals

Simplify.

1) $\sqrt{25}$

2) $\sqrt{121}$

3) $\sqrt{169}$

4) $\sqrt{196}$

5) $\sqrt{289}$

6) $\sqrt{400}$

7) $\sqrt{18}$

8) $\sqrt{48}$

9) $\sqrt{20}$

10) $\sqrt{45}$

11) $\sqrt{72}$

12) $3\sqrt{12}$

13) $-6\sqrt{32}$

14) $9\sqrt{40}$

15) $\sqrt{125}$

16) $-7\sqrt{50}$

17) $\sqrt{192}$

18) $3\sqrt{144}$

19) $\sqrt{150}$

20) $\sqrt{900}$

21) $\sqrt{200}$

22) $3\sqrt{160}$

23) $5\sqrt{98}$

24) $\sqrt{88}$

25) $\sqrt{135}$

26) $\sqrt{128}$

27) $3\sqrt{75}$

28) $\sqrt{700}$

29) $\sqrt{245}$

30) $2\sqrt{96}$

$\sqrt{\text{SIMPLIFYING RADICALS WITH VARIABLES}}$

In a **square root** the **index** is 2

$\sqrt[2]{x}$

In a **cube root** the **index** is 3

$\sqrt[3]{x}$

In a **fourth root** the **index** is 4

$\sqrt[4]{x}$

To simplify radicals with variables look at the radical as a "jail" with the variables trying to "break out". The index indicates how many must be in a group to "break out". For instance, if the index is 3 then there must be 3 of the same thing to escape.

$\sqrt[3]{x^3} = \sqrt[3]{x \cdot x \cdot x} = x$

$\sqrt[4]{x^4} = \sqrt[4]{x \cdot x \cdot x \cdot x} = x$

Take note of this one: $\sqrt[3]{x^6} = \sqrt[3]{x \cdot x \cdot x \cdot x \cdot x \cdot x} = x^2$ (Notice the square means two groups).

But, watch what happens when there is an extra variable.....

$\sqrt{x^5}$ (which really means $\sqrt[2]{x^5}$) $= \sqrt{x \cdot x \cdot x \cdot x \cdot x} = x^2\sqrt{x}$

To figure the answer without drawing all the x's, simply divide the index into the exponent. The number of times the answer comes out evenly, is the exponent of the variable on the outside and the remainder is the exponent under the radical in the answer.

$\sqrt[3]{x^4}$ $(\frac{4}{3} = 1$ remainder 1) $= x\sqrt[3]{x}$	$\sqrt{x^7}$ $(\frac{7}{2} = 3$ remainder 1) $= x^3\sqrt{x}$
$\sqrt{x^{16}}$ $(\frac{16}{2} = 8$, no remainder) $= x^8$	$\sqrt[4]{x^{14}}$ $(\frac{14}{4} = 3$, remainder 2) $= x^3\sqrt[4]{x^2}$

Try these:

1. $\sqrt{x^5}$ _____ 2. $\sqrt{x^9}$ _____ 3. $\sqrt[3]{x^7}$ _____ 4. $\sqrt{x^{13}}$ _____ 5. $\sqrt[4]{x^{20}}$ _____ 6. $\sqrt[3]{x^{17}}$ _____

Simplifying Radicals with Variables

Simplify.

1) $\sqrt{x^8}$

2) $\sqrt{x^{10}}$

3) $\sqrt{y^9}$

4) $\sqrt{x^{13}}$

5) $\sqrt{x^6y^6}$

6) $\sqrt{x^{10}y^4}$

7) $\sqrt{9x^6}$

8) $\sqrt{16y^{10}}$

9) $\sqrt{36x^2}$

10) $\sqrt{x^3y^9}$

11) $\sqrt{a^{13}b^3}$

12) $\sqrt{32y^5}$

13) $\sqrt{27x^5}$

14) $\sqrt{80y^5}$

15) $\sqrt{48x^9y^5}$

16) $\sqrt{96x^2y^4}$

17) $\sqrt{36a^2b^6}$

18) $\sqrt{100x^4y^{16}}$

19) $\sqrt{24x^3y^5}$

20) $\sqrt{90x^9y^5}$

21) $4\sqrt{16a^6b^3}$

22) $3\sqrt{81a^2b^5}$

23) $x\sqrt{x^2y^4}$

24) $y\sqrt{x^5y^8}$

25) $3\sqrt{9a^3b^2}$

26) $4\sqrt{36a^6b^9}$

27) $4y^2\sqrt{32x^2y^3}$

28) $\sqrt{100(a+5)^4}$

29) $\sqrt{64(x+y)^4}$

30) $\sqrt{49(x-3)^2}$

31) $\sqrt{4(x-1)^2}$

32) $\sqrt{x^2+12x+36}$

33) $\sqrt{b^2-10b+25}$

Addition and Subtraction of Radicals

Simplify.

1) $\sqrt{3}+2\sqrt{3}$

2) $4\sqrt{2}+7\sqrt{2}$

3) $-2\sqrt{6}+3\sqrt{6}$

4) $3\sqrt{7}-9\sqrt{7}$

5) $-2\sqrt{2}-7\sqrt{2}$

6) $-4\sqrt{5}-6\sqrt{5}$

7) $3\sqrt{y}+7\sqrt{y}$

8) $5\sqrt{x}-9\sqrt{x}$

9) $6\sqrt{x}-11\sqrt{x}$

10) $5\sqrt{3a}-6\sqrt{3a}$

11) $-4\sqrt{7b}-8\sqrt{7b}$

12) $-6\sqrt{2a}-4\sqrt{2a}$

13) $-4\sqrt{3a}+3\sqrt{3a}$

14) $3y\sqrt{2}-8y\sqrt{2}$

15) $3a\sqrt{5a}-6a\sqrt{5a}$

16) $-4x\sqrt{2y}-3x\sqrt{2y}$

17) $2\sqrt{xy}-10\sqrt{xy}$

18) $-5\sqrt{xy}+7\sqrt{xy}$

19) $3\sqrt{192}-2\sqrt{48}$

20) $4\sqrt{72}-4\sqrt{50}$

21) $6\sqrt{20}-3\sqrt{45}$

22) $4\sqrt{16x}-3\sqrt{25x}$

23) $-2\sqrt{36y}+6\sqrt{64y}$

24) $2\sqrt{2x^2}-3\sqrt{18x^2}$

25) $3\sqrt{16x}-2\sqrt{25x}$

26) $-4\sqrt{36y}+6\sqrt{64y}$

27) $2\sqrt{2y^2}-3\sqrt{32y^2}$

28) $3\sqrt{2}-6\sqrt{2}+7\sqrt{2}$

29) $2\sqrt{5}-7\sqrt{5}-10\sqrt{5}$

30) $2\sqrt{20x}-3\sqrt{45x}+\sqrt{80x}$

31) $3\sqrt{12}-5\sqrt{3}+6\sqrt{27}$

32) $4\sqrt{50}-2\sqrt{32}-\sqrt{18}$

33) $4\sqrt{a^3b}+2a\sqrt{9ab}-\sqrt{16a^3b}$

Multiplication of Radicals

Simplify.

1) $\sqrt{3}\sqrt{27}$

2) $\sqrt{5}\sqrt{20}$

3) $\sqrt{z}\sqrt{z}$

4) $\sqrt{xy^5}\sqrt{x^3y}$

5) $\sqrt{a^2b}\sqrt{a^4b^3}$

6) $\sqrt{8a^3b}\sqrt{2ab^5}$

7) $\sqrt{2xy^3}\sqrt{18x^5y}$

8) $\sqrt{5a^6b^3}\sqrt{20a^2b}$

9) $\sqrt{5ab^3}\sqrt{15a^9b^5}$

10) $\sqrt{6}(\sqrt{6}-\sqrt{3})$

11) $4(\sqrt{8}-\sqrt{2})$

12) $\sqrt{2}(\sqrt{2}-\sqrt{8})$

13) $\sqrt{x}(\sqrt{x}+\sqrt{y})$

14) $\sqrt{3}(\sqrt{12}-\sqrt{x})$

15) $\sqrt{8}(\sqrt{y}-\sqrt{2})$

16) $\sqrt{6}(\sqrt{2}-\sqrt{6})$

17) $\sqrt{5}(\sqrt{20}-\sqrt{a})$

18) $(\sqrt{x}-4)^2$

19) $(3\sqrt{x}-y)^2$

20) $\sqrt{2a}(\sqrt{2a}-\sqrt{80})$

21) $\sqrt{3x}(\sqrt{6x}-\sqrt{3x})$

22) $(2\sqrt{x}-3y)(3\sqrt{x}-5y)$

23) $(4\sqrt{x}+\sqrt{y})(5\sqrt{x}-\sqrt{y})$

24) $(2\sqrt{x}-\sqrt{y})(2\sqrt{x}+\sqrt{y})$

25) $(\sqrt{5x}+y)(\sqrt{5x}-y)$

26) $(3\sqrt{x}+\sqrt{y})(5\sqrt{x}-3\sqrt{y})$

27) $(4\sqrt{x}-3\sqrt{y})(2\sqrt{x}-5\sqrt{y})$

Division of Radicals

Simplify.

1) $\frac{\sqrt{50}}{\sqrt{2}}$

2) $\frac{\sqrt{54}}{\sqrt{6}}$

3) $\frac{\sqrt{80}}{\sqrt{5}}$

4) $\frac{\sqrt{162}}{\sqrt{2}}$

5) $\frac{\sqrt{108x}}{\sqrt{3x}}$

6) $\frac{\sqrt{75x^3}}{\sqrt{3x}}$

7) $\frac{\sqrt{24xy^3}}{\sqrt{6xy}}$

8) $\frac{1}{\sqrt{5}}$

9) $\frac{1}{\sqrt{2}}$

10) $\frac{5}{\sqrt{x}}$

11) $\frac{6}{\sqrt{3x}}$

12) $\frac{\sqrt{48x^4y}}{\sqrt{3x^2y^3}}$

13) $\frac{\sqrt{3xy^5}}{\sqrt{27x^3}}$

14) $\frac{\sqrt{32xy^5}}{\sqrt{2x^3y}}$

15) $\frac{\sqrt{30xy}}{\sqrt{10x^3y^3}}$

16) $\frac{1}{\sqrt{3}-1}$

17) $\frac{3}{\sqrt{5}-2}$

18) $\frac{9}{3+\sqrt{6}}$

19) $\frac{6}{\sqrt{2}-5}$

20) $\frac{2}{\sqrt{3}-3}$

21) $\frac{\sqrt{x}}{\sqrt{x}+\sqrt{y}}$

22) $\frac{3\sqrt{3}-6\sqrt{3}}{\sqrt{3}}$

23) $\frac{9\sqrt{2}-3\sqrt{2}}{3\sqrt{2}}$

24) $\frac{3\sqrt{27}+\sqrt{3}}{\sqrt{12}}$

25) $\frac{8x\sqrt{2}-3\sqrt{128}}{\sqrt{32}}$

26) $\frac{2y\sqrt{y^3}+7\sqrt{y^5}}{\sqrt{y}}$

27) $\frac{9a^2\sqrt{a}-3a\sqrt{a^3}}{\sqrt{4a}}$

Imaginary Numbers

$$\sqrt{-1} = i$$

Examples:

$\sqrt{-9} = \sqrt{-1} \cdot \sqrt{9}$ $\downarrow \quad \downarrow$ $i \cdot 3 = 3i$	$\sqrt{-36} = \sqrt{-1} \cdot \sqrt{36}$ $\downarrow \quad \downarrow$ $i \cdot 6 = 6i$
$\sqrt{-27} = \sqrt{-1} \cdot \sqrt{9} \cdot \sqrt{3}$ $\downarrow \quad \downarrow \quad \downarrow$ $i \cdot 3 \cdot \sqrt{3} = 3i\sqrt{3}$	$\sqrt{-50} = \sqrt{-1} \cdot \sqrt{25} \cdot \sqrt{2}$ $\downarrow \quad \downarrow \quad \downarrow$ $i \cdot 5 \cdot \sqrt{2} = 5i\sqrt{2}$

Now Try These:

1) $\sqrt{-81}$ _____

2) $\sqrt{-49}$ _____

3) $\sqrt{-20}$ _____

4) $\sqrt{-28}$ _____

5) $\sqrt{-75}$ _____

6) $\sqrt{-1}$ _____

7) $5\sqrt{-12}$ _____

8) $-4\sqrt{-24}$ _____

9) $3\sqrt{-45}$ _____

10) $-7\sqrt{-60}$ _____

Equations with Radicals

Solve and check.

1) $\sqrt{x} = 6$

2) $\sqrt{y} = 8$

3) $\sqrt{a} = 16$

4) $\sqrt{a} = 10$

5) $\sqrt{3x} = 3$

6) $\sqrt{2x} = 4$

7) $\sqrt{3x} = 6$

8) $5 - \sqrt{5x} = 0$

9) $\sqrt{6x} - 12 = 0$

10) $3 - \sqrt{2x} = 0$

11) $\sqrt{3x} + 1 = 4$

12) $\sqrt{2x - 3} = 5$

13) $\sqrt{4x + 5} = 1$

14) $\sqrt{2x + 7} = 3$

15) $\sqrt{2x - 1} = 3$

16) $\sqrt{2x - 3} = 1$

17) $\sqrt{6x + 1} = 5$

18) $\sqrt{5x - 4} = 6$

19) $0 = 3 - \sqrt{2 + x}$

20) $0 = 5 - \sqrt{5 + x}$

21) $\sqrt{2x} - 1 = 3$

22) $5 = 7 - \sqrt{2c}$

23) $10 = 3 + \sqrt{2x + 1}$

24) $\sqrt{2x + 5} = 0$

25) $\sqrt{3x - 2} = 0$

26) $0 = \sqrt{3x - 6} - 9$

27) $\sqrt{2x} - 4 = -6$

28) $\sqrt{2x + 5} - 3 = 0$

29) $\sqrt{3x - 6} = \sqrt{9x - 12}$

Applications with Radicals

Solve.

1) Six added to the square root of the product of two and a number is equal to ten. Find the number.

2) The product of a number and the square root of three is equal to the square root of twelve. Find the number.

3) How far would a submarine periscope have to be above the water to locate a ship 4 miles away? The equation for the distance in miles that the lookout can see is $d = 1.4\sqrt{h}$, where h is the height in feet above the surface of the water. Round to the nearest hundredth.

4) How far would a submarine periscope have to be above the water to locate a ship 4.2 miles away? The equation for the distance in miles that the lookout can see is $d = 1.4\sqrt{h}$, where h is the height in feet above the surface of the water. Round to the nearest hundredth.

5) An object is dropped from a helicopter. Find the distance the object has fallen when the speed reaches 96 ft/s. The equation for the distance is $v = \sqrt{64\,d}$, where v is the speed of the object and d is the distance.

6) An object is dropped from a television transmitter tower. Find the distance the object has fallen when the speed reaches 112 ft/s. The equation for the distance is $v = \sqrt{64\,d}$, where v is the speed of the object and d is the distance.

7) A stone is dropped from a bridge and hits the water 2 seconds later. How high is the bridge? The equation for the distance an object falls in T seconds is given by $T = \sqrt{\frac{d}{16}}$, where d is the distance in feet.

8) A stone is dropped into a mine shaft and hits the bottom 4 seconds later. How deep is the mine shaft? The equation for the distance an object falls in T seconds is given by $T = \sqrt{\frac{d}{16}}$, where d is the distance in feet.

9) Find the length of a pendulum that makes one swing in 4 seconds. The equation for the time of one swing of a pendulum is given by $T = 2\pi\sqrt{\frac{L}{32}}$, where T is the time in seconds and L is the length in feet. Use 3.14 for π. Round to the nearest hundredth.

10) Find the length of a pendulum that makes one swing in 2.4 seconds. The equation for the time of one swing of a pendulum is given by $T = 2\pi\sqrt{\frac{L}{32}}$, where T is the time in seconds and L is the length in feet. Use 3.14 for π. Round to the nearest hundredth.

Solving by factoring

Solve by factoring.

1) $x^2 - 2x - 24 = 0$

2) $t^2 + 3t - 18 = 0$

3) $z^2 - 6z + 5 = 0$

4) $v^2 - 7v + 6 = 0$

5) $X^2 + 9x + 8 = 0$

6) $v^2 - 10v + 9 = 0$

7) $x^2 - 10x + 25 = 0$

8) $y^2 - 12y + 36 = 0$

9) $10y^2 + 5y = 0$

10) $8x^2 - 12x = 0$

11) $x^2 - 21 = 4x$

12) $t^2 - 20 = t$

13) $2x^2 + x = 10$

14) $4x^2 + 12x = -9$

15) $5r^2 = 12 + 11r$

16) $3x^2 = x + 2$

17) $5y^2 = 11y - 2$

18) $16v^2 - 8v + 1 = 0$

19) $25y^2 - 10y + 1 = 0$

20) $x^2 - 1 = 0$

21) $t^2 - 25 = 0$

22) $16y^2 - 1 = 0$

23) $4z^2 - 25 = 0$

24) $x + 28 = x(x - 2)$

25) $y + 12 = y(y-3)$

26) $x^2 - 4x - 2 = 3x^2 - 5x - 3$

27) $x^2 + 2x + 3 = (2x-1)(x+5)$

28) $x^2 + x - 1 = (2x+1)(x+2)$

Solving by Taking Square Roots

Solve by taking square roots.

1) $x^2 = 4$

2) $y^2 = 16$

3) $v^2 - 81 = 0$

4) $Z^2 - 144 = 0$

5) $9x^2 - 25 = 0$

6) $16w^2 - 81 = 0$

7) $16w^2 = 25$

8) $4x^2 = 81$

9) $25v^2 - 16 = 0$

10) $36x^2 - 49 = 0$

11) $4x^2 - 9 = 0$

12) $9x^2 - 100 = 0$

13) $x^2 + 9 = 0$

14) $y^2 + 100 = 0$

15) $w^2 - 8 = 0$

16) $v^2 - 18 = 0$

17) $x^2 - 50 = 0$

18) $(x + 1)^2 = 9$

19) $(y - 2)^2 = 36$

20) $3(x + 5)^2 = 27$

21) $5(z - 2)^2 = 80$

22) $4(x - 1)^2 - 25 = 0$

23) $9(y + 2)^2 - 100 = 0$

24) $16(w + 3)^2 - 49 = 0$

25) $25(y - 1)^2 - 36 = 0$

26) $(x - 3)^2 - 32 = 0$

27) $(y + 4)^2 - 75 = 0$

28) $(x - 1)^2 - 50 = 0$

29) $(x + 1)^2 - 80 = 0$

Solving by Completing the Square

Solve by completing the square.

1) $x^2 + 6x - 7 = 0$

2) $x^2 + 4x - 21 = 0$

3) $x^2 - 6x - 27 = 0$

4) $w^2 + 2w - 35 = 0$

5) $x^2 = 6x - 9$

6) $z^2 = 6z + 16$

7) $v^2 + 2v - 3 = 0$

8) $y^2 - 4y - 7 = 0$

9) $x^2 + 8x = 7$

10) $w^2 - 6w = 1$

11) $z^2 = 4z + 3$

12) $y^2 = 8y - 16$

13) $p^2 + 2p = 3$

14) $r^2 + 3r = 3$

15) $t^2 - 4t = -3$

16) $2y^2 - 5y - 12 = 0$

17) $2x^2 + 3x - 5 = 0$

18) $2x^2 - 5x = -3$

19) $2r^2 + 9r = 5$

20) $3x^2 = 7x - 2$

21) $3y^2 - y = 2$

22) $4x^2 + 7x - 2 = 0$

23) $3y^2 - 4y = 4$

24) $3t^2 - 2t = 5$

25) $4v^2 - 3v = 1$

26) $3z^2 - 4z = -1$

27) $3t^2 = 2t + 1$

28) $6t^2 - 5t + 1 = 0$

29) $4z^2 - 8z - 5 = 0$

30) $6z^2 = z + 2$

Solving with the Quadratic Formula

Solve by using the quadratic formula.

1) $x^2 - 3x - 4 = 0$

2) $y^2 + 6y + 5 = 0$

3) $z^2 - 3z - 10 = 0$

4) $v^2 - 6v + 5 = 0$

5) $z^2 + 5z - 6 = 0$

6) $x^2 + 3x - 18 = 0$

7) $t^2 - t - 20 = 0$

8) $x^2 - x - 6 = 0$

9) $y^2 = 8y + 9$

10) $w^2 = 6w + 16$

11) $r^2 = 9 - 8r$

12) $z^2 = 10 - 9z$

13) $3y^2 - 2y - 1 = 0$

14) $2t^2 - t - 10 = 0$

15) $x^2 - 6x + 6 = 0$

16) $x^2 - 6x + 2 = 0$

17) $p^2 - 4p = 0$

18) $3v^2 - v = 0$

19) $9t^2 - 25 = 0$

20) $16s^2 - 9 = 0$

21) $3y^2 + y = 14$

22) $2x^2 - 7x = 4$

23) $3x^2 = -2x + 8$

24) $2x^2 + x = 10$

25) $5x^2 - 13x = 6$

26) $3v^2 = -v + 2$

27) $8x^2 + 2x - 3 = 0$

28) $t^2 - t - 3 = 0$

29) $x^2 + 6x = 4$

30) $t^2 + 5t - 1 = 0$

Basic Algebra II Worksheets - Answer Key

Worksheet 1

1. x^6
2. b^{11}
3. y^{10}
4. $40x^6$
5. $25y^{11}$
6. $-14a^7$
7. $-35y^{17}$
8. $-24x^{15}$
9. $-6b^6c^9$
10. $-18m^2$
11. $32x^{11}$
12. $-28x^3y^6$
13. $48x^{10}$
14. x^{11}
15. $14y^5z^5$
16. $-15x^8y^4$
17. $-64x^{17}$
18. $44x^{13}$
19. $60c^9$
20. $4b^{12}$
21. $a^{11}b^{15}$
22. x^7y^7
23. $-7a^{11}$
24. $14x^{13}$
25. $4a^7b^4c^6$
26. $56m^3n^3$
27. $-60a^7b^3$
28. $-48x^6y^2$
29. $6x^7y^5$
30. $-3y^{10}$
31. $90x^{12}\ y^5z^5$
32. $100z^{24}$
33. $-6x^{15}yz^5$

Worksheet 2

1. $9r^4s^2$
2. $125x^9y^9$
3. $-64x^9y^6$
4. $64x^6y^8$
5. $36a^6b^{10}$
6. $16x^{16}y^{16}$
7. $1000x^3$
8. $81a^8$
9. $8y^{15}z^3$
10. $-64a^3b^6$
11. a^9b^9
12. $-125x^3y^6$
13. $x^{30}y^{12}$
14. $4a^2b^{16}$
15. $9a^6$
16. x^{25}
17. z^{21}
18. a^6
19. $m^{15}n^{20}$
20. x^2y^4
21. $-p^{21}q^{21}$
22. $-a^{25}b^{30}$
23. v^{36}
24. b^9c^9
25. $a^5b^5c^{15}$
26. $x^{40}y^{16}$
27. $-a^{20}b^{20}$
28. $-a^{28}c^{28}$
29. $x^{12}y^{12}z^8$
30. d^{30}
31. $256x^{12}$
32. f^{42}
33. $343x^6y^9$
34. $x^{10}y^{10}$
35. $-125a^{15}b^{12}$
36. $-m^{35}n^{14}$

Worksheet 3

1. $64x^9y^3$
2. $216x^{14}y^9$
3. $4a^7$
4. $-3x^5y^{10}$
5. $-1152x^{12}$
6. $-32x^5$
7. $64a^4b^{10}$
8. $32x^{13}y^2$
9. $-512x^{11}y^6$
10. $4m^4n^8$
11. $25b^{21}$
12. x^8y^{12}
13. $x^{12}y^{12}$
14. $-3x^4y^5$
15. $-64a^3b^{11}$
16. $50x^7y^{15}$
17. $64x^7y^5$
18. $256a^8b^{10}$
19. $-27x^7y^{31}$
20. $16x^{22}$
21. $-2m^{17}n$
22. $128p^7q^7$
23. $-27x^{35}y^{54}$
24. $r^{30}s^{24}$

Worksheet 4

1. x
2. a
3. $-c^3$
4. $-\frac{1}{r}$
5. $\frac{1}{z^3}$
6. $\frac{1}{b^2}$
7. $\frac{b^2}{a^2}$
8. $\frac{r^3}{s^3}$
9. $-\frac{b^2}{a^3}$
10. $\frac{x}{y^6}$
11. $\frac{b^3}{3}$
12. $\frac{1}{3y^3}$
13. $\frac{x^2}{2y^2}$
14. $\frac{2a^4}{7c^4}$
15. $\frac{x^2}{3}$
16. $-\frac{a}{5b^2}$
17. $-\frac{z^4}{3a^2}$
18. $\frac{a^2}{2b}$
19. $\frac{1}{x^4}$
20. $\frac{a^2}{5b^2}$
21. $\frac{x^{12}}{3}$
22. $-\frac{3y^2}{2z^2}$
23. $-a^2b^5$
24. $-c^{12}$

Worksheet 5

1. $\frac{1}{16}$
2. $\frac{1}{49}$
3. $\frac{1}{25}$
4. $\frac{1}{a^3}$
5. $5y^3$
6. $8x^4$
7. z^3
8. x^7
9. m^6
10. $4x^4$
11. $12z^6$
12. a^7b^7
13. $a^{10}b^8$
14. x^3y^{10}
15. r^9s^3
16. a^9
17. $\frac{1}{x^6y^6}$
18. m^9n^3
19. x^9y^7
20. $\frac{q^7}{p^3}$
21. $a^{14}b^2$
22. $13a^3b^5$
23. m^2n^2
24. $\frac{1}{x^3y^3}$
25. a^9b^5
26. $\frac{1}{Y^3z^2}$
27. x^3y^7
28. $8ab^4$

Worksheet 6

1. $\frac{1}{9}$
2. $\frac{1}{8}$
3. $\frac{1}{4}$
4. $\frac{1}{10}$
5. $\frac{1}{36}$
6. $\frac{1}{8}$
7. $\frac{1}{64}$
8. $\frac{1}{243}$
9. $\frac{1}{64}$
10. $\frac{1}{81}$
11. $\frac{1}{16}$
12. $\frac{1}{125}$
13. $\frac{1}{x}$
14. $\frac{1}{y^9}$
15. $\frac{1}{a^5}$
16. $\frac{1}{b^3}$
17. $\frac{b}{a}$
18. $\frac{x}{y^2}$
19. 1
20. $\frac{b^2}{a^3}$
21. $\frac{1}{x^2y}$
22. $\frac{1}{x^4}$
23. $\frac{1}{x^3}$
24. $\frac{1}{a^6}$
25. $\frac{1}{a^{14}}$
26. $\frac{y^2}{x^2}$
27. $\frac{x^2}{y^2}$
28. $\frac{1}{x^8}$
29. y^6
30. a^8
31. 1
32. 1

Worksheet 7

1. $\frac{1}{y^2}$
2. $\frac{1}{a^{15}}$
3. $\frac{1}{x^4}$
4. $\frac{w^{12}}{v^3}$
5. $\frac{z^{20}}{16}$
6. $x^{25}y^{25}$
7. y^{16}
8. $\frac{m^{10}}{n^{14}}$
9. $\frac{1}{c^{50}s^{15}}$
10. m^2n^2

Worksheet 8

1. 1
2. 1
3. 1
4. 1
5. 1
6. 1
7. b
8. c
9. x
10. a^2
11. $\frac{1}{c}$
12. $\frac{b}{5}$
13. a^2x^2
14. 3x
15. -x
16. 1
17. 3
18. -3

Worksheet 9

1. $-x^2 + 2x$
2. $y^2 + y - 5$
3. $2x^2 + 3x + 3$
4. $x^3 + 3x^2 - 6x$
5. $y^3 - 3$
6. $x^3 - x^2 + 4x$
7. $-9x - 5$
8. $3x^3 + 2x^2 + 3x + 1$
9. $4y^3 - y^2 + 4y - 7$
10. $x^3 - 4x - 1$
11. $2y^3 - 12xy - 5$
12. $3x^2 - 5xy$
13. $x^2 - 5x + 2$
14. $4x^2 + 7x - 8$
15. $3x^3 - 2x^2 - 8x - 5$

Worksheet 10

1. $x^2 + x$
2. $2y - y^2$
3. $-x^2 - 2x$
4. $-8y + y^2$
5. $2a^2 - 2a$
6. $3b^2 + 15b$
7. $-2x^3 + 2X^2$
8. $-4y^3 - 24y^2$
9. $-6y^4 + 6y^3$
10. $-2x^4 + 3x^2$
11. $10x^3 - 4x^2$
12. $6y^2 - 3y^3$
13. $8x^2 - 12x$
14. $2y^2 - y$
15. $2x^2 - 3x$
16. $6x^2 - 3x$
17. $-x^3y + x^2y^3$
18. $-2x^2y^2 + xy^3$
19. $x^3 - 2x^2 + x$
20. $x^3 - 3x^2 - 2x$
21. $-y^3 + 4y^2 - 3y$
22. $-y^3 + 5y^2 + 6y$
23. $-a^3 + 6a^2 + a$
24. $-2b^3 - 3b^2 + 6b$
25. $2x^4 - 3x^3 - 2x^2$
26. $-3y^4 - 5y^3 - 3y^2$
27. $-x^5 - 5x^4 - 6x^3$
28. $-2y^5 - 3y^4 - 4y^3$
29. $-4y^4 - 10y^3 + 16y^2$
30. $12x^4 - 6x^3 + 21x^2$
31. $20x^4 - 4x^3 - 36x^2$
32. $-5y^4 + 15y^3 - 30y^2$
33. $x^3y - x^2y^2 + xy^3$
34. $a^3b - 3a^2b^2 - 4ab^3$
35. $x^3y - 2x^2y^2 + 2xy^3$
36. $a^3b + 5a^2b^2 - 7ab^3$

Worksheet 11

1. $x^2 + 5x + 4$
2. $y^2 + 5y + 6$
3. $a^2 + 3a - 10$
4. $b^2 - b - 20$
5. $y^2 - 5y - 14$
6. $x^2 + 5x - 36$
7. $y^2 - 8y + 12$
8. $a^2 - 15a + 56$
9. $a^2 - 10a + 16$
10. $x^2 + 8x - 33$
11. $2x^2 + 13x + 6$
12. $3y^2 + 5y + 2$
13. $2x^2 + 3x - 9$
14. $5x^2 + 13x - 6$
15. $3x^2 - 17x + 10$
16. $6x^2 - 13x + 5$
17. $2y^2 - 7y - 9$
18. $4y^2 + y - 14$
19. $9x^2 + 33x + 28$
20. $30a^2 + 17a + 2$
21. $12a^2 - 56a + 65$
22. $10a^2 - 53a + 63$
23. $15b^2 + 43b - 44$
24. $12a^2 + 31a - 30$
25. $x^2 + 3xy + 2y^2$
26. $2a^2 + 5ab + 2b^2$
27. $2x^2 - 5xy + 3y^2$
28. $2a^2 - 3ab - 9b^2$
29. $8a^2 + 18ab - 5b^2$
30. $2x^2 + xy - y^2$
31. $9x^2 - 9xy - 10y^2$
32. $30x^2 + 17xy + 2y^2$

Worksheet 12

1. $x^3 - 4x^2 + 5x + 2$
2. $x^3 - 4x^2 + 8x - 5$
3. $a^3 - 3a^2 + 4a - 4$
4. $x^3 - 7x^2 + 7x + 15$
5. $a^3 - 6a^2 + 8a - 3$
6. $3x^3 - 16x^2 - 7x + 4$
7. $-2b^3 + 7b^2 + 6b - 8$
8. $-2a^3 + 5a^2 + 6a - 9$
9. $-2x^3 + 15x^2 - 24x + 9$
10. $-3a^3 - 4a^2 + 5a + 2$
11. $x^3 + x^2 - 4x - 4$
12. $y^3 - 9y$
13. $x^3 - 5x^2 + 3x - 15$
14. $2y^3 + 2y^2 - 4y$
15. $x^4 - 3x^3 - 2x^2 + 7x - 3$
16. $2y^4 + y^3 - 6y^2 - 10y + 15$
17. $8y^3 - 14y^2 - 51y + 7$
18. $9y^3 - 3y^2 - 11y + 6$
19. $3a^4 - 6a^3 - 2a^2 + 7a - 6$
20. $4y^3 + 6y^2 - 8y + 2$
21. $4a^4 - 12a^3 - 4a^2 + 13a - 3$
22. $6b^4 - 11b^3 + 3b^2 + 18b - 6$
23. $y^4 + 2y^3 - y^2 + 2$
24. $3a^4 - 8a^3 + 13a^2 - 9a + 2$

Worksheet 13

1. $x^2 - 1$
2. $x^2 - 9$
3. $x^2 - 25$
4. $x^2 - 49$
5. $4x^2 - 1$
6. $9x^2 - 1$
7. $16x^2 - 9$
8. $x^2 + 10x + 25$
9. $y^2 - 8y + 16$
10. $9y^2 - 6y + 1$
11. $x^2 - 2x + 1$
12. $x^2 - 6x + 9$
13. $x^2 + 14x + 49$
14. $x^2 + 18x + 81$
15. $x^2 - 2xy + y^2$
16. $4a^2 - 20a + 25$
17. $25x^2 - 40x + 16$
18. $9x^2 - 42x + 49$
19. $9a^2 - 25$
20. $36x^2 - 25$
21. $4x^2 + 20x + 25$
22. $81x^2 - 36x + 4$
23. $a^2 - 4ab + 4b^2$
24. $x^2 + 4xy + 4y^2$
25. $25x^2 - 36$
26. $b^2 - 36a^2$
27. $x^2 + 10xy + 25y^2$
28. $4 - 28y + 49y^2$
29. $9 - 30y + 25y^2$
30. $9 - 25y^2$
31. $16x^2 - 1$
32. $4a^2 + 12ab + 9b^2$
33. $x^2 + 12xy + 36y^2$

Worksheet 14

1. $9x^2 - 3x$
2. $3x^2 - 4x - 4$
3. $6x^2 - x - 1$
4. $4x^2 + 31x + 21$
5. $x^2 + 6x + 9$
6. $x^2 - 16x + 64$
7. $4x^2 + 4x + 1$
8. $9x^2 - 24x + 16$
9. $\pi(x^2 + 8x + 16)$
10. $\pi(x^2 - 6x + 9)$
11. $\pi(x^2 + 12x + 36)$
12. $\pi(4x^2 + 4x + 1)$

Worksheet 15

1. $x + 1$
2. $y + 1$
3. $2a - 3$
4. $3a - 7$
5. $2a - 3$
6. $2b - 5$
7. $2a + 3$
8. $5y + 3$
9. $3b^2 - 2$
10. $2y - 1$
11. $x^2 + 2x - 4$
12. $a^2 - 4a + 5$
13. $x^2 - 2x - 1$
14. $a^3 - 6a^2 - 4a$
15. $xy - 3$
16. $xy + 5$
17. $-2y^2 + 5$
18. $-4x^2 + 3$
19. $-y + 4$
20. $-2 + 3x^2$
21. $-7x - 4 - \frac{6}{x}$
22. $4x + 1 - \frac{5}{x}$
23. $6y + 4 - \frac{5}{y}$
24. $-2x + 5 - \frac{4}{x}$
25. $4a - 5 + 6b$
26. $2xy - 1 - 3y$
27. $2a - 3 + 5b$
28. $2a + 1 - 4b$
29. $5x + 6 - 2y$
30. $b - 4 + 3ab$

Worksheet 16

1. $x + 3$
2. $x + 4$
3. $x - 6$
4. $a - 7$
5. $x - 4$
6. $y + 5$
7. $x + 3$
8. $y - 2$
9. $2x + 3$
10. $2y - 7$
11. $2y - 5$
12. $y + 7$
13. $x + 3$
14. $2y - 5$
15. $x - 1$
16. $x - 3 + \frac{18}{x+3}$
17. $2x - 2 - \frac{6}{2x-3}$
18. $2y + 2 + \frac{2}{2x-1}$
19. $a - 2 + \frac{20}{a+5}$
20. $b - 3 + \frac{-20}{b-4}$
21. $5y + 3$
22. $6a - 12 + \frac{30}{2a-3}$
23. $2a + 5$
24. $3x + 4$

Worksheet 17

1. $4(a + 1)$
2. $6(c - 1)$
3. $4(2 - a^2)$
4. $3(3 + 7x^2)$
5. $3(x + 3)$
6. $5(2a^2 + 3)$
7. $8(3a - 1)$
8. $12(2x + 1)$
9. $3(3x - 2)$
10. $8a(2a - 1)$
11. $4y(3x - 4)$
12. $b^2 (6 - 5b)$
13. $4x^2 (5x - 6)$
14. $12a^2 (a^3 - 3)$
15. $6a^2b^2(4ab^2 - 3)$
16. $2ab(2a^4 + 3b^3)$
17. $a^3b^2 (1 + ab)$
18. $5x^2y(5y - 3x)$
19. $xy(3x - 5y)$
20. $4a^2b(2a - 3b^2)$
21. $x^2y^2 (x - y^2)$
22. $x(x^2 - 5x + 7)$
23. $y(y^2 - 6y - 8)$
24. $4(x^2 - 4x + 5)$
25. $3(2y^2 - 3y + 4)$
26. $3(x^2 - 3x + 6)$
27. $b^2(b^2 - 3b + 7)$
28. $4x^2(1 - 2x + 3x^3)$
29. $4(3y^2 - 4y + 12)$
30. $5y(y^3 + 2y^2 - 7)$
31. $4x^2(x^2 + 3x - 7)$
32. $15ab(3a^3b - 5a^2 + 2b^3)$
33. $16x^2y^2(2x^2 - 6y^2 - 3x^4)$

Worksheet 18

1. $(a + b)(x + 3)$
2. $(x - y)(a + 5)$
3. $(b - 1)(x - y)$
4. $(c - d)(a + b)$
5. $(a - 1)(y - 1)$
6. $(y + 3)(a - 1)$
7. $(y - 2)(x - 1)$
8. $(Y - 7)(3x - 1)$
9. $(x - 5)(2x - 1)$
10. $(x - 3)(5y + 1)$
11. $(a - 2b)(x + y)$
12. $(a + 1)(4a - 1)$
13. $(x - 9)(a - 1)$
14. $(a - 5)(b - 2)$
15. $(x - 3y)(c + d)$
16. $(a + 1)(3x + 4)$
17. $(x - 1)(a - 3)$
18. $(a - 4)(x - y)$
19. $(a - 3b)(x - y)$
20. $(a - 9)(x + 1)$
21. $(x - 7)(a - 3)$
22. $(x - 6)(2x - 1)$
23. $(e - 5)(d - 1)$
24. $(a - 4)(2x + y)$
25. $(a - 9)(m + n)$
26. $(n - 2)(3m + 1)$
27. $(b - 1)(2a - c)$
28. $(a - 2)(x - 3y)$
29. $(y - 5)(x - 1)$
30. $(a - b)(2 - c)$
31. $(n - 1)(m - 2)$
32. $(c - 7)(x + y)$
33. $(b - c)(a + 3)$
34. $(x - y)(5 + z)$

Worksheet 19

1. $(y - 3)(2x + 3)$
2. $(x + y)(x + 2)$
3. $(a + 5)(x + 6)$
4. $(2x + 1)(x - y)$
5. $(2x+2y)(3x+1) = 2(x+ y)(3x+1)$
6. $(4a + 3)(a + b)$
7. $(a - 2)(x - 7)$
8. $(2x - 5)(x + y)$
9. $(a - 2)(x + l)$
10. $(y - 3)(3x + 1)$
11. $(a - 2b)(3a - 1)$
12. $(x - 3)(2a + 1)$
13. $(x - 2)(2a - 3)$
14. $(5a - 3x)(a - 3)$
15. $(x - y)(y - 3)$
16. $(7a - y)(a - b)$
17. $(2x - y)(3x - 2)$
18. $(4a - b)(a + 3)$
19. $(a - 4x)(2a + 3)$
20. $(x - 2)(3x - y)$
21. $(2a + y)(4x - 1)$

Worksheet 21

1. $(a - 7)(a + 5)$
2. $(a - 3)(a - 1)$
3. $(a + 5)(a - 2)$
4. $(a - 3)(a - 2)$
5. $(b - 5)(b - 2)$
6. $(b + 5)(b + 3)$
7. $(b+8)(b-2)$
8. $(y - 6)(y + 5)$
9. $(y + 8)(y - 5)$
10. $(y + 11)(y - 6)$
11. $(x - 10)(x + 6)$
12. $(y - 2)(y - 5)$
13. $(y - 6)(y - 3)$
14. $(x - 6)(x - 6)$
15. $(x - 12)(x + 8)$
16. $(a + 7)(a - 4)$
17. $(x + 8)(x + 2)$
18. $(b - 20)(b + 9)$
19. $(x + 5)(x + 5)$
20. $(x - 7)(x - 7)$
21. $(b + 3)(b + 4)$
22. $(b + 8)(b + 2)$
23. $(x - 12)(x + 3)$
24. $(x - 12)(x + 5)$
25. $(x + 14)(x - 4)$
26. $(x - 16)(x + 8)$
27. $(x - 11)(x + 7)$
28. $(b - 14)(b - 6)$
29. $(b - 9)(b - 12)$
30. $(b - 12)(b - 15)$
31. $(a + 7)(a + 9)$
32. $(x - 15)(x - 4)$
33. $(x - 21)(x - 4)$

Worksheet 21a

1. (x - 2)(2x - 1)
2. (3x + 1)(x - 1)
3. (2a + 1)(a + 3)
4. (3x - 2)(x + 1)
5. (2b - 1)(b - 6)
6. (3a - 1)(a - 2)
7. (3x - 1)(x - 4)
8. (2x + 3)(2x - 1)
9. (5a - 3)(a + 1)
10. (5a - 2)(a + 3)
11. (3y - 2)(2y + 3)
12. (6x - 5)(x + 1)
13. (5x + 2)(x - 1)
14. (7x - 1)(x - 2)
15. (7y + 1)(y + 1)
16. (7x - 1)(2x - 1)
17. (7y + 4)(y + 2)
18. (3a - 1)(3a + 2)
19. (4x + 1)(2x - 7)
20. (3a + 4)(a - 3)
21. (3x + 2)(x - 4)
22. (3x + 2)(2x - 3)
23. (4y - 5)(y + 5)
24. (7x - 1)(x + 3)
25. (5x + 7)(x - 1)
26. (5x + 2)(2x - 3)
27. (5x - 2)(3x + 4)
28. (4x - 3)(2x - 5)
29. (4x - 5)(3x + 2)
30. (3x - 5)(3x + 1)
31. (4x + 5)(2x - 3)
32. (5x + 2)(2x - 5)
33. (5x - 2)(3x - 4)
34. (6y + 1)(2y - 3)

Worksheet 22

1. 2(x + 3)(x + 1)
2. 3(x + 4)(x + 1)
3. 3(a + 4)(a - 3)
4. 4(x - 3)(x + 1)
5. a(b - 2)(b + 1)
6. a(b + 7)(b - 2)
7. x(y - 2)(y-5)
8. x(y+7)(y+4)
9. x(x - 6)(x - 2)
10. 2a(a - 4)(a - 1)
11. 3y(y - 5)(y - 1)
12. 5y(y + 4)(y - 2)
13. 4(x + 8)(x - 1)
14. 2(x - 3)(x + 2)
15. 3(z - 8)(z +1)
16. z(z + 9)(z + 5)
17. (a - 8b)(a + 6b)
18. c(c +15)(c + 2)
19. (x - 7y)(x - 2y)
20. 4(y - 8)(y + 4)
21. 3(x + 3)(x - 2)
22. 2(x - 10)(x + 3)
23. 4(a + 8)(2a - 1)
24. 3(y - 5)(y - 3)
25. 5(y - 1)(y - 9)
26. 2x(x + 3)(x - 2)
27. (y + 8x)(y - x)
28. (a - 5b)(a - 7b)
29. x(x + 8)(x - 5)
30. 10x(x+2)(x+4)
31. 2y(x -18)(x +1)
32. $(x^2 + 9)(x^2 - 7)$
33. 2(x + 15)(x - 8)
34. $a^2(a + 9)(a - 8)$

Worksheet 23

1. $3(x-1)^2$
2. 2(4x-1)(x+3)
3. 2(3y-7)(y-1)
4. x(3x-2)(x-2)
5. 5(3y+2)(y-1)
6. 2b(a+6)(a-2)
7. (14x - y)(x – 6y)
8. (3y +5z)(y + z)
9. (x - 6y)(x - 5y)
10. x(2x + 1)(x - 4)
11. (5 + x)(3 + x)
12. (7-x)(3+x)
13. (8 - x)(4 + x)
14. (2y+1)(2y-3)
15. 3(2y-1)(2y+3)
16. (4x - 1)(3x - 4)
17. (9 + x)(3 - x)
18. 5(x - 8)(x + 1)
19. 2(2y -3)(2y +1)
20. x(5x - 1)(x - 1)
21. (3x - 4)(x - 5)
22. 3(2y -3)(2y +1)
23. 2(2x -1)(2x +3)
24. 2(x + 5)(3x - 4)
25. (7a +b)(5a -3b)
26. 5a(2a -5)(a +1)
27. x(3x+1)(2x+5)
28. 2x(3x -1)(x -4)
29. 2(5b -3)(b+2)
30. z(11 + z)(2 - z)
31. (5a -3b)(2a-5b)
32. 2x(3x - 1)(x - 5)
33. (19 - x)(1 + x)
34. (13x - 3)(x + 4)

Worksheet 24

1. (x + 4)(x - 4)
2. (x + 5)(x - 5)
3. (x + 8)(x - 8)
4. (3x + 1)(3x - 1)
5. (4x + 5)(4x - 5)
6. (3x + 7)(3x - 7)
7. $(x^2 + 2)(x^2 - 2)$
8. $(x^4+10)(x^4-10)$
9. (6x + 1)(6x - 1)
10. (9x + 1)(9x -1)
11. (1+10x)(1-10x)
12. (1 +9x)(1 -9x)
13. $(y^2+11)(y^2-11)$
14. (1+12x)(1-12x)
15. Irreducible over the integers
16. Irreducible over the integers
17. $(x + y^3)(x - y^3)$
18. $(x^2+ y^4)$ $(x- y^2)(x- y^2)$
19. (1 + 5x)(1 - 5x)
20. (1 +6x)(1 -6x)
21. (2 + 3x)(2 - 3x)
22. (4 + 7x)(4 - 7x)
23. (b+12c)(b-12c)
24. (a +7b)(a -7b)
25. (xy+10)(xy -10)
26. $(x^3+ 9)(x^3 - 9)$
27. (3x+4y)(3x-4y)
28. (5x+12)(5x-12)
29. (xy + 1)(xy - 1)
30. (x + 20)(x - 20)
31. (6a + 1)(6a - 1)
32. (7x + 2)(7x - 2)
33. $(x^2 + 2)(x^2 - 2)$

Worksheet 25

1. $(x+y)(x^2-xy+x^2)$
2. $(x+2)(x^2-2x+4)$
3. $(x-y)$ (x^2+xy+y^2)
4. $(x-2)(x^2+2x+4)$
5. $(x-1)(x^2+x+1)$
6. $(a-1)(a^2+a+1)$
7. $(3a+1)$ $(9a^2-3a+1)$
8. $(2x+1)$ $(4X^2-2x+1)$
9. $(2x-y)$ $(4x^2+2xy+y^2)$
10. $(3a-b)$ $(9a^2+3ab+b^2)$
11. $(3a+2b)$ $(9a^2-6ab+4b^2)$
12. $(2x + 5y)$ $(4x^2-10xy+25y^2)$
13. $(5a-3b)$ $(25a^2+15ab+9b^2)$
14. $(4x-3y)$ $(16x^2+12xy+9y^2)$
15. $(x-y^2)$ $(x^2+xy^2+y^4)$
16. $(2a-5)$ $(4a^2 + 10a+25)$
17. $(2a^2 + b)$ $(4a^4-2a^2b+b^2)$
18. $(4x^2+1)$ $(16x^4-4x^2+1)$
19. $(a-1)$ (a^2+a+1) (a^6+a^3+1)
20. $(a+1)$ (a^2-a+1) (a^6-a^3+1)

Worksheet 26

1. $3(x+2)(x-2)$
2. $2(x-5)(x+5)$
3. $x(x+1)^2$
4. $y(y-4)^2$
5. $x^2(x+3)(x-2)$
6. $a^3(a-8)(a+5)$
7. $3(b+3)(b+7)$
8. $(5a-3)(a+2)$
9. $4(y-1)(y-7)$
10. $2(a-11)(a+2)$
11. $x(x-10)(x+2)$
12. $b(b-6)(b+1)$
13. $a^2(a-14)(a+2)$
14. $5a(a-3)^2$
15. $2(2x-1)(x-1)$
16. $x^2(2x-1)(x-5)$
17. $x^2(x+4)(x-4)$
18. (a^2+9) $(a+3)(a-3)$
19. $3(5x-1)(x-1)$
20. $3a(3a+1)(a+4)$
21. $x(3y-4)(y+5)$
22. $-3(x-4)(x+2)$
23. $b^2(a+8)(a-1)$
24. $4y(x+2)(x+1)$
25. $2(36+a^2)$
26. $18a(a-2)(a-1)$
27. $y^2(x+1)(x-1)$
28. $5(x+3y)(x-3y)$
29. $x^2(x+3)(x-3)$
30. (a^2+1) $(a+1)(a-1)$
31. $3(x-1)(a+b)$
32. $(a+b)$ $(y+2)(y-2)$

Worksheet 27

1. -1, -2
2. 4, 6
3. 6, 1
4. -7, 5
5. 0, 8
6. 0, -1
7. 0, 4
8. 0, -7
9. $0, -\frac{2}{3}$
10. $0, \frac{5}{2}$
11. $0, \frac{1}{2}$
12. $0, -\frac{3}{4}$
13. -1, 4
14. -4, 7
15. -4, 4
16. -3, 7
17. -8, 2
18. -1, 6
19. 9, -2
20. -1, 9
21. -2, 7
22. $-1, \frac{3}{2}$
23. $\frac{1}{4}, 3$
24. $-3, \frac{2}{5}$
25. $-\frac{1}{2}, -2$
26. -11, 1
27. 3, 6
28. -10, 5
29. 6, 5
30. $-4, \frac{7}{2}$
31. -4, 12
32. -3. 4
33. $-4, \frac{3}{2}$

Worksheet 28

1. 4
2. -6
3. 2, 5
4. 4, 6
5. 3, 9
6. 3
7. 5, 6
8. 10, 12
9. base: 24 ft.; height: 8 ft.
10. length: 24 in.; width: 6 in.
11. length: 15 ft.; width: 8 ft.
12. 8 in.

Worksheet 29

1. $\frac{2}{3x^2}$ 2. $\frac{3x}{4y^2}$
3. $\frac{1}{(x+1)^2}$
4. $(3x - 2)^2$
5. -1 6. $\frac{3}{4y}$
7. $\frac{1}{3y}$ 8. $\frac{4x^2}{3}$
9. $\frac{x^2}{2}$ 10. $-\frac{2x^2}{3}$
11. $\frac{a}{c}$ 12. $\frac{x}{3}$
13. $-\frac{2}{x}$ 14. $\frac{x+5}{x-7}$
15. $\frac{x-3}{x-6}$ 16. $\frac{x-3}{x+3}$
17. $\frac{x+2}{x-1}$ 18. $\frac{x-4}{x-5}$
19. $\frac{x-2}{x+1}$
20. $\frac{x+3}{x-1}$
21. $\frac{x-6}{4-x}$
22. $\frac{-1(y+3)}{y+5}$
23. $\frac{3(x-1)}{x-2}$
24. $\frac{1}{2(x+1)}$
25. $\frac{5x-1}{5x+3}$
26. $\frac{x-3}{x+5}$
27. $-\frac{x-4}{x-2}$
28. $\frac{2(x+2)}{x+6}$
29. $\frac{2x-7}{x+3}$
30. 2(x - 1)

Worksheet 30

1. $\frac{2b}{25ax^2y}$
2. $\frac{ay}{20x}$
3. $\frac{15}{4}$
4. $\frac{3}{4}$
5. $\frac{x+1}{x+4}$
6. $\frac{x}{x+2}$
7. $\frac{x+2}{x(x-2)}$
8. $\frac{x(x+4)}{y(x-5)}$
9. $\frac{y}{2}$
10. $\frac{x-1}{x^2(x+8)}$
11. 1
12. $\frac{x-5}{x+5}$
13. $\frac{x-9}{x+9}$
14. $\frac{x+9}{x+2}$
15. $\frac{x+7}{x-3}$
16. $\frac{x+6}{x+2}$
17. x + 1
18. $\frac{2x-3}{x+6}$
19. $\frac{3x-1}{2x+3}$
20. $\frac{x+3}{2x+3}$

Worksheet 31

1. $\frac{9xy^2}{20ab}$
2. $\frac{8b}{3}$
3. $\frac{1}{2}$
4. $\frac{5(2x-1)}{8(x+3)}$
5. $\frac{x}{x+2}$
6. $\frac{x-3}{x+1}$
7. $\frac{x(x+1)}{(x+2)}$
8. x + 3
9. $\frac{x(x+5)}{y^2(x-5)}$
10. $3ay^2$
11. 1
12. $\frac{x-5}{x-7}$
13. 1
14. $\frac{x-1}{x+1}$
15. $\frac{x+4}{x-4}$
16. $\frac{x+6}{x-6}$
17. $\frac{(x+4)(x+2)}{(x+3)^2}$
18. $\frac{x+5}{x-12}$
19. $\frac{1}{x-2}$
20. $\frac{3y-2}{3y-1}$

Worksheet 32

1. $\frac{7}{x^2}$
2. $\frac{6}{xy}$
3. $\frac{-5}{x+3}$
4. $\frac{x-3}{x-5}$
5. $\frac{10x}{3x+2}$
6. $\frac{-3y}{5y-1}$
7. $\frac{2(2x+3)}{x-2}$
8. $\frac{8x-5}{2x-1}$
9. 2
10. 3
11. $\frac{-2(2x+1)}{3x+5}$
12. $\frac{-3n+5}{2n+9}$
13. $\frac{1}{x-5}$
14. $\frac{2}{x+3}$
15. $\frac{1}{x-2}$
16. $\frac{1}{x-2}$
17. $\frac{1}{y+5}$
18. $\frac{5}{2x+3}$
19. $\frac{4}{x-5}$
20. $\frac{x-2}{x+7}$

Worksheet 33

1. $\frac{20}{3x}$
2. $\frac{5}{6a}$
3. $\frac{17}{18x}$
4. $\frac{75+y}{15y^2}$
5. $\frac{4x-5}{6x}$
6. $\frac{11x+2}{24x}$
7. $\frac{7x-1}{36x}$
8. $\frac{x+1-3xy+3y}{xy^2}$
9. $\frac{3x^2-5x-9}{6x^2}$
10. $\frac{4x^2+13x+12}{36x^2}$
11. $\frac{x+11}{(x-1)(x+2)}$
12. $\frac{3(y-9)}{(y+6)(y-3)}$
13. $\frac{2(x^2+2x-32)}{(x-8)(x-2)}$
14. $\frac{3(x^2-7x+2)}{(2x-1)(x-3)}$
15. $\frac{-x-4}{(x+2)(x-2)}$
16. $\frac{7x+6}{(x-4)(x+2)}$
17. $\frac{4}{x-3}$
18. $\frac{x^2+5x-5}{(x-2)(x-1)}$

Worksheet 34

1. $\frac{x}{x+5}$
2. $\frac{2x(x+3)}{(x+6)(x-6)}$
3. $\frac{1}{2}$
4. 3
5. $\frac{5(2x-1)}{3(2x-5)}$
6. $\frac{2}{3}$
7. $\frac{x-9}{x+3}$
8. $\frac{x+2}{x-7}$
9. $\frac{x-6}{x+3}$
10. $\frac{x+2}{x-3}$
11. $\frac{x}{x+6}$
12. $\frac{x^2-x+1}{x}$
13. $\frac{x^2+5x+18}{x^2+4x+1}$
14. $\frac{x+3}{x+4}$
15. $\frac{x+2}{7x+6}$
16. $\frac{-2}{3(3x+2)}$

Worksheet 35

1. 2
2. 3
3. 3
4. 1
5. $\frac{4}{3}$
6. 4
7. 3
8. 7
9. 3
10. 2
11. -4
12. $\frac{1}{2}$
13. 4
14. 5
15. 6
16. -4 and 2
17. 2 and 3
18. -6 and 4
19. $\frac{3}{2}$ and 2
20. 1 and 12

Worksheet 36

1. 15
2. 30
3. 25
4. 16
5. 12
6. 24
7. 40
8. 6
9. 17
10. 28
11. 11
12. 17
13. 18
14. 64
15. 1
16. 27
17. 34
18. $-\frac{1}{2}$
19. -1
20. $-\frac{8}{3}$
21. 7
22. -1
23. $-\frac{3}{2}$
24. 21
25. $\frac{7}{2}$
26. 2
27. $-\frac{3}{5}$
28. 13
29. $\frac{8}{3}$
30. $-\frac{1}{9}$

Worksheet 37

1. $217.36
2. $1500
3. 216 sq ft
4. 16,000 voters
5. 90 clocks
6. $96
7. 250 deer
8. 50 shares
9. 1 oz
10. 3 gallons

Worksheet 38

1. $y = 8 - 2x$
2. $y = 5x - 2$
3. $y = -\frac{2}{5}x + 4$
4. $y = \frac{5}{2}x - 5$
5. $y = -\frac{1}{2}x + 3$
6. $y = \frac{1}{2}x - 3$
7. $y = -\frac{1}{4}x + 2$
8. $y = \frac{2}{3}x - 6$
9. $y = 3x + 9$
10. $x = -4y + 6$
11. $x = 5y + 7$
12. $x = \frac{1}{2}y + 2$
13. $x = -\frac{1}{3}y + 4$
14. $x = -\frac{3}{2}y + 4$
15. $x = -\frac{5}{2}y + 5$
16. $x = \frac{5}{3}y + 10$
17. $x = \frac{7}{2}y + 7$
18. $x = \frac{4}{5}y - 1$
19. $b = \frac{2a}{h}$
20. $a = P - b - c$
21. $r = \frac{d}{t}$
22. $P = \frac{nRT}{V}$
23. $W = \frac{P - 2l}{2}$
24. $C = Rt + S$
25. $r = \frac{E - IR}{I}$
26. $m = \frac{RK + T}{B}$

Worksheet 38

27. $m = \frac{K}{r - v}$
28. $l = \frac{\pi r^2 - S}{\pi r}$
29. $r = \frac{A - P}{Pt}$
30. $c = \frac{A - P}{A}$

Worksheet 39

1. $1\frac{1}{2}$ hours
2. 6 hours
3. 15 minutes
4. 24 minutes
5. 12 minutes
6. 24 hours
7. $20\frac{4}{5}$ hours
8. 21 hours
9. 8 hours
10. 31 hours

Worksheet 40

1. freight train: 35 mph; express train: 55 mph
2. commuter plane: 180 mph; jet plane: 380 mph
3. car: 50 mph; plane: 400 mph
4. single-engine plane: 97.5 mph; jet plane: 390 mph
5. 7 mph
6. 50 mph
7. 27 mph
8. 30 mph
9. 2 mph
10. 40 mph

Worksheet 41

1. \$96
2. \$380
3. 440 mi
4. 3150 mi
5. $s = kx$
6. $r = \frac{k}{s}$
7. $V = kt$
8. $m = \frac{k}{n}$
9. $y = \frac{k}{x^2}$
10. $V = kr^3$
11. $k = 9$
12. $k = 18$
13. $k = 21$
14. $k = 50$
15. $k = 500$
16. $k = 6$
17. $k = 36$
18. $k = 7$

Worksheet 42

a)
1. 11
2. 5
3. 7
4. 10
5. 6
6. 3
7. 8
8. 9

b)
1. $6\sqrt{2}$
2. $2\sqrt{3}$
3. $5\sqrt{2}$
4. $2\sqrt{5}$
5. $3\sqrt{3}$
6. $2\sqrt{2}$
7. $4\sqrt{3}$
8. $3\sqrt{5}$

Worksheet 42a

1. 5
2. 11
3. 13
4. 14
5. 17
6. 20
7. $3\sqrt{2}$
8. $4\sqrt{3}$
9. $2\sqrt{5}$
10. $3\sqrt{5}$
11. $6\sqrt{2}$
12. $6\sqrt{3}$
13. $-24\sqrt{2}$
14. $18\sqrt{10}$
15. $5\sqrt{5}$
16. $-35\sqrt{2}$
17. $8\sqrt{3}$
18. 36
19. $5\sqrt{6}$
20. 30
21. $10\sqrt{2}$
22. $12\sqrt{10}$
23. $35\sqrt{2}$
24. $2\sqrt{22}$
25. $3\sqrt{15}$
26. $8\sqrt{2}$
27. $15\sqrt{3}$
28. $10\sqrt{7}$
29. $7\sqrt{5}$
30. $8\sqrt{6}$

Worksheet 43

1. $x^2\sqrt{x}$
2. $x^4\sqrt{x}$
3. $x^2\sqrt[3]{x}$
4. $x^6\sqrt{x}$
5. x^5
6. $x^5\sqrt[3]{x^2}$

Worksheet 43a

1. x^4
2. x^5
3. $y^4\sqrt{y}$
4. $x^6\sqrt{x}$
5. x^3y^3
6. x^5y^2
7. $3x^3$
8. $4y^5$
9. $6x$
10. $xy^4\sqrt{xy}$
11. $a^6b\sqrt{ab}$
12. $4y^2\sqrt{2y}$
13. $3x^2\sqrt{3x}$
14. $4y^2\sqrt{5y}$
15. $4x^4y^2\sqrt{3xy}$
16. $4xy^2\sqrt{6}$
17. $6ab^3$
18. $10x^2y^8$
19. $2xy^2\sqrt{6xy}$
20. $3x^4y^2\sqrt{10xy}$
21. $16a^3b\sqrt{b}$
22. $27ab^2\sqrt{b}$
23. x^2y^2
24. $x^2y^5\sqrt{x}$
25. $9ab\sqrt{a}$
26. $24a^3b^4\sqrt{b}$
27. $16xy^3\sqrt{2y}$
28. $10(a+5)^2$
29. $8(x+y)^2$
30. $7(x-3)$
31. $2(x-1)$
32. $x+6$
33. $b-5$

Worksheet 44

1. $3\sqrt{3}$
2. $11\sqrt{2}$
3. $\sqrt{6}$
4. $-6\sqrt{7}$
5. $-9\sqrt{2}$
6. $-10\sqrt{5}$
7. $10\sqrt{y}$
8. $-4\sqrt{x}$
9. $-5\sqrt{x}$
10. $-\sqrt{3a}$
11. $-12\sqrt{7b}$
12. $-10\sqrt{2a}$
13. $-\sqrt{3a}$
14. $-5y\sqrt{2}$
15. $-3a\sqrt{5a}$
16. $-7x\sqrt{2y}$
17. $-8\sqrt{xy}$
18. $2\sqrt{xy}$
19. $16\sqrt{3}$
20. $4\sqrt{2}$
21. $3\sqrt{5}$
22. $\sqrt{x}$
23. $36\sqrt{y}$
24. $-7x\sqrt{2}$
25. $2\sqrt{x}$
26. $24\sqrt{y}$
27. $-10y\sqrt{2}$
28. $4\sqrt{2}$
29. $-15\sqrt{5}$
30. $-\sqrt{5x}$
31. $19\sqrt{3}$
32. $9\sqrt{2}$
33. $6a\sqrt{ab}$

Worksheet 45

1. 9
2. 10
3. z
4. x^2y^3
5. a^3b^2
6. $4a^2b^3$
7. $6x^3y^2$
8. $10a^4b^2$
9. $5a^5b^4\sqrt{3}$
10. $6-3\sqrt{2}$
11. $4\sqrt{2}$
12. -2
13. $x+\sqrt{xy}$
14. $6-\sqrt{3x}$
15. $2\sqrt{2y}-4$
16. $2\sqrt{3}-6$
17. $10-\sqrt{5a}$
18. $x-8\sqrt{x}+16$
19. $9x-6y\sqrt{x}+y^2$
20. $2a-4\sqrt{10a}$
21. $3x\sqrt{2}-3x$
22. $6x-19y\sqrt{x}+15y^2$
23. $20x+\sqrt{xy}-y$
24. $4x-y$
25. $5x-y^2$
26. $15x-4\sqrt{xy}-3y$
27. $8x-26\sqrt{xy}+15y$

Worksheet 46

1. 5
2. 3
3. 4
4. 9
5. 6
6. $5x$
7. $2y$
8. $\frac{\sqrt{5}}{5}$
9. $\frac{\sqrt{2}}{2}$
10. $\frac{5\sqrt{x}}{x}$
11. $\frac{2\sqrt{3x}}{x}$
12. $\frac{4x}{y}$
13. $\frac{y^2\sqrt{y}}{3x}$
14. $\frac{4y^2}{x}$
15. $\frac{\sqrt{3}}{xy}$
16. $\frac{\sqrt{3}+1}{2}$
17. $3(\sqrt{5}+2)$
18. $3(3-\sqrt{6})$
19. $\frac{6(\sqrt{2}+5)}{-23}$
20. $\frac{\sqrt{3}+3}{-3}$
21. $\frac{x-\sqrt{xy}}{x-y}$
22. -3
23. 2
24. 5
25. 2x-6
26. $9y^2$
27. $3a^2$

Worksheet 47

1. $9i$
2. $7i$
3. $2i\sqrt{5}$
4. $2i\sqrt{7}$
5. $5i\sqrt{3}$
6. i
7. $10i\sqrt{3}$
8. $-8i\sqrt{6}$
9. $9i\sqrt{5}$
10. $-14i\sqrt{15}$

Worksheet 48

1. 36
2. 64
3. 256
4. 100
5. 3
6. 8
7. 12
8. 5
9. 24
10. $\frac{9}{2}$
11. 3
12. 14
13. -1
14. 1
15. 5
16. 2
17. 4
18. 8
19. 7
20. 20
21. 8
22. 2
23. 24
24. $-\frac{5}{2}$
25. $\frac{2}{3}$
26. 29
27. No solution
28. 2
29. 1

Worksheet 49

1. 8
2. 2
3. 8.16 feet
4. 9 feet
5. 144 feet
6. 196 feet
7. 64 feet
8. 256 feet
9. 12.98 feet
10. 4.67 feet

Worksheet 50

1. 6, -4
2. -6 +3
3. 5, 1
4. 6.1
5. -8, -1
6. 9, 1
7. 5
8. 6
9. $-\frac{1}{2}$, 0
10. 0, $\frac{3}{2}$
11. -3, 7
12. -4, 5
13. $-\frac{5}{2}$, 2
14. $-\frac{3}{2}$
15. $-\frac{4}{5}$, 3
16. $-\frac{2}{3}$, 1
17. $\frac{1}{5}$, 2
18. $\frac{1}{4}$
19. $\frac{1}{5}$
20. -1, 1
21. - 5, 5
22. $-\frac{1}{4}$, $\frac{1}{4}$
23. $-\frac{5}{2}$, $\frac{5}{2}$
24. -4, 7
25. -2, 6
26. $-\frac{1}{2}$, 1
27. -8, 1
28. -3, -1

Worksheet 51

1. -2, 2 **2.** -4, 4
3. -9, 9 **4.** -12.12
5. $-\frac{5}{3}, \frac{5}{3}$
6. $-\frac{9}{4}, \frac{9}{4}$
7. $-\frac{5}{4}, \frac{5}{4}$
8. $-\frac{9}{2}, \frac{9}{2}$
9. $-\frac{4}{5}, \frac{4}{5}$
10. $-\frac{7}{6}, \frac{7}{6}$
11. $-\frac{3}{2}, \frac{3}{2}$
12. $-\frac{10}{3}, \frac{10}{3}$
13. No real number solution
14. No real number solution
15. $-2\sqrt{2}, 2\sqrt{2}$
16. $-3\sqrt{2}, 3\sqrt{2}$
17. $-5\sqrt{2}, 5\sqrt{2}$
18. -4, 2
19. -4, 8
20. -2, -8
21. 6, -2
22. $-\frac{3}{2}, \frac{7}{2}$
23. $-\frac{16}{3}, \frac{4}{3}$
24. $-\frac{5}{4}, -\frac{19}{4}$
25. $-\frac{1}{5}, \frac{11}{5}$
26. $3+4\sqrt{2}, 3-4\sqrt{2}$
27. $-4+5\sqrt{3}, -4-5\sqrt{3}$
28. $1-5\sqrt{2}, 1+5\sqrt{2}$
29. $-1-4\sqrt{5}, -1+4\sqrt{5}$

Worksheet 52

1. -7, 1 **2.** -7, 3
3. -3, 9 **4.** -7, 5
5. 3 **6.** -2, 8
7. -3, 1
8. $2+\sqrt{11}, 2-\sqrt{11}$
9. $4+\sqrt{23}, 3-\sqrt{23}$
10. $3+\sqrt{10}, 3-\sqrt{10}$
11. $2+\sqrt{7}, 2-\sqrt{7}$
12. 4
13. -3, 1
14. $\frac{-3+\sqrt{21}}{2}, \frac{-3-\sqrt{21}}{2}$
15. 3, 1
16. 4, $-\frac{3}{2}$
17. $-\frac{5}{2}$, 1
18. $\frac{3}{2}$, 1
19. -5, $\frac{1}{2}$
20. $\frac{1}{3}$, 2
21. $-\frac{2}{3}$, 1
22. -2, $\frac{1}{4}$
23. 2, $-\frac{2}{3}$
24. -1, $\frac{5}{3}$
25. $-\frac{1}{4}$, 1
26. $\frac{1}{3}$, 1
27. $-\frac{1}{3}$, 1
28. $\frac{1}{3}, \frac{1}{2}$
29. $-\frac{1}{2}, \frac{5}{2}$
30. $-\frac{1}{2}$ $\frac{2}{3}$

Worksheet 53

1. -1, 4 **2.** -5, -1
3. -2, 5 **4.** 1, 5
5. -6, 1 **6.** -6, 3
7. -4, 5 **8.** -2, 3
9. -1, 9 **10.** -2, 8
11. -9, 1
12. -10, 1
13. $-\frac{1}{3}$, 1
14. -2, $\frac{5}{2}$
15. $3+\sqrt{3}, 3-\sqrt{3}$
16. $-3+\sqrt{7}, 3-\sqrt{7}$
17. 0, 4
18. 0, $-\frac{1}{3}$
19. $-\frac{5}{3}, \frac{5}{3}$
20. $-\frac{3}{4}, \frac{3}{4}$
21. $-\frac{7}{3}$, 2
22. $-\frac{1}{2}$, 4
23. -2, $\frac{4}{3}$
24. $-\frac{5}{2}$,2
25. $-\frac{2}{5}$, 3
26. -1, $\frac{2}{3}$
27. $-\frac{3}{4}, \frac{1}{2}$
28. $\frac{1+\sqrt{13}}{2}, \frac{1-\sqrt{13}}{2}$
29. $-3+\sqrt{13}, -3-\sqrt{13}$
30. $\frac{-5+\sqrt{29}}{2}, \frac{-5-\sqrt{29}}{2}$

Table of Measurement Abbreviations

U.S. Customary System

Length		Capacity		Weight		Area		Rate	
in.	inches	oz	ounces	oz	ounces	in^2	square inches	ft/s	feet per second
ft	feet	c	cups	lb	pounds	ft^2	square feet	mph	miles per hour
yd	yards	qt	quarts			yd^2	square yards		
mi	miles	gal	gallons			mi^2	square miles		

Metric System

Length		Capacity		Weight		Area		Rate	
mm	millimeters	ml	milliliters	mg	milligrams	cm^2	square centimeters	m/s	meters per second
cm	centimeters	cl	centiliters	cg	centigrams	m^2	square meters	km/s	kilometers per second
m	meters	L	liters	g	grams	km^2	square kilometers	km/h	kilometers per hour
km	kilometers	kl	kiloliters	kg	kilograms				

Time

h hours min minutes s seconds

Table of Equations and Geometric Formulas

Perimeter

Triangle: $P = a + b + c$

Rectangle: $P = 2L + 2W$

Square: $P = 4s$

Circle: $C = \pi d$ or $C = 2\pi r$

Volume

Rectangular solid: $V = LWH$

Cube: $V = s^3$

Sphere: $V = \frac{4}{3}\pi r^3$

Right circular cylinder: $V = \pi r^2 h$

Right circular cone: $V = \frac{1}{3}\pi r^2 h$

Regular pyramid: $V = \frac{1}{3}s^2 h$

Triangles

$\angle A + \angle B + \angle C = 180°$

Slope-Intercept Form of a Straight Line

$y = mx + b$, where m = slope and b = y-intercept

Slope of a Line

$m = \frac{y_2 - y_1}{x_2 - x_1}, x_1 \neq x_2$

Area

Triangle: $A = \frac{1}{2}bh$

Rectangle: $A = LW$

Square: $A = s^2$

Circle: $A = \pi r^2$

Parallelogram: $A = bh$

Trapezoid: $A = \frac{1}{2}h(b_1 + b_2)$

Surface Area

Rectangular solid: $S = 2LW + 2LH + 2WH$

Cube: $S = 6s^2$

Sphere: $S = 4\pi r^2$

Right circular cylinder: $S = 2\pi r^2 + 2\pi rh$

Right circular cone: $S = \pi r^2 + \pi rl$

Regular pyramid: $S = s^2 + 2sl$

Pythagorean Theorem

$c^2 = a^2 + b^2$

Point-Slope Formula

$y - y_1 = m(x - x_1)$

Quadratic Formula

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

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Table of Properties

Properties of Real Numbers

Commutative Property of Addition
If a and b are real numbers, then $a + b = b + a$.

Associative Property of Addition
If a, b, and c are real numbers, then $(a + b) + c = a + (b + c)$.

Addition Property of Zero
If a is a real number, then $a + 0 = 0 + a = a$.

Inverse Property of Addition
If a is a real number, then $a + (-a) = (-a) + a = 0$.

Multiplication Property of One
If a is a real number, then $a \cdot 1 = 1 \cdot a = a$.

Commutative Property of Multiplication
If a and b are real numbers, then $a \cdot b = b \cdot a$.

Associative Property of Multiplication
If a, b, and c are real numbers, then $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

Multiplication Property of Zero
If a is a real number, then $a \cdot 0 = 0 \cdot a = 0$.

Inverse Property of Multiplication
If a is a real number and $a \neq 0$, then $a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$.

Distributive Property
If a, b, and c are real numbers, then $a(b + c) = ab + ac$.

Properties of Exponents

If m and n are integers, then $x^m \cdot x^n = x^{m+n}$.

If m and n are integers, then $(x^m)^n = x^{mn}$.

If x is a real number and $x \neq 0$, then $x^0 = 1$.

If m and n are integers and $x \neq 0$, then $\frac{x^m}{x^n} = x^{m-n}$.

If m, n, and p are integers, then $(x^m \cdot y^n)^p = x^{mp}y^{np}$.

If n is a positive integer and $x \neq 0$, then $x^{-n} = \frac{1}{x^n}$ and $\frac{1}{x^{-n}} = x^n$.

Properties of Equations

Addition Property of Equations
If $a = b$, then $a + c = b + c$.

Multiplication Property of Equations
If $a = b$ and $c \neq 0$, then $a \cdot c = b \cdot c$.

Properties of Inequalities

Addition Property of Inequalities
If $a > b$, then $a + c > b + c$.
If $a < b$, then $a + c < b + c$.

Multiplication Property of Inequalities
If $a > b$ and $c > 0$, then $ac > bc$.
If $a < b$ and $c > 0$, then $ac < bc$.
If $a > b$ and $c < 0$, then $ac < bc$.
If $a < b$ and $c < 0$, then $ac > bc$.

Properties of Radical Expressions

If a and b are positive real numbers, then $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$ and $\sqrt{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$.

A radical expression is in simplest form when:
- the radicand has no perfect-square factors
- no radicals appear in the denominator
- no fractions appear in the radicand



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